CS140: Algorithms

Fall 2000

Assignment 4

This assignment is due by the start of class on Wed., Sept. 20. You may collaborate with your classmates to complete this assignment but you must reference your collaborators. In addition, you must do your own write-up. This assignment is worth 100 points (65 points if handwritten).

1. $S = S[0], S[1], \ldots, S[n-1]$ is an array of distinct integers. S is said to be *unimodal* if there is some index i (where $0 \leq i \leq n-1$) such that $S[0], ..., S[i]$ is a strictly increasing sequence and $S[i], ..., S[n-1]$ is a strictly decreasing sequence. For example, $S = 2, 4, 8, 7, 6, 5, 3, 1, 0, -1$ is unimodal but $S = 2, 3, 1, 4$ is not.

   Consider the problem of finding the maximum element in a unimodal array S when $n = 2^m$. Assume an algorithm is only allowed access to S through queries of the form "Is $S[j] < S[k]$?" Give the best upper and lower bounds you can on the number of queries needed to solve this problem. Your answer should not use big-O notation; i.e. you can get exact bounds for this problem. Prove your answers; i.e. give an algorithm that achieves the upper bound and an adversary that achieves the lower bound.

2. The input in this problem is a binary vector $b_0, \ldots, b_{n-1}$. The problem is to determine whether the vector contains two consecutive 0's. You are allowed to make queries of the form "What is bit i?" Let $T(n)$ denote the number of queries needed to solve the problem on a vector of n bits. Give the best upper and lower bounds you can on $T(n)$. Prove your answers; i.e. give an algorithm that achieves the upper bound and an adversary that achieves the lower bound.

   Hint: Consider finding a bound for the following as well. Let $S(n)$ denote the number of queries needed to determine whether a bit vector of length n satisfies (at least) one of the following conditions:

   (a) It contains two consecutive 0's

   (b) The first bit is a 0.