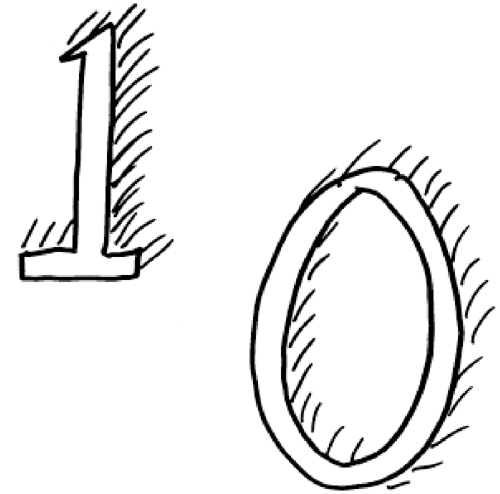
Fuzzy Logic



“Soft Computing”

- Neural networks
- Fuzzy logic
- Neuro-Fuzzy control
- Genetic algorithms

Reference

Neuro-Fuzzy and Soft Computing: A Computational Approach to Learning and Machine Intelligence

Jyh-Shing Roger Jang, et al, Prentice-Hall, 1996

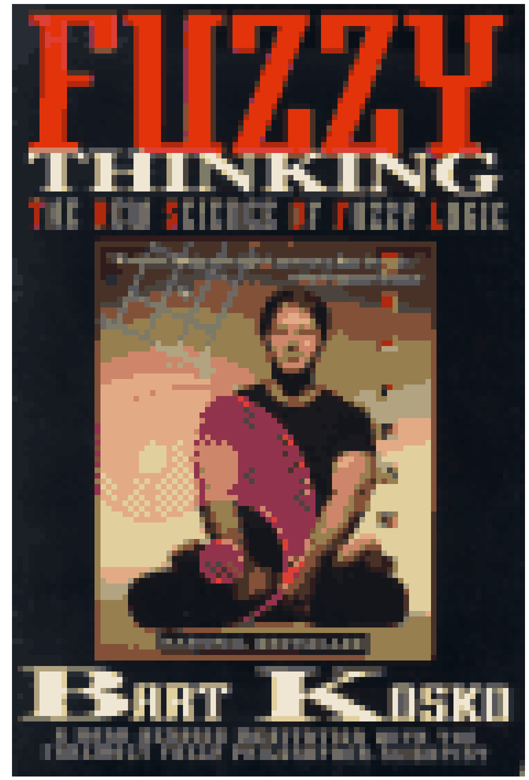
Fuzzy Logic History

- 1920, Jan Łukasiewicz, Multi-valued logic
- 1937, Max Black: Vagueness, an exercise in logical analysis, Phil. of Science, **4**, 427-455
- 1967, Lotfi Zadeh, UCB: Fuzzy Sets, in Information and Control J.
- 1974, E.H. Mamdani, Control Systems
- 1980's-90's: Bart Kosko, USC

Fuzzy Founders/Followers

Jan Łukasiewicz
(1878-1956)

Lotfi Zadeh
(1921-)



Bart Kosko

Prof. Zadeh:

As the complexity of a system increases,

our ability to make precise and yet significant statements about its behavior diminishes

until a threshold is reached beyond which precision and significance (or relevance)

become almost mutually exclusive characteristics.

Fuzzy Jounrals

- IEEE Trans. on Fuzzy Systems
- International J. of Approximate Reasoning
- Intelligent and Fuzzy Systems
- Journal of Cybernetics

Fuzzy Logic Applications

- Subway ride smoothness control
- Camcorder auto-focus and jiggle control
- Braking systems
- Saturn automobile transmission
- Copier quality control
- Rice-cooker temperature control

Neuro Fuzzy Rice Cooker

(Zojirushi Corporation)

The *Neuro Fuzzy*® Rice Cooker & Warmer features advanced *Neuro Fuzzy*® logic technology, which allows the rice cooker to 'think' for itself and make fine adjustments to temperature and heating time to cook perfect rice every time. The spherical inner cooking pan and heating system allows the heat to distribute evenly and cook rice perfectly. It also features different settings for cooking white rice, sushi rice, brown rice and porridge. Other features

Anti-Lock Brakes

- A typical ABS is composed of a central electronic unit, four speed sensors (one for each wheel), and two or more hydraulic valves on the brake circuit.
- The electronic unit constantly monitors the rotation speed of each wheel. When it senses that any number of wheels are rotating considerably slower than the others (a condition that will bring it to lock[1]) it moves the valves to decrease the pressure on the braking circuit, effectively reducing the braking force on that wheel.
- The wheel(s) then turn faster and when they turn too fast, the force is reapplied. This process is repeated continuously, and this causes the characteristic pulsing feel through the brake pedal.
- A typical anti-lock system can apply and release braking pressure up to 20 times a second.

More Applications of Fuzzy Logic

- Automatic control of dam gates for hydroelectric-powerplants (Tokyo Electric Power)
- Camera-aiming for the telecast of sporting events (Omron)
- Cruise-control for automobiles (Nissan, Subaru)
- Positioning of wafer-steppers in the production of semiconductors (Canon)
- Prediction system for early recognition of earthquakes (Inst. of Seismology Bureau of Metrology, Japan)
- Controlling of subway systems in order to improve driving comfort, precision of halting and power economy (Hitachi)

Fuzzy Silver Bullet?

- Fuzzy logic may not provide any new mechanisms that weren’t there before.
- It provides a **viewpoint**, that helps expedite problem solving.
- Analogy: Object-Oriented Programming didn’t create any new computable functions.

Fuzzy Set Basics

- Classical ("crisp") sets:
  - Membership in a set is all or nothing
  - Characteristic function c_S: Universe $\rightarrow \{0, 1\}$
  - $c_S(x) = 1$ iff $x \in S$

range of real numbers

- Fuzzy sets:
  - Membership in a set is a degree
    - membership function c_S: Universe $\rightarrow [0, 1]$

Linguistic Characterizations of Degree of Membership

- Consider the set of “hot” days in Claremont in 2004.
- Was Oct. 29 “hot”? It might have been called one of:
  - “very hot”
  - “sort of hot”
  - “not hot”
- The answer depends on the observer, time, etc.

Air-Conditioning System (Mitsubishi)

Problem description:

Industrial air-conditioning system shall be able to react flexibly to changing ambient conditions

Realization:

- 50 rules
- 6 linguistic variables
- Resolution: 8 bit
- Input variables: room temperature, wall temperature and temporal evaluation of these signals.

Development:

- 4 days to create the prototype.
- 20 days for testing and integration.
- 80 days for optimization with real test objects.
- Implementation as pure software solution on standard microcontroller.

Results:

- Reduction of starting processes down to 40 percent of the standard solution.
- Sustaining of the temperature even with interference factors (like open window, etc.) substantially improved.
- Fewer sensors required, Established energy saving by testing: 24 percent.

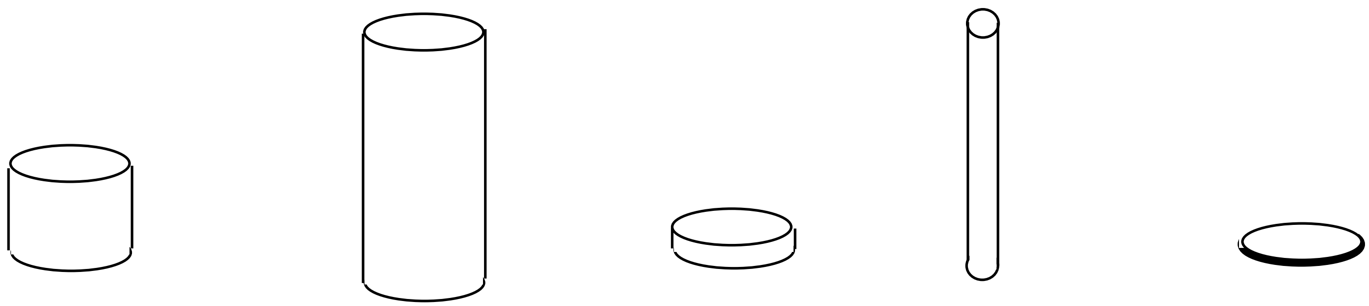
Sounds similar to probability, but isn’t

- Probability deals with *uncertainty* or *likelihood* of occurrence.

- Fuzzy logic deals with *ambiguity*, *vagueness* of description.

Fuzzy Membership

Are these disks, cylinders, or rods?



In other words, which object is a member of class disk, etc.?

