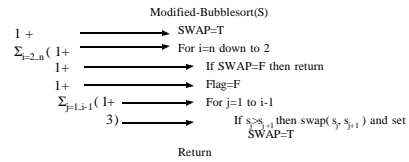


Run Time Analysis

- Iterative algorithm → Loop counting
- Recursive algorithm → Recurrence relations

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Loop counting



$$1 + \sum_{i=2..n} (1 + \sum_{j=1..i-1} 1))$$

actually $c_1 + \sum_{i=2..n} (c_2 + \sum_{j=1..i-1} c_3)$

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Summation

$$\begin{aligned} \sum_{i=2..n} \sum_{j=1..i-1} c &= c \mathbf{S}_{i=2..n}(i-1) \\ &= c (\mathbf{S}_{i=1..n}(i-1)) - c \\ &= c (\mathbf{S}_{i=1..n} i - \mathbf{S}_{i=1..n} 1) - c \\ &= c(n(n+1)/2 - n) - c \\ &= O(n^2) \end{aligned}$$

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Series

- A series is a summation of terms
- Common series
 - Arithmetic series: $1+2+\dots+n$
 - Geometric series: $1+a+a^2+\dots+a^n$

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Series Things we want to do:

- Solve exactly
- Bound above or below
- Prove a solution (or bound) is correct

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Closed form solutions to some common series

- $f(n) = 1+2+\dots+n = n(n+1)/2$
- $f(n) = 1+a+a^2+\dots+a^n = n$ if $a=1$
 $= (a^{n+1}-1)/(a-1)$ else

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Bounds on series

For any constant k:

$$\begin{aligned}\sum_{i=1..n} i^k &\leq \sum_{i=1..n} n^k \\ &= (n^{k+1}) \\ &= O(n^{k+1})\end{aligned}$$

CS140 47

Recursive Algorithms

What about Sort2?

```
Sort2(S)
  If ||S|| ≤ 1
    Return: S
  Else
    Return: Sort2(S|max-element(S)),max-element(S)
```

The number of steps of Sort2 satisfies:

$$\begin{aligned}T(1) &= c_2 \\ T(n) &= c_1 + T(n-1), n > 1\end{aligned}$$

CS140 48

Recurrence Relations

Methods to solve or bound:

- Guess and prove
- Unwinding
- Master method
- **WORK TREES**

CS140 49

Guess and Prove

- $T(0) = 1$
- $T(1) = 1$
- $T(n) = T(n-1) + T(n-2)$

- Closed form: TO DO

CS140 410

Unwinding

- $T(1)=1$
- $T(n) = 2T(n/2) + n$
 $= 2(2T(n/4) + n/2) + n$
 $= 2(2(T(n/8) + n/4) + n/2) + n$

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Master Theorem

- Read the book

CS140 412

Recursive Algorithm A

- On input size $n \leq d$, algorithm A performs c steps.
- On input size $n > d$, algorithm A makes k recursive calls on inputs of size n_1, n_2, \dots, n_k and then takes $f(n)$ steps to produce its solution.

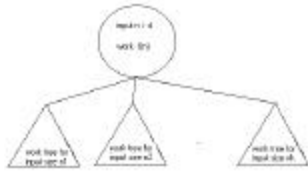
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Work Trees for A $n \leq d$



CS140 4-14

Work Trees for A $n > d$



CS140 4-15