CS140: Algorithms
Fall 2000

Solutions to Assignment 5

1. Problem 10.3-4 on page 192 of the text.

   Let s be the ith smallest element of the input set S. Suppose the claim (in the problem statement) is false. Then there must be some element t such that $t < s$ and $t > s$ are both consistent with the order information the algorithm has collected (via it's queries) about S. But then there is a total order consistent with the algorithm's information in which t is the ith smallest element of S. This contradicts the correctness of the algorithm.

2. Problem 10.3-6 on page 192 of the text. (Recall that the k-th order statistic of a set is just the k-th smallest element of the set.)

   Assume that the input set S, when in sorted order, is $s_1, \ldots, s_n$. (To simplify the analysis a bit we'll also assume that the elements of S are distinct.) The k-th quartiles of this set are the elements $s_{\lceil \frac{i(n+1)}{k} \rceil}$, where $i = 1, \ldots, k-1$. Here is an algorithm for the problem.

   Algorithm for k-th quartiles

   If k is even, call Select$(S, \lceil (n-k+1)/2 \rceil)$ to find $s_{\lceil (n-k+1)/2 \rceil}$. Partition S around $s_{\lceil (n-k+1)/2 \rceil}$ then recursively find the $k/2$ quartiles in each set.

   If k is odd, call Select$(S, \lceil d_i \rceil)$ for $d_0 = (n-k+1)(k-1)/2k$ and $d_1 = (n-k+1)(k+1)/2k$. Recursively find the $(k-1)/2$ quartiles of the sets $\{s \in S | s < s_{d_0}\}$ and $\{s \in S | s > s_{d_1}\}$.

   The running time of the algorithm is then $T(n, k) \leq 2T(n/2, k/2) + cn$. The recursion halts at depth $\lg k$ so the running time is $O(n \lg k$.

3. Problem 10.3-8 on page 192 of the text.

   To simplify the analysis assume n is a power of 2 and that the elements of the lists are distinct. If $n = 1$ then we can choose either element as the median. Now suppose that $n > 1$. If $s_{n/2} < t_{n/2}$ then the median of the original set is the median of the set $\{s_{n/2+1} \ldots s_n\} \cup \{t_1 \ldots t_{n/2}\}$. If $s_{n/2} > t_{n/2}$ then the median of the original set is the median of the set $\{t_{n/2+1} \cdots t_n\} \cup \{s_1, \ldots s_{n/2}\}$. Using the obvious recursive algorithm we get the recurrence relation $T(2n) = T(n) + 1 = O(\lg n)$.