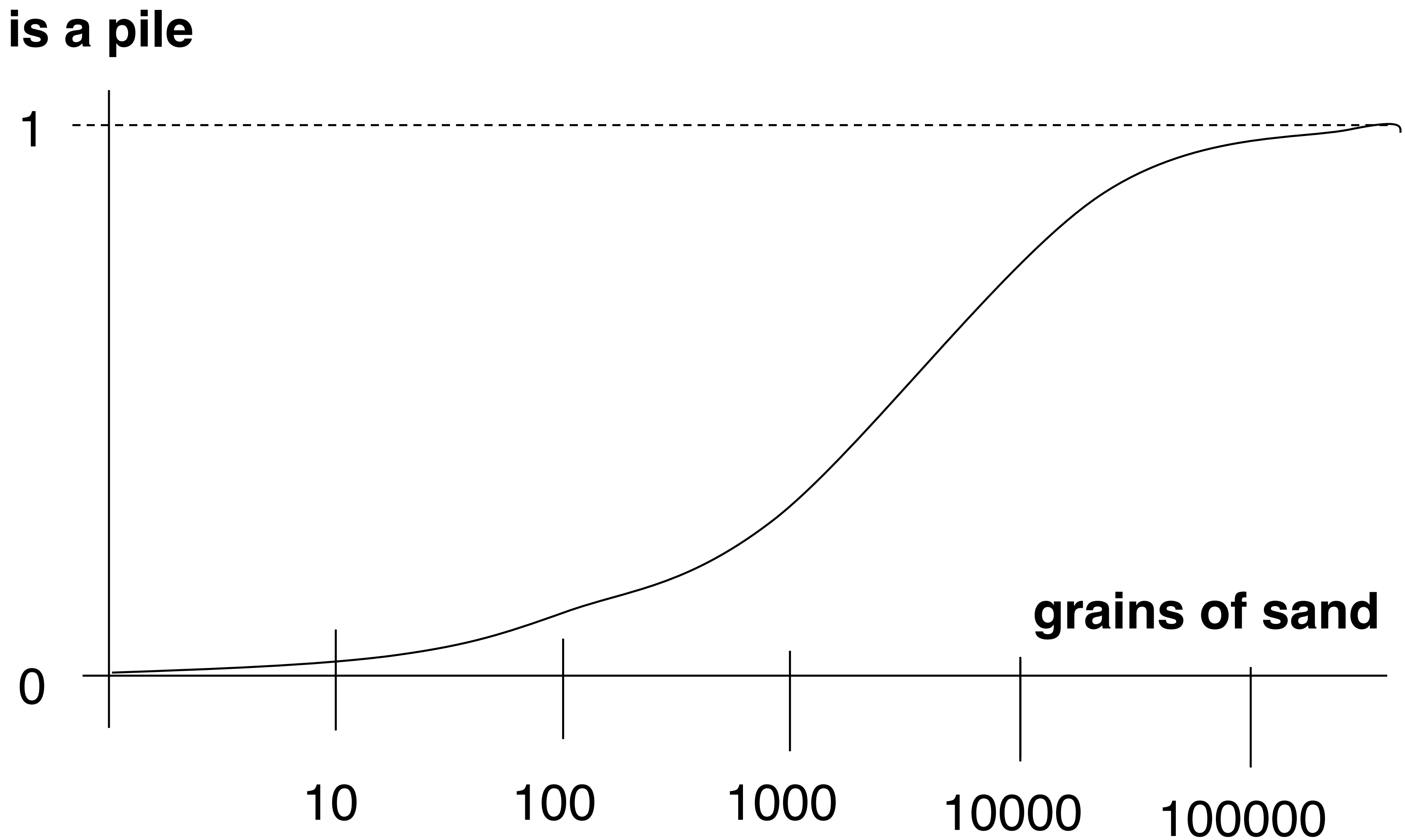
Fuzzy Membership

Which of these is a pile of sand?

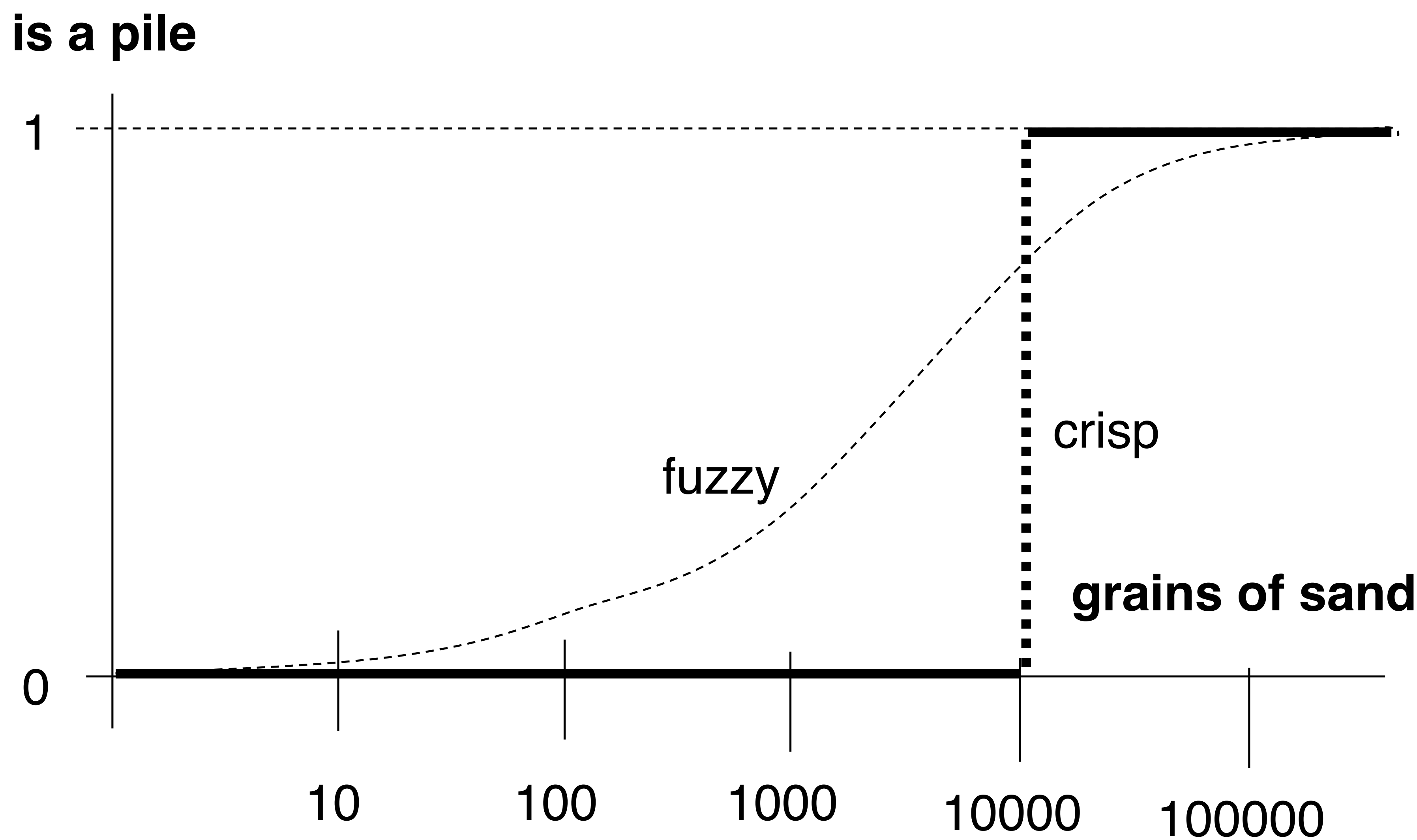
Where does the line get drawn about what is or is not a “pile”?

