CS140: Algorithm
Assignment 5
Assigned 10/20/99, Due 10/27/99

1. Let $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ and $R(x) + b_n x^n + b_{n-1} x^{n-1} + \ldots + b_1 x + b_0$, where $a_n \neq 0$ and $b_n \neq 0$; i.e. P and R are polynomials of degree n. Then $P(x) \cdot R(x)$ is a polynomial of degree $2n$ and the coefficient of its x^i term is
$$c_i = \sum_{k,\ell : k+\ell=i, 0 \leq k,\ell \leq n} a_k b_\ell$$
We can obviously compute this product in time $\theta(n^2)$ but we would like to do better.
   (a) Show how to multiply $ax + b$ and $cx + d$ using three scalar multiplication.
   (b) Show that two degree-n polynomials can be multiplied in time $\theta(n^{\lg 3})$. You may assume that n is a power of 2. (Hint: Divide and Conquer.)
2. Problem 16-2 on page 325 of the text.
3. I shop for groceries at a store a couple of block from my house. I don't have a car so I usually walk and then have to carry the groceries home. I never buy an item that weighs more than 25 lbs. And when the grocery clerk packs my groceries I ask her/him to use two bags and to distribute the weight between the bags as evenly as possible. The last time I shopped I bought four items weighing 5 lbs., 4 lbs., 3 lbs. and 2 lb. The clerk packed the bags so that one weighed 8 lbs. and the other 6 lbs.
   (a) How could the groceries be packed to distribute the weight more evenly?
   (b) Design an algorithm the clerk can use to find an optimal way to pack my groceries. The algorithm should take as input the number of items n and their weights $w_1, w_2, \ldots, w_n$. We don't want to hold up the line so the algorithm should run in time $O(n \lg(n))$. (Hint: Dyanmic Programming.)