JAPE Quantifier Proofs

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$\forall x.P(x) \vdash \forall y.P(y)$

1:	$\forall x.P(x)$	premise
2:	actual i	assumption
3:	$P(i)$	$\forall$ elim 1,2
4:	$\forall y.P(y)$	$\forall$ intro 2-3

$\exists x.P(x) \vdash \exists y.P(y)$

1:	$\exists x.P(x)$	premise
2:	actual i, $P(i)$	assumptions
3:	$\exists y.P(y)$	hyp 1
4:	$\exists y.P(y)$	$\exists$ elim 1,2-3

$\forall x.\forall y.P(x,y) \vdash \forall y.\forall x.P(x,y)$

1:	$\forall x.\forall y.P(x,y)$	premise
2:	actual i	assumption
3:	actual i1	assumption
4:	$\forall y.P(i1,y)$	$\forall$ elim 1,3
5:	$P(i1,i)$	$\forall$ elim 4,2
6:	$\forall x.P(x,i)$	$\forall$ intro 3-5
7:	$\forall y.\forall x.P(x,y)$	$\forall$ intro 2-6

$\exists x.\exists y.P(x,y) \vdash \exists y.\exists x.P(x,y)$

1:	$\exists x.\exists y.P(x,y)$	premise
2:	actual i, $\exists y.P(i,y)$	assumptions
3:	actual i1, $P(i,i1)$	assumptions
4:	$\exists x.P(x,i1)$	$\exists$ intro 3.2,2.1
5:	$\exists y.\exists x.P(x,y)$	$\exists$ intro 4,3.1
6:	$\exists y.\exists x.P(x,y)$	$\exists$ elim 2.2,3-5
7:	$\exists y.\exists x.P(x,y)$	$\exists$ elim 1,2-6

$\exists x.\forall y.P(x,y) \vdash \forall y.\exists x.P(x,y)$

1:	$\exists x.\forall y.P(x,y)$	premise
2:	actual i, $\forall y.P(i,y)$	assumptions
3:	actual i1	assumption
4:	$P(i,i1)$	$\forall$ elim 2.2,3
5:	$\exists x.P(x,i1)$	$\exists$ intro 4,2.1
6:	$\forall y.\exists x.P(x,y)$	$\forall$ intro 3-5
7:	$\forall y.\exists x.P(x,y)$	$\exists$ elim 1,2-6

$\exists x.\forall y.P(x,y) \vdash \exists x.P(x,x)$

1:	$\exists x.\forall y.P(x,y)$	premise
2:	actual i, $\forall y.P(i,y)$	assumptions
3:	$P(i,i)$	$\forall$ elim 2.2,2.1
4:	$\exists x.P(x,x)$	$\exists$ intro 3,2.1
5:	$\exists x.P(x,x)$	$\exists$ elim 1,2-4

$P(x) \vdash \forall y.P(x)$

1:	$P(x)$	premise
2:	actual i	assumption
3:	$P(x)$	hyp 1
4:	$\forall y.P(x)$	$\forall$ intro 2-3

$\forall x.P(x,x) \vdash \forall x.\exists y.P(x,y)$

1:	$\forall x.P(x,x)$	premise
2:	actual i	assumption
3:	$P(i,i)$	$\forall$ elim 1,2
4:	$\exists y.P(i,y)$	$\exists$ intro 3,2
5:	$\forall x.\exists y.P(x,y)$	$\forall$ intro 2-4