Questions not addressed by Fuzzy Logic

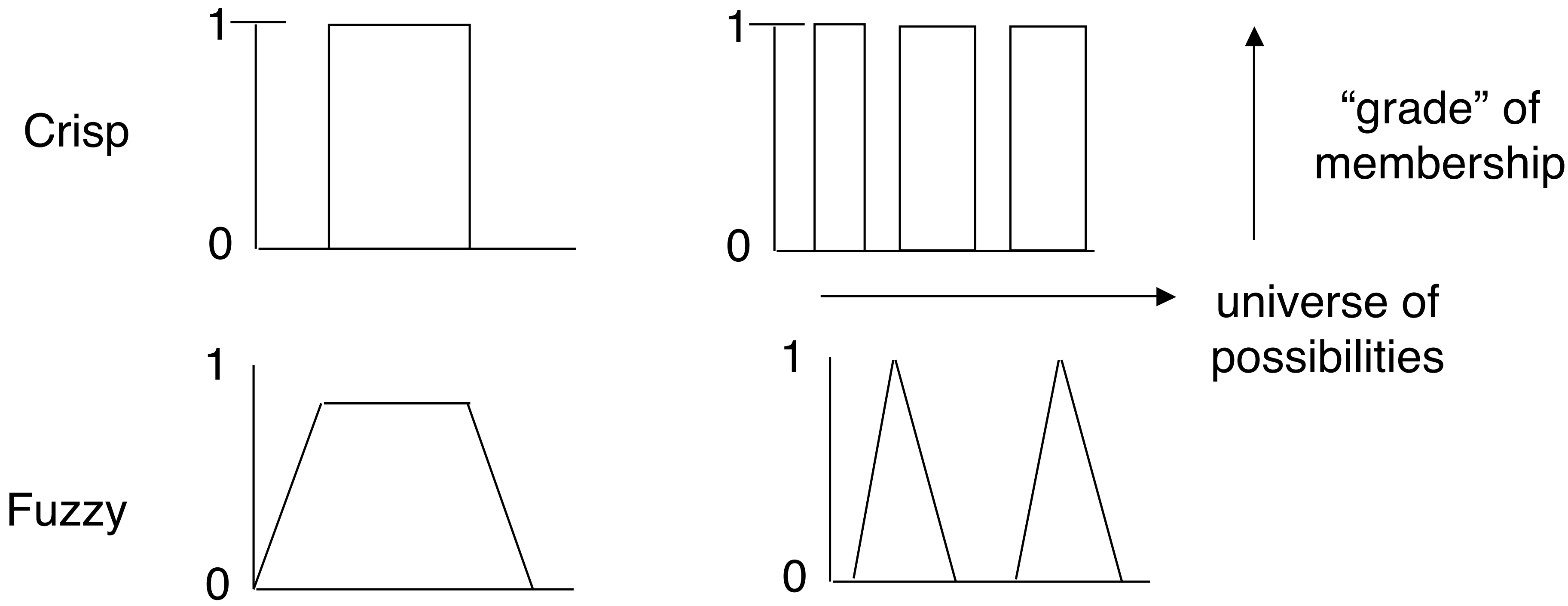
Membership function plots



Crisp vs. Fuzzy Membership Functions

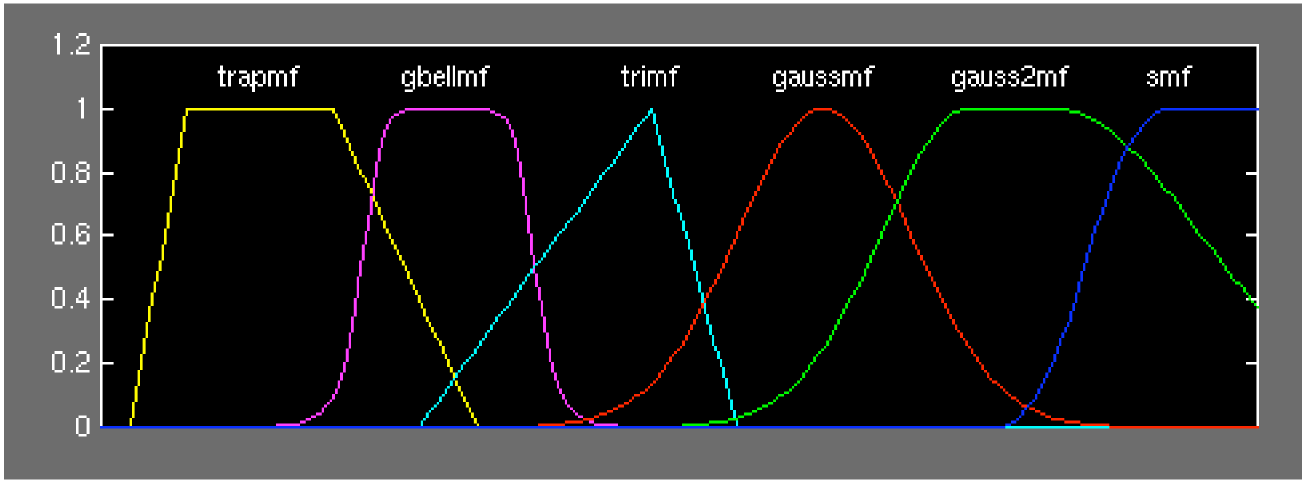


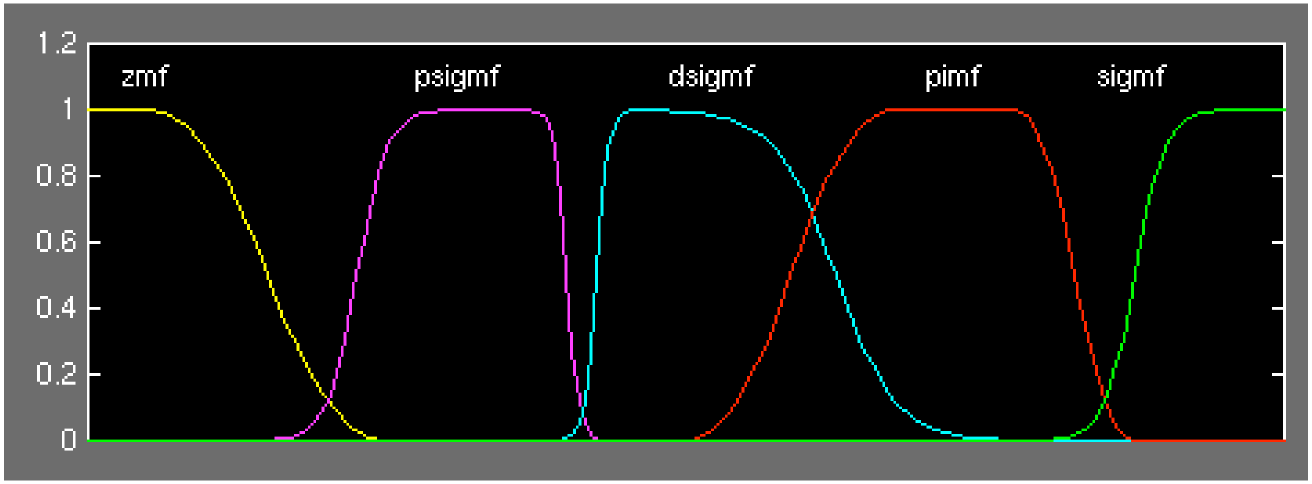
More Crisp vs. Fuzzy Membership Functions



Note: Universe can be continuous or discrete, ordered or unordered.

Some of Matlab's Builtin Membership Functions





Fuzzification: Degree of Membership derivable quantitatively from attributes

- (Inspired by Prof. Zadeh)
- Is X a goth? (Degree of membership = gothicity)
  - Always wears black: add .2 to gothicity
  - Wears a black cape: add .05
  - Heavy dark makeup: add .1
  - Visible fangs: add .2
  - Wears chains: add.05*number of chains
  - Wears amulets: add .03*number of amulets
  - Wears spikes around neck: add .3
  - Piercings: add .1*number of piercings
  - Frequents death raves: add .1*frequency (in DRPM)
  - Lives in dungeon-like environment: add .2
  - Has a pet vampire bat: add .1

Defuzzification:
Those agree/disagree questionnaires

Degree of membership translates into numbers:

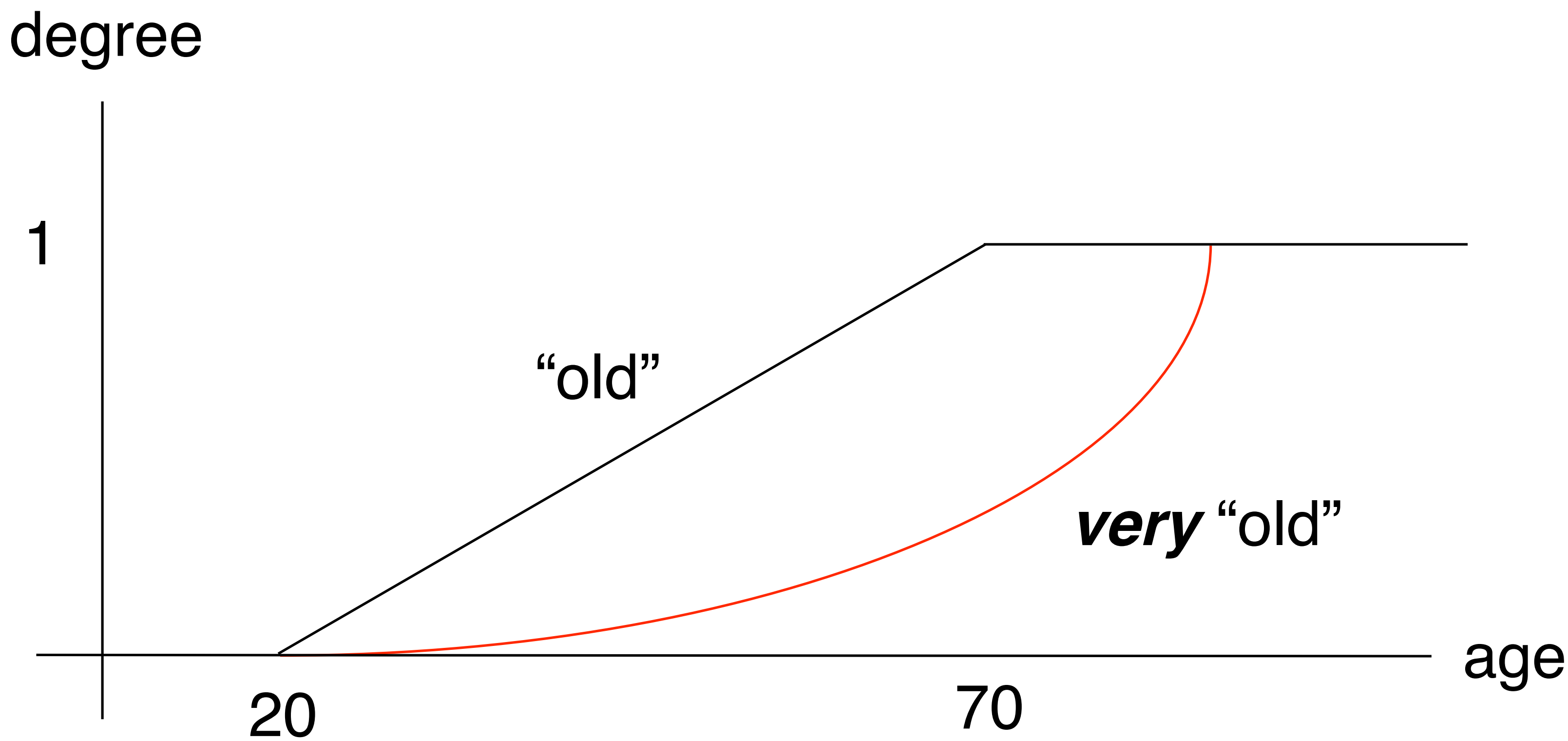
1. This course was effectively organized:
   ___ strongly disagree ___ disagree ___ neutral ___ agree ___ strongly agree

2. This course stimulated my interest in the subject matter:
   ___ strongly disagree ___ disagree ___ neutral ___ agree ___ strongly agree

