CS 81 Logic and Computability

Spring 2008

HW 6 due 3/13/08

1. Construct a DFA that recognizes each the following languages. Justify your answer by describing, in English, the purpose of each state.
    a. The strings over $\{p,q,r,\neg,\wedge,\vee,(,)\}$ that represent well formed formula in conjunctive normal form (CNF) with no unnecessary parenthesis. In other words the strings have the form $C_1 \wedge C_2 \wedge \ldots \wedge C_n$ where each C_i has the form $(x_1 \vee x_2 \vee \ldots x_k)$, with x_i in $\{p,\neg p,q,\neg q,r,\neg r\}$. The empty clause is acceptable as is a formula with an empty clause ().
    b. The strings over {a,b} in which the parities of a and b are the same; i.e. a string in the language has an even number of both a and b or an odd number of both a and b.
    c. The strings over {a,b} in which each consecutive block of 5 symbols has at least two b's; i.e. if $s_0 s_1 \ldots s_n$ is in the language then for any i, $0 \le i \le n-4$, the substring $s_i s_{i+1} s_{i+2} s_{i+3} s_{i+4}$ contains at least 2 b's.

2. Construct an NFA that recognizes the following languages. Justify your answer by describing, in English, the purpose of each state.
    a. The strings over {a,b} containing at least 2 a's and such that some pair of a's is separated by a string whose length is a multiple of 3.
    b. The strings over{a,b,c} that have the same value when multiplied from the left as from the right according to the following (non-associative) multiplication table:

	a	**b**	**c**
a	a	a	c
b	c	a	b
c	b	c	a

    For example, aaa is in the language because multiplication from the left yields (aa)a=aa=a and from the right yields a(aa)=aa=a. But bac is not because (ba)c=cc=a but b(ac)=bc=b.

3. Construct a regular expression for each of the following languages. Provide justification of correctness.
    a. The set of strings over {0,1} that represent, in binary, a number that is equivalent to zero modulo 3.
    b. The strings over {a,b} with an equal number of a's and b's such that in every prefix the number of a's and the numbers of b's differs by at most 2; i.e. if $s_0 s_1 \ldots s_n$ is in the language then for any i, $0 \le i \le n$, the prefix $s_0 s_1 \ldots s_i$ has the property that the number of a's and b's differ by at most 2.