CS140: Algorithms

Fall 2000

Solutions 1

1. State whether each of the following is true or false.

   (a) If $f(n) = O(g(n))$ then $lim_{n\to\infty} f(n)/g(n) = c$ for some constant c.
   False: The ratio $f(n)/g(n)$ may be bounded by a constant but oscillate so there is no limit.

   (b) $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$.
   True

   (c) If $f(n) = o(g(n))$ then $f(n) = O(g(n))$.
   True

   (d) If $f(n) = O(g(n))$ then $f(n) = o(g(n))$.
   False: For example $f(n) = O(f(n))$ but $f(n) \neq o(f(n))$.

2. Problem 2-2 on page 38 of the text.

$f(n)$	$g(n)$	O	o	Ω	ω	θ
$\lg^k n$	n^ϵ	yes	yes	no	no	no
n^k	c^n	yes	yes	no	no	no
$\sqrt{n}$	$n^{\sin(n)}$	no	no	no	no	no
2^n	$2^{n/2}$	no	no	yes	yes	no
$n^{\lg(m)}$	$m^{\lg(n)}$	yes	no	yes	no	yes
$\lg(n!)$	$\lg(n^n)$	yes	no	yes	no	yes

3. Problem 2-3 on page 39 of the text.

   1, $n^{1/\lg n}$

   $\lg(\lg^* n)$

   $\lg^*(n)$, $\lg^*(\lg n)$

   $2^{\lg^* n}$

   $\ln \ln n$

   $\sqrt{\lg n}$

   $\ln n$

   $\lg^2(n)$

   $2^{\sqrt{2 \lg n}}$

   $(\sqrt{2})^{\lg n}$

$n,\ 2^{\lg n}$

$n \lg n,\ \lg(n!)$

$n^2,\ 4^{\lg n}$

n^3

$(\lg n)!$

$n^{\lg \lg n},\ (\lg n)^{\lg n}$

$(\frac{3}{2})^n$

2^n

$n2^n$

e^n

$n!$

$(n+1)!$

2^{2^n}

$2^{2^{n+1}}$