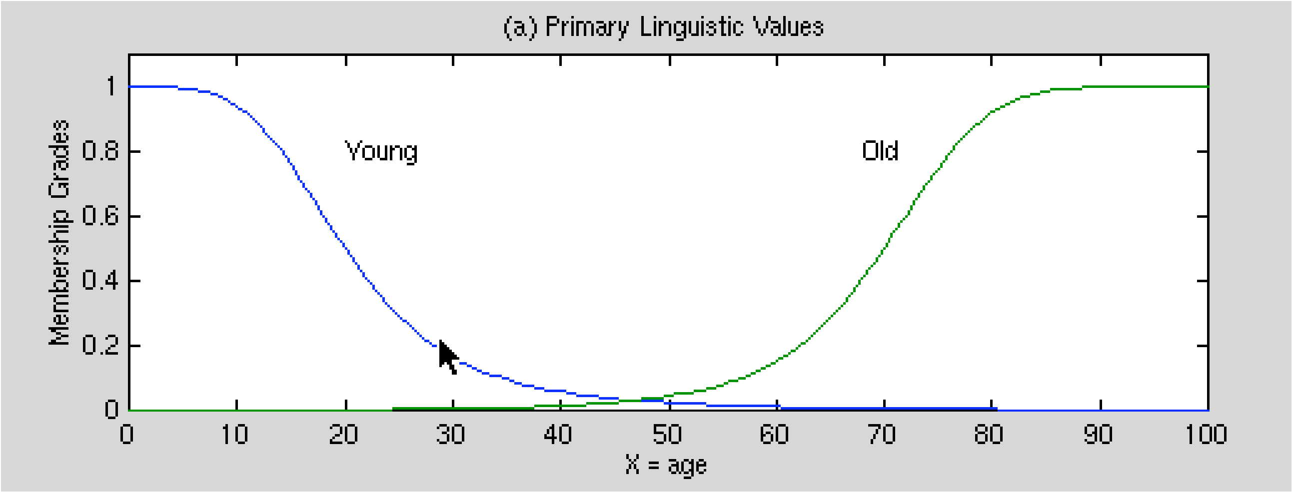
etc.

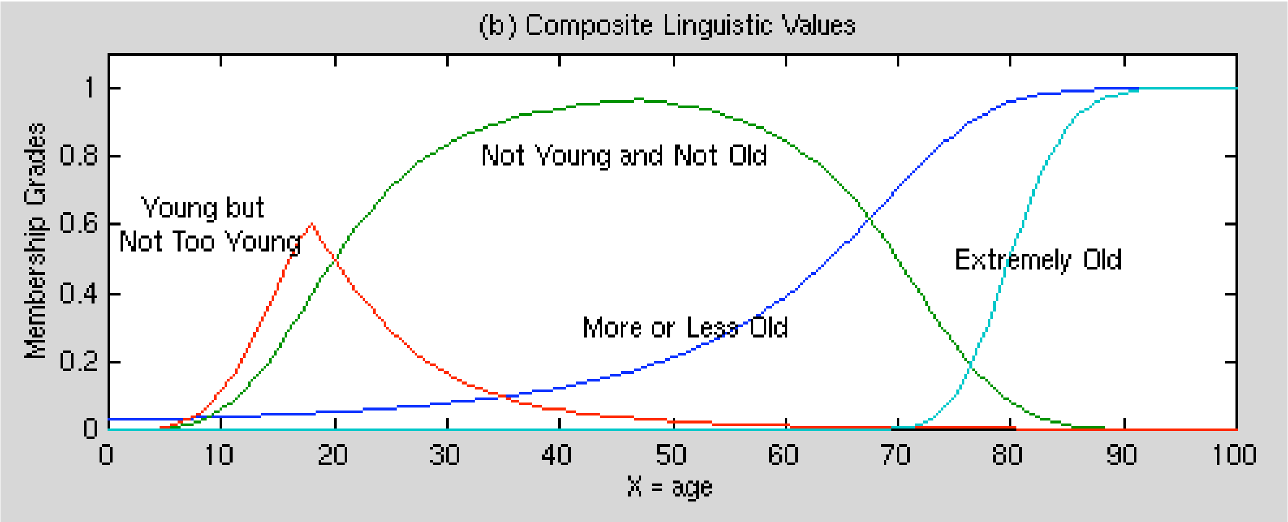
Ultimately a number is produced that will determines someone's salary.

Linguistic Modifiers



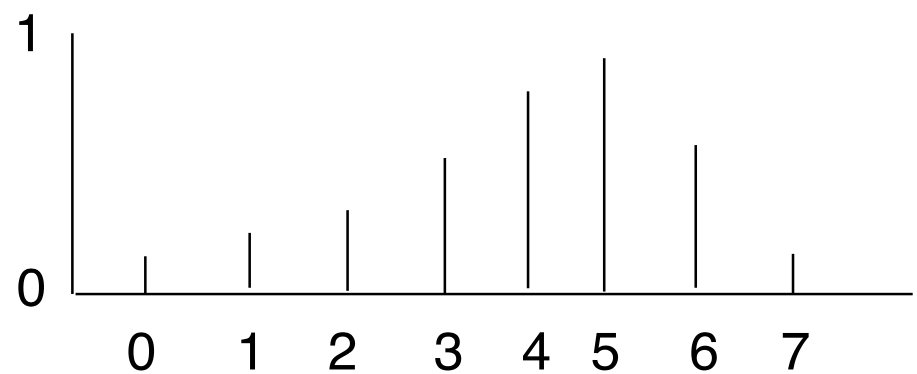
Example: Composites of Young and Old



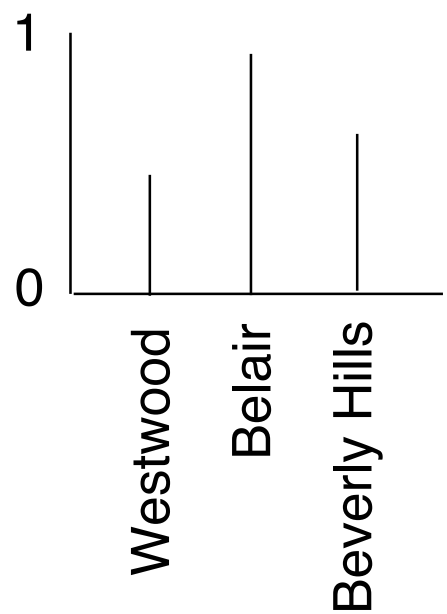


Discrete Universe Examples

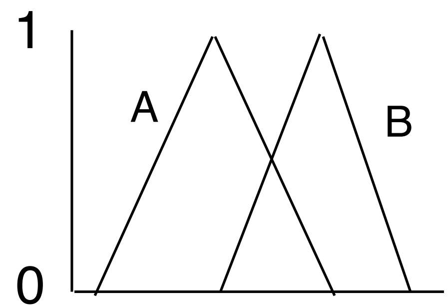
Ideal number of courses to take (ordered universe)



Desirable place to live (unordered universe)



Fuzzy-Set Operations
expressed using membership functions

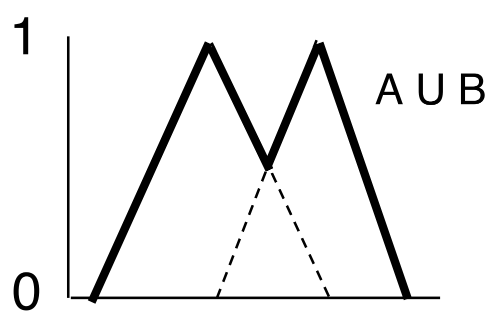


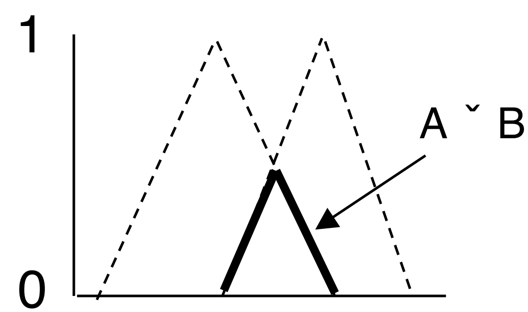
Fuzzy OR (union)

Fuzzy AND (intersection)

$c_{A\ U\ B}(x) = max(c_A\ (x), c_B(x))$

$c_{A \text{ ˇ } B}(x) = min(c_A\ (x), c_B(x))$





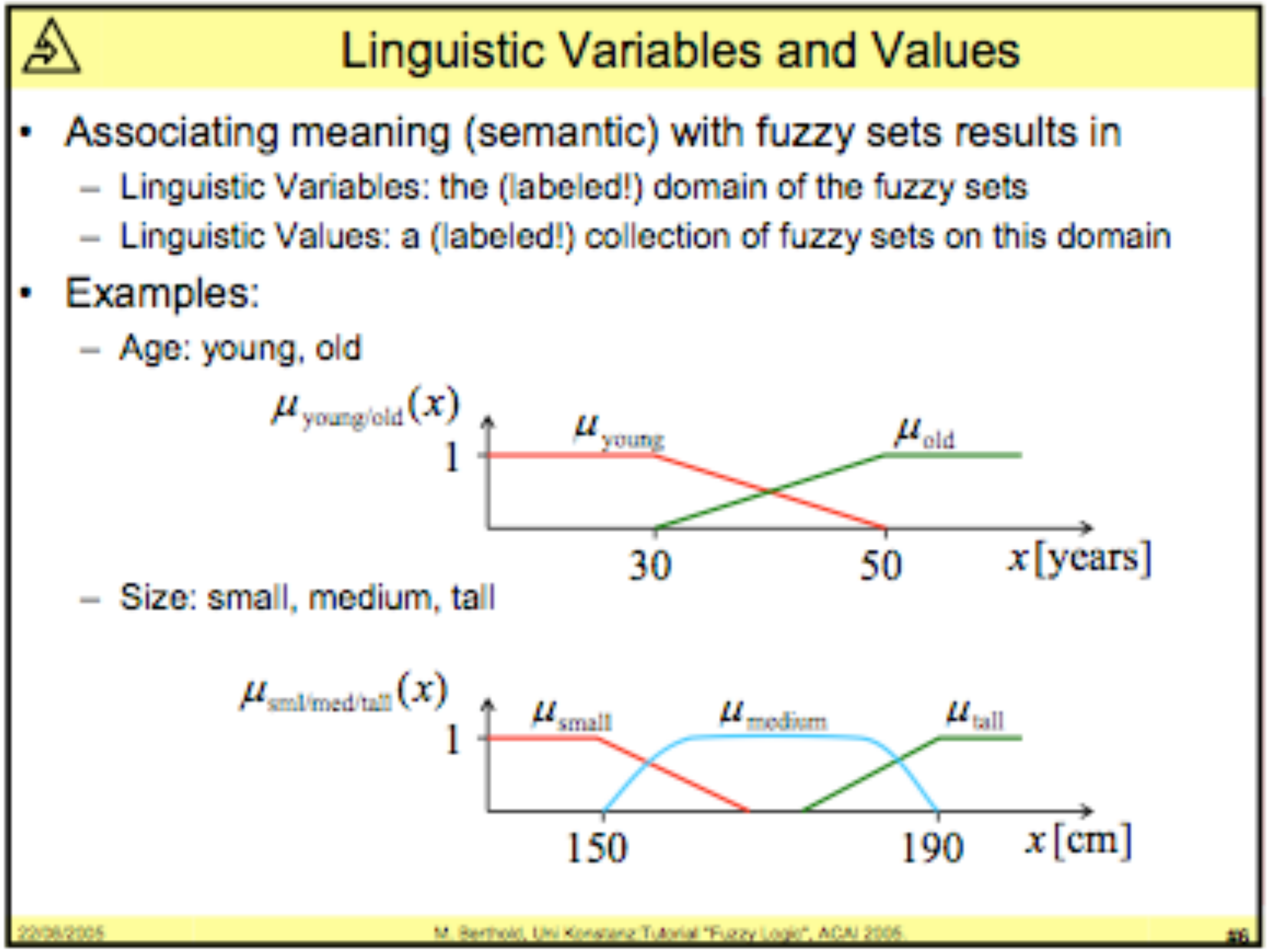
Fuzzy intersection (for the coninuous case) is a “t-norm”

A t-norm is a function T: [0, 1] × [0, 1] → [0, 1] satisfying:

- Commutativity: T(a, b) = T(b, a)
- Monotonicity: T(a, b) ≤ T(c, d) if a ≤ c and b ≤ d
- Associativity: T(a, T(b, c)) = T(T(a, b), c)
- The number 1 acts as identity element: T(a, 1) = a

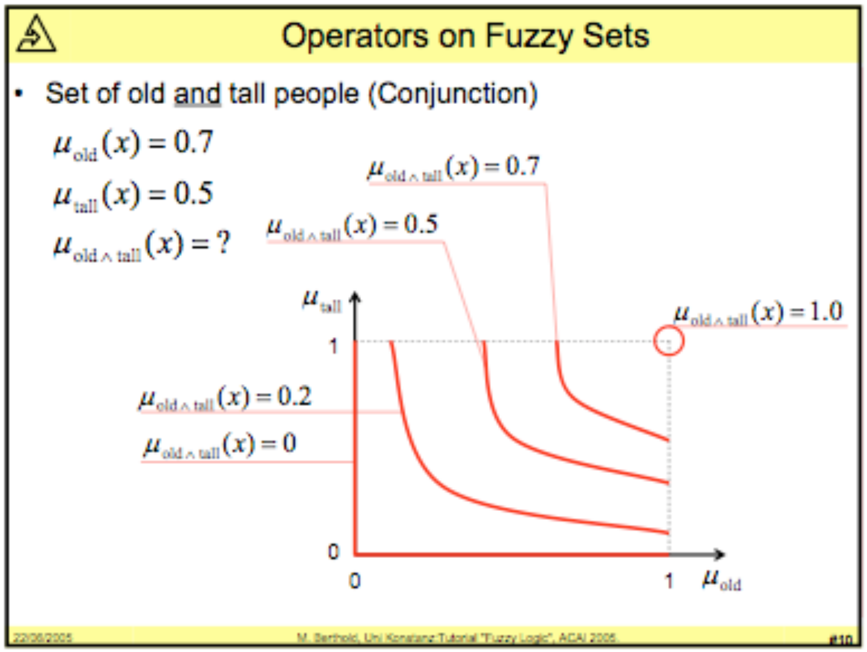
cf. Wikipedia article for more examples and results

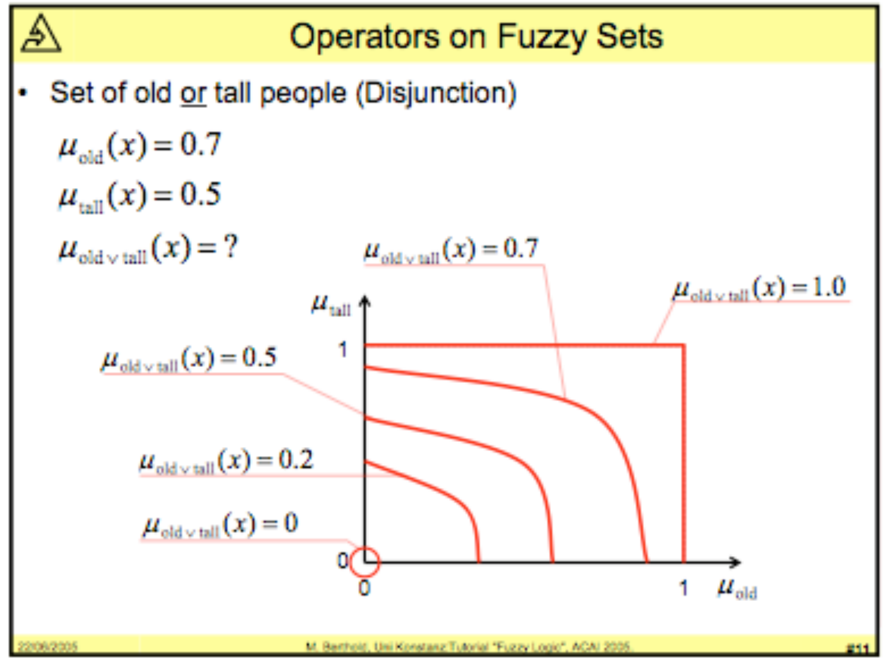
Operator Example



M. Berthold, Uni Konstanz:Tutorial "Fuzzy Logic", ACAI 2005.

Operator Example: Membership contours

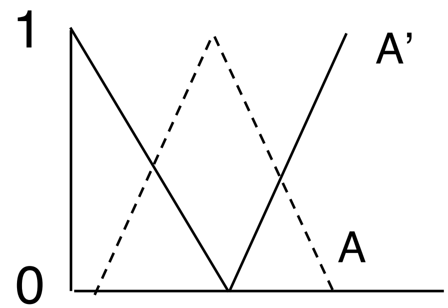




M. Berthold, Uni Konstanz:Tutorial "Fuzzy Logic", ACAI 2005.

Fuzzy Complement

$c_{A'}(x) = 1 - c_A(x)$.



Which Set-Theoretic Rules Hold?

$A \cup B = B \cup A$

$A \cap B = B \cap A$

$(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

$(A \cap B)' = A' \cup B'$

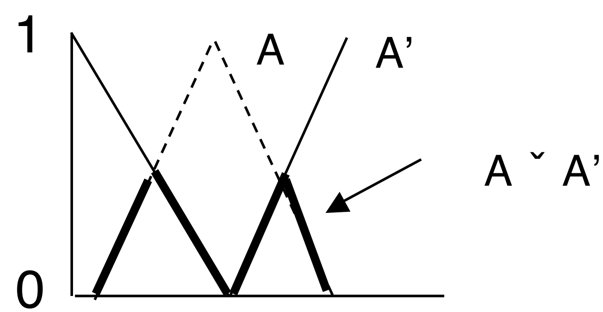
$(A \cup B)' = A' \cap B'$

etc.

Fuzzy Anomaly?

The intersection of a set with its complement is not necessarily empty.

$c_{A'}(x) = 1 - c_A(x)$.



Fuzzy Implication

There is no single standard. A variety of versions exists:

Larsen:	$x \rightarrow y = xy$
Lukasiewicz:	$x \rightarrow y = \min(1, 1-x+y)$
Mamdani:	$x \rightarrow y = \min(x, y)$
Standard strict:	$x \rightarrow y = x \leq y \; ? \; 1 : 0$
Goedel:	$x \rightarrow y = x \leq y \; ? \; 1 : y$
Gaines:	$x \rightarrow y = x \leq y \; ? \; 1 : y/x$
Kleene-Dienes:	$x \rightarrow y = \max(1-x, y)$
Kleene-Dienes-Luk:	$x \rightarrow y = 1-x+xy$

Fuzzy Implication

If we use the definition from ordinary logic:

$$x \to y = (\neg x) \vee y$$

we would be led to:

$$x \to y = \max(1-x, y)$$

the Kleene-Dienes version.

Fuzzy Query Languages

- FSQL (Fuzzy SQL)

  http://www.lcc.uma.es/~ppgg/FSQL/
- SQLf
- FQuery

a) numeric fuzzy values (e.g., *young*),
b) modifiers (e.g., *very* young),
c) fuzzy relations (e.g., age *around* 30),
d) fuzzy sets of scalar values
e) fuzzy (linguistic) quantifiers,
f) importance coefficients

Linguistic Rules

- In Fuzzy Logic, rules are expressed **qualitatively and linguistically**, rather than quantitatively.
- The result is qualitatively understandable, yet can be **interpreted quantitatively** when desired.
- The interpretive framework can be **adjusted to suit**.

Mamdani and Sugeno

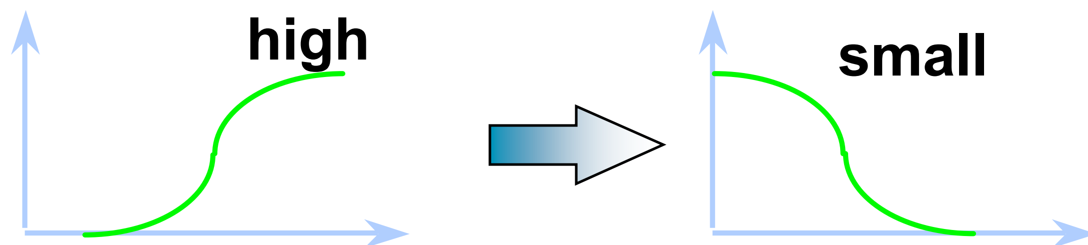
Ebrahim MAMDANI
Electrical and Electronic
Engineering Professor,
Imperial College of Science,
Technology and Medicine,
University of London

Michio Sugeno
Head of the Laboratory for
Language-Based Intelligent Systems
Brain Science Institute, RIKEN
Japan

Fuzzy If-Then Rules

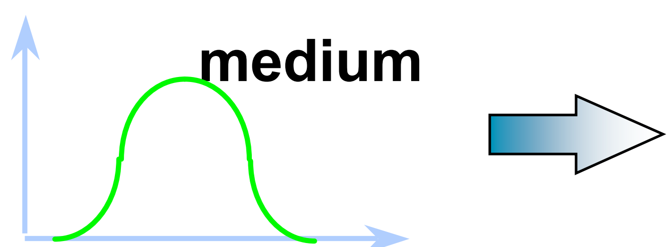
- **Mamdani style**

  If pressure is high then volume is small



- **Sugeno style (uses *equations* on rhs)**

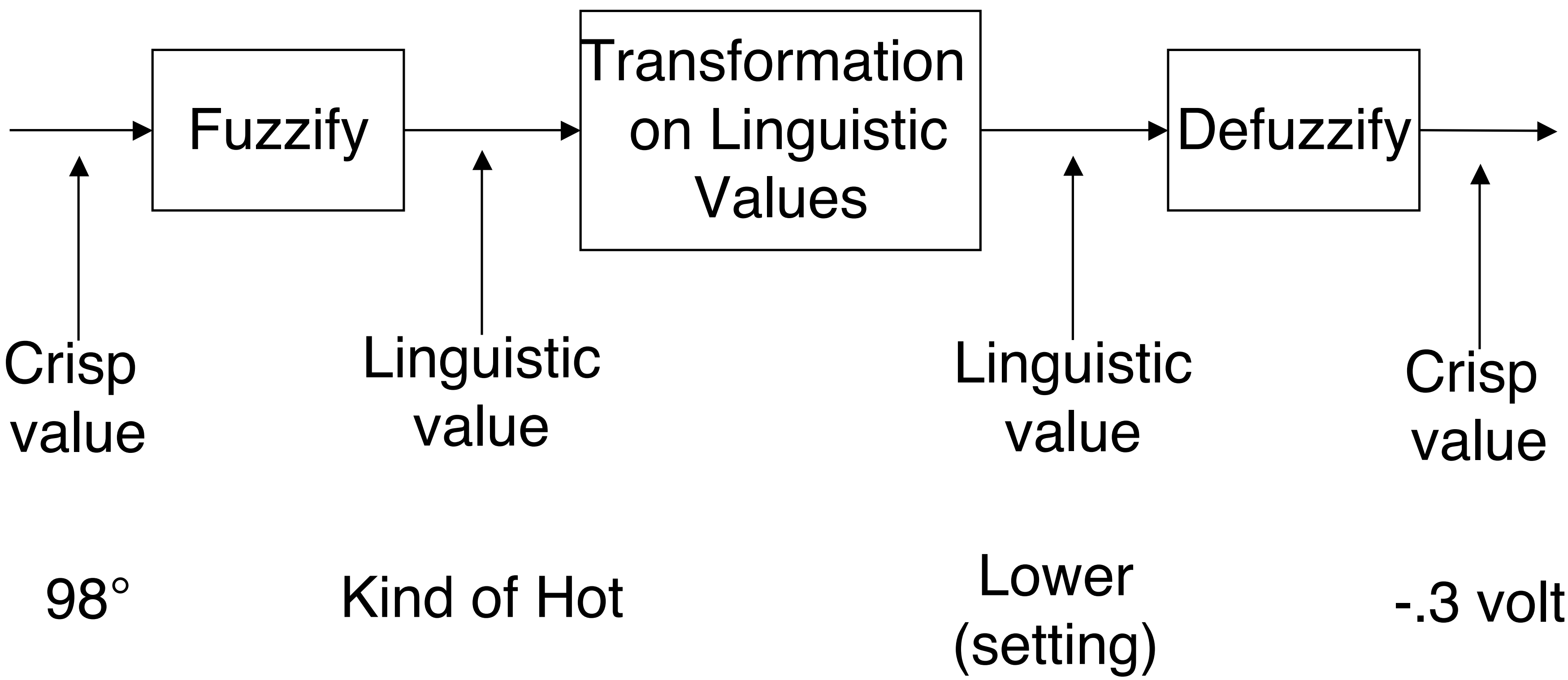
  If speed is medium then resistance = 5*speed



resistance = 5*speed

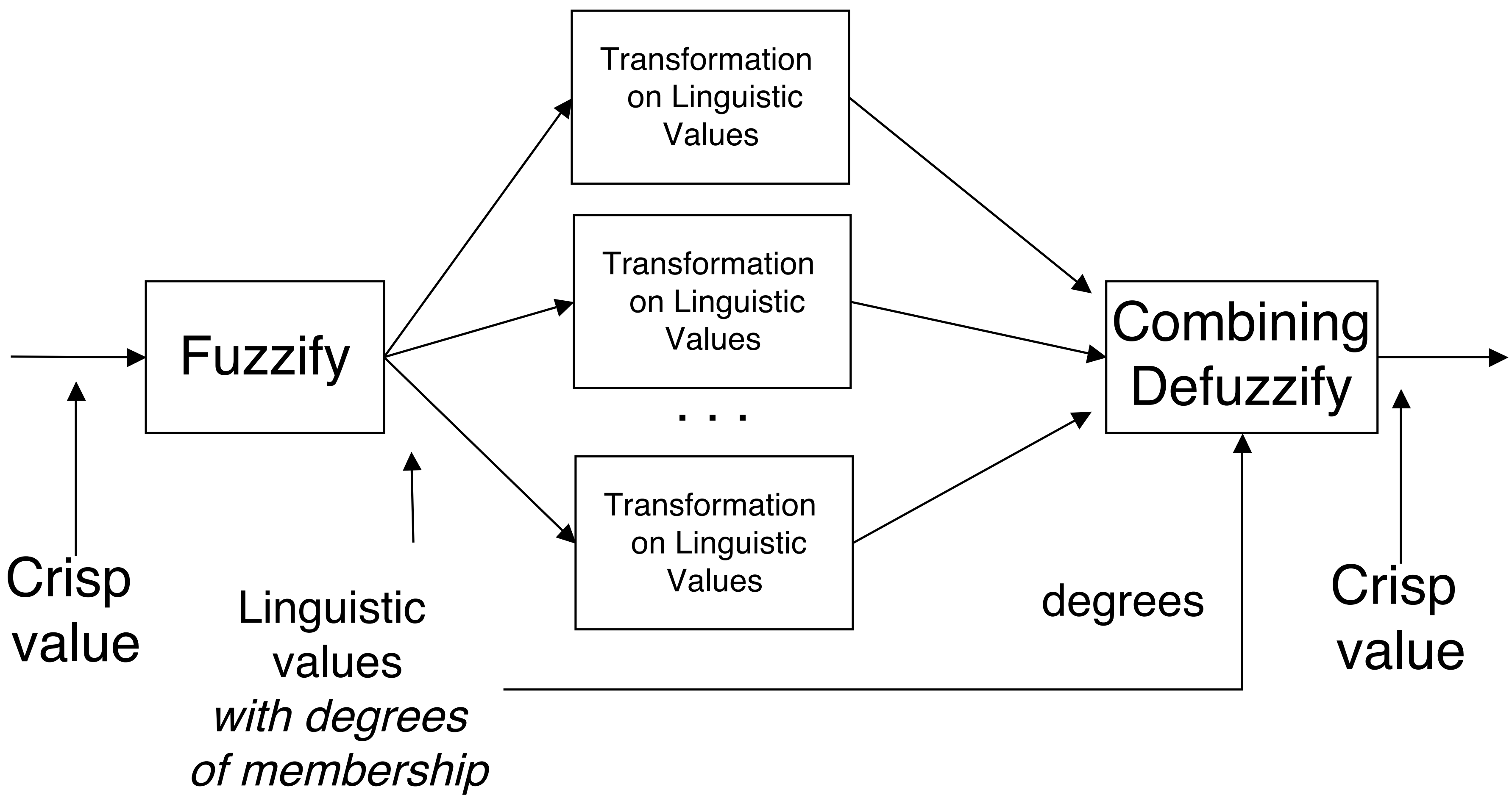
Fuzzification/Defuzzification in Function Implementation

Over-Simplified Picture

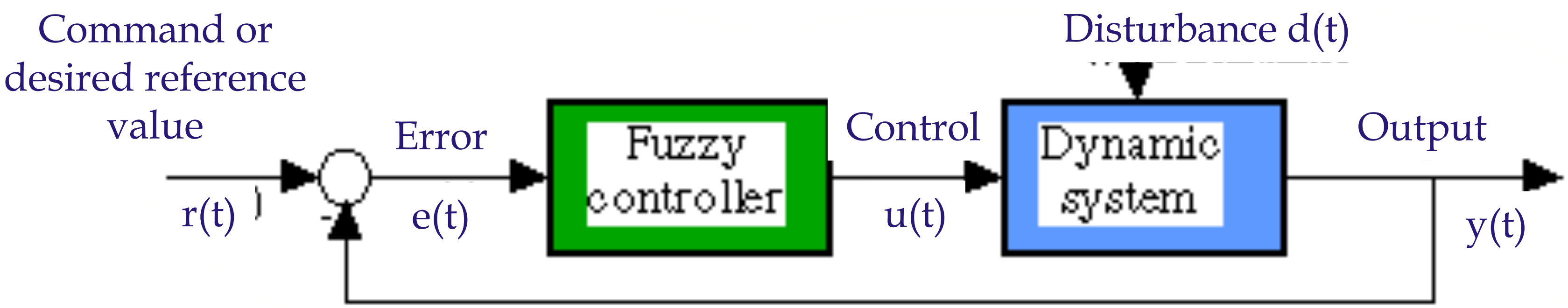


Fuzzification/Defuzzification in Function Implementation

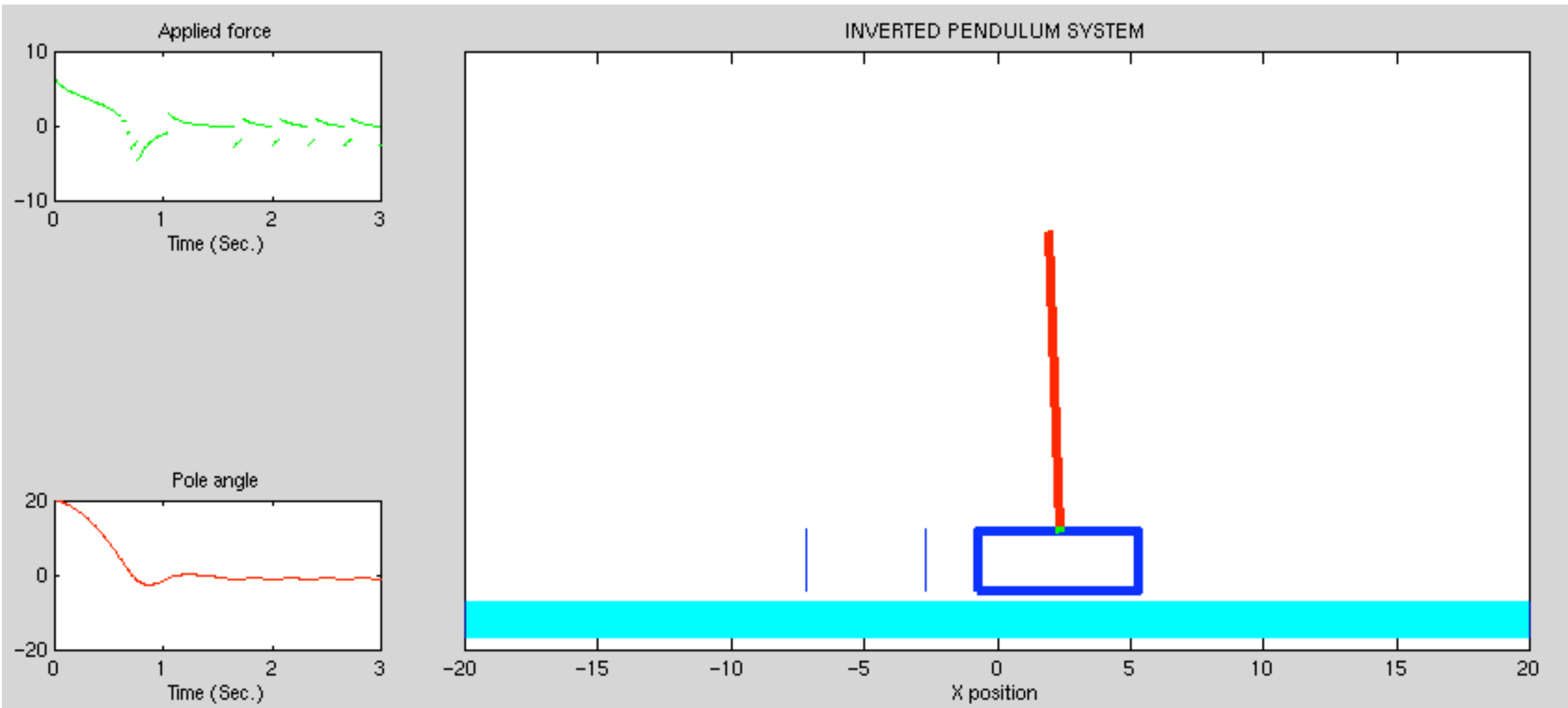
More-Realistic Picture (multiple applicable rules)



Mamdani Control Model (next several slides)



Pole-on-Cart (Inverted Pendulum) Example

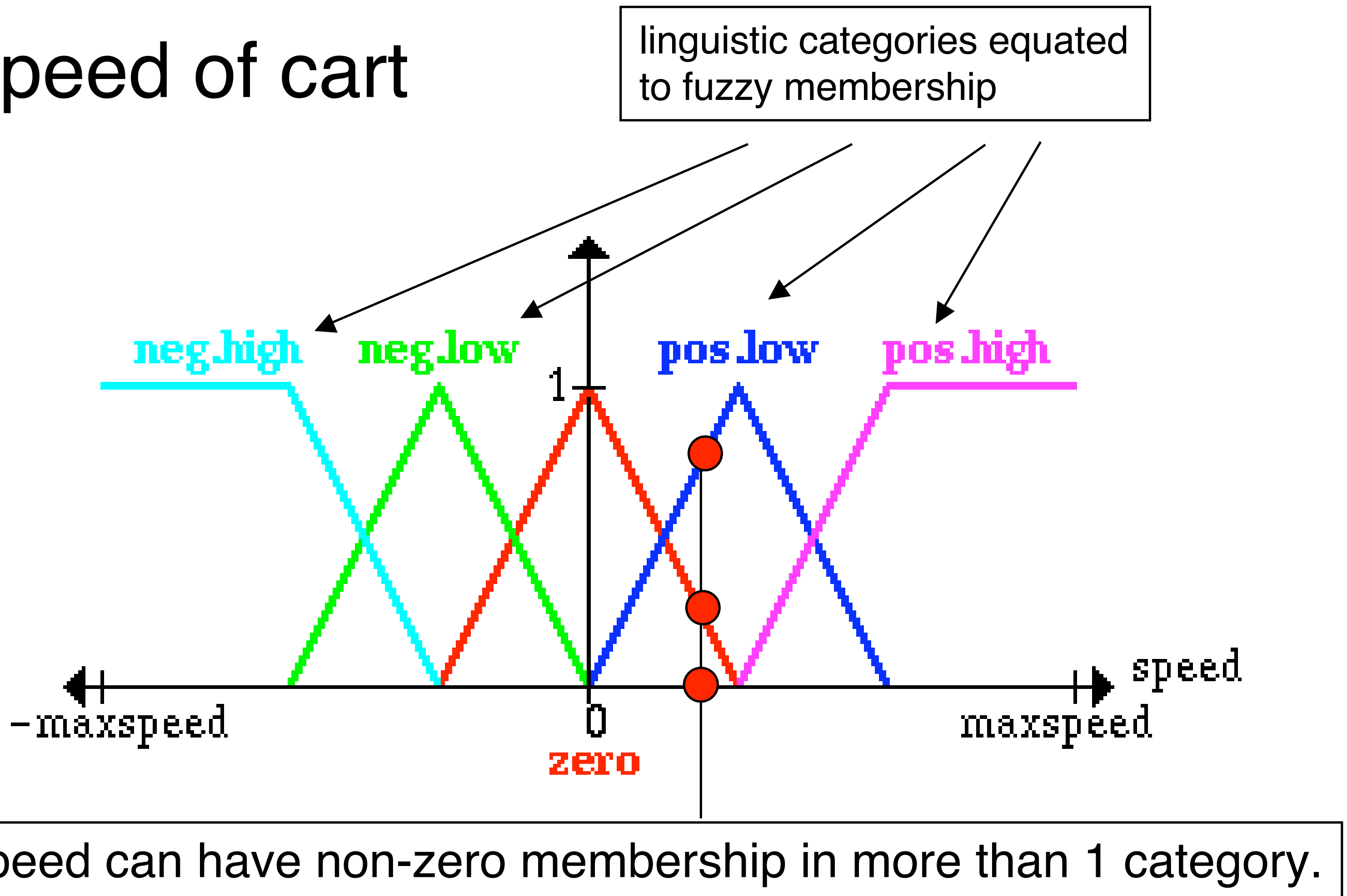


Problem

- Determine a fuzzy transformation adequate to specify the pole-cart controller.

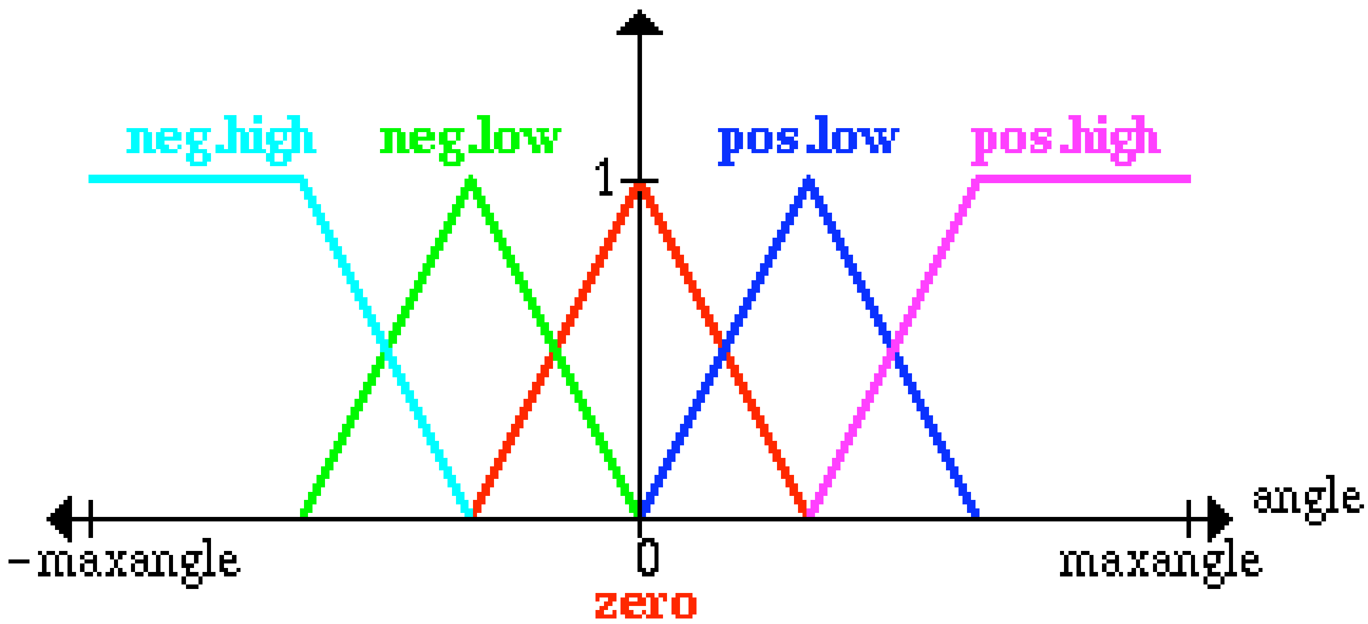
Pole-on-Cart Example

- Speed of cart



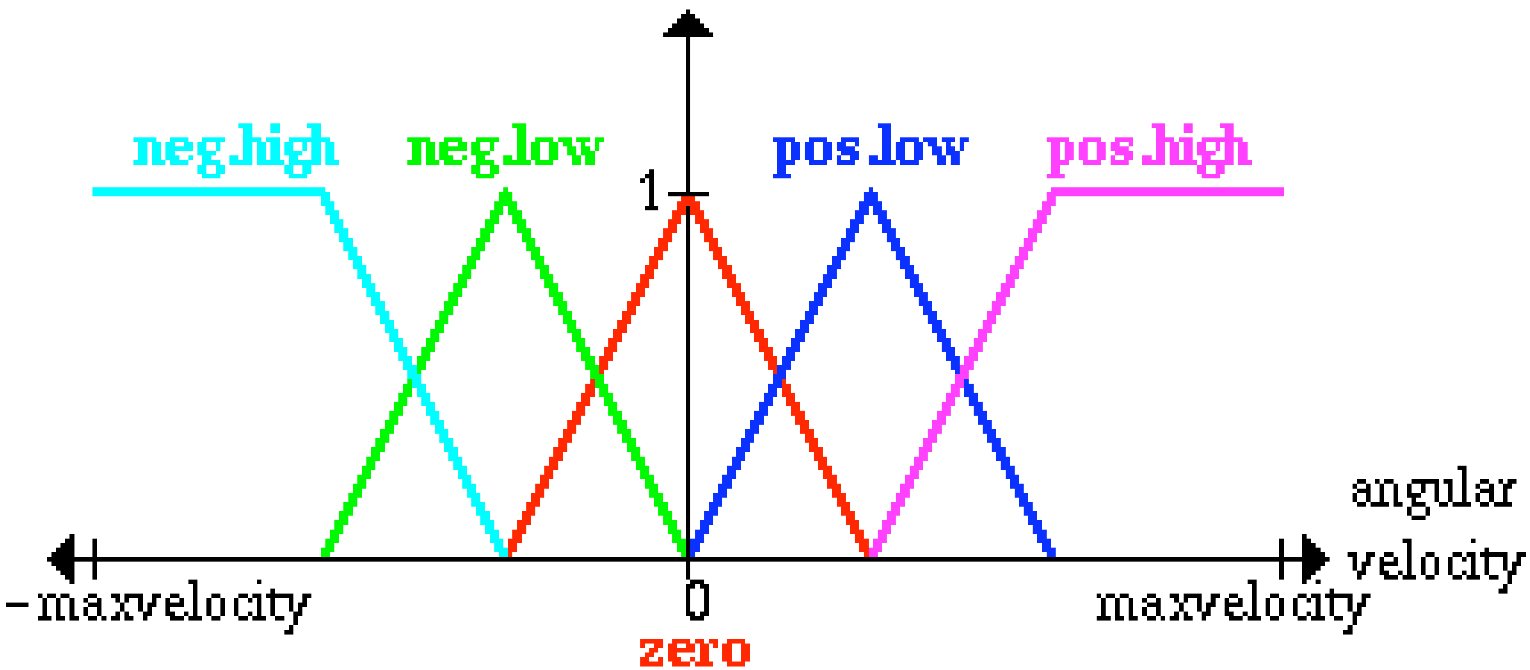
Pole-on-Cart Balancing Example

- Angle of pole



Pole-on-Cart Balancing Example

- Angular velocity of pole



Transformation Rep. by "Fuzzy Rule Base"
(Kosko: FAM = Fuzzy Associative Memory)

Example of a fuzzy-logic rule represented in this table:
"**If** the angular velocity is pos. low **and** the angle is zero, **then** set the speed at low."

angle

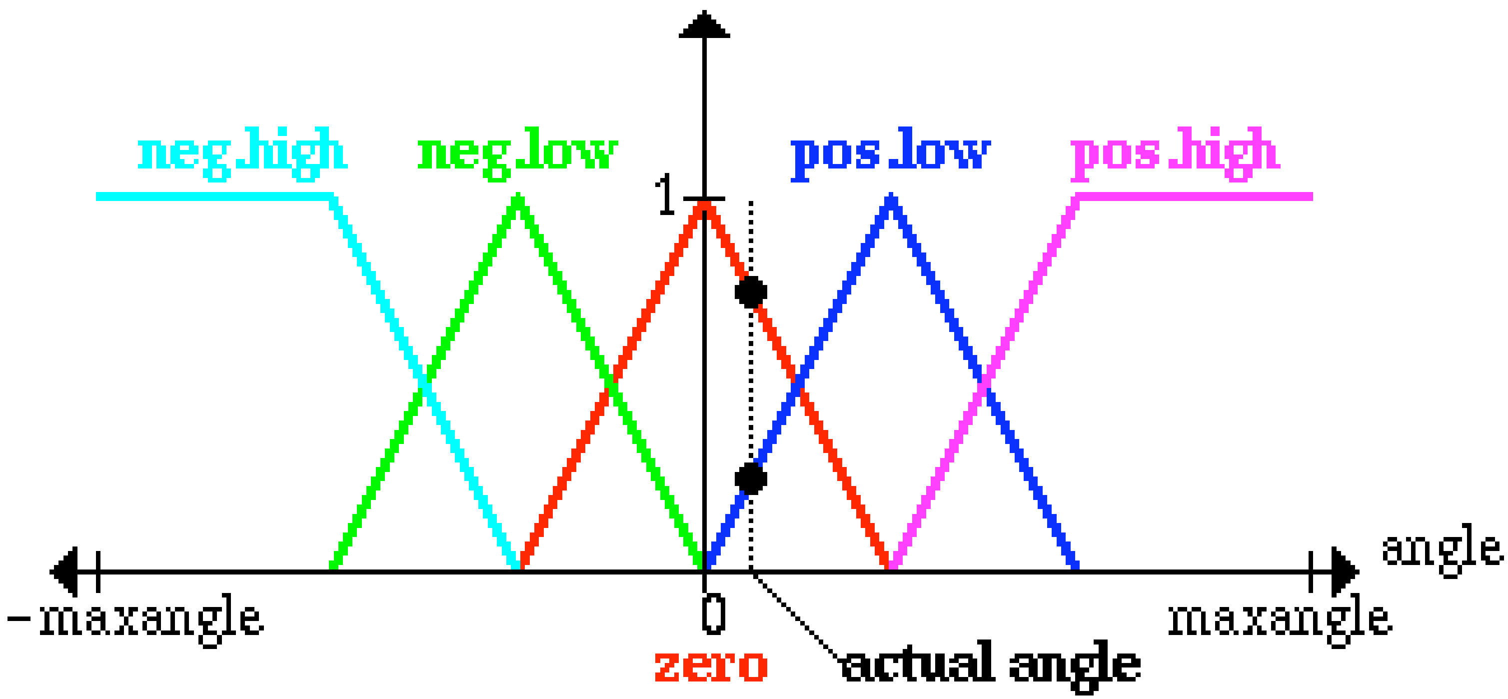
angular velocity	**neg. high**	**neg. low**	**zero**	**pos. low**	**pos. high**
neg. high			neg. high		
neg. low			neg. low		
zero	neg. high	neg. low	zero	pos. low	pos. high
pos. low		zero	low		
pos. hi		high			

control speed
as a function of angle and angular velocity

Inference in a Mamdani-style Fuzzy System

