CS140: Algorithms

Fall 2000

Solutions to Assignment 2

1. Prove $\sum_{i=0}^{n} i2^i = (n-1)2^{n+1} + 2$ by induction on n.

   If $n = 0$ then the left-hand side of the equation is $\sum_{i=0}^{0} i = 0$ and the right-hand side is $(-2 + 2 = 0$. So the claim holds when $n = 0$. Now suppose the claim holds for some $n \geq 0$. Then

   $$\sum_{i=0}^{n+1} i2^i = (n+1)2^{n+1} + \sum_{i=0}^{n} i2^i = (n+1)2^{n+1} + (n-1)2^{n+1} + 2 = n2^{n+1} + 2$$

2. Compute the running times of the following program.

   `algorithm` what(n)

   ```
   for i = 1 to ⌊√n⌋ do
      for j = ⌊√n⌋ downto 1 do
         for k = 1 to j do
            { O(1) time sequence of statements }
         end
      end
   end
   ```

   `end algorithm`

   $$\begin{aligned} \sum_{i=1}^{\lfloor\sqrt{n}\rfloor} \sum_{j=1}^{\lfloor\sqrt{n}\rfloor} \sum_{k=1}^{j} c &= \sum_{i=1}^{\lfloor\sqrt{n}\rfloor} \sum_{j=1}^{\lfloor\sqrt{n}\rfloor} j \\ &= \sum_{i=1}^{\lfloor\sqrt{n}\rfloor} \lfloor\sqrt{n}\rfloor(\lfloor\sqrt{n}\rfloor + 1)/2 \\ &\leq \lfloor\sqrt{n}\rfloor\lfloor\sqrt{n}\rfloor(\lfloor\sqrt{n}\rfloor + 1)/2 \\ &= O(n^{3/2}) \end{aligned}$$

3. CrazySort is a sorting algorithm that sorts a list of n integers as follows. If the list has 1 element, the algorithm returns the list as sorted. If the list has 2 elements, the algorithm compares the elements and swaps them if necessary. If the list has 3 or more elements, the algorithm recursively sorts the first 2/3rds of the input list, then recursively sorts the second 2/3rds, and then recursively sorts the first 2/3rds again. (You might want to convince yourself that the algorithm is correct.) Use the work tree approach to analyze the running time of CrazySort on inputs of size 3^m, $m = 0, 1, \ldots$. In particular, please answer the following questions.

(a) How many nodes are at level i, where the root is at level 0? 3^i

(b) What is the instance size associated with a node at level i? $(\frac{2}{3})^i n$

(c) How much work is associated with a node at level i? c

(d) How much work is done at the nodes in level i? $c3^i$

(e) How many levels are in the work tree?
$1 + log_{\frac{3}{2}} n$

(f) What is the running time of CrazySort? $O(n^{\log_{\frac{3}{2}} 3})$