Finding the min and max

- Lower Bound Theorem: Finding the minimum and maximum in a set of n integers requires at least ceil(3n/2) - 2 comparisons.
- Proof: Adversary argument

MIN/MAX Adversary - Accounting

- Adversary = interactive comparison oracle
- Accounting scheme: For x in S

$b_{MAX}(x)=$ 1 if the algorithm can rule out x as the largest integer; 0 otherwise

$b_{MIN}(x)=$ 1 if the algorithm can rule out x as the smallest integer; 0 otherwise

MIN/MAX Adversary- Strategy

- On query x<y ?
- Answer consistently with previous answers
- If yes and no both consistent then answer so as to minimize the changes in the MAX/MIN variables

MIN/MAX Adversary - Analysis

Consider query x<y?

- If YES: bMIN(x)→1 and bMAX(y)→1
- If NO: bMAX(x)→1 and bMIN(y)→1

MIN/MAX Adversary – proof of bound

- Claim: At most floor(n/2) queries can result in the change of two b variable
- Claim: 2n-2 changes must occur before the algorithm concludes

⇒ floor(n/2) + 2n-2 - 2floor(n/2) queries are needed before the algorithm can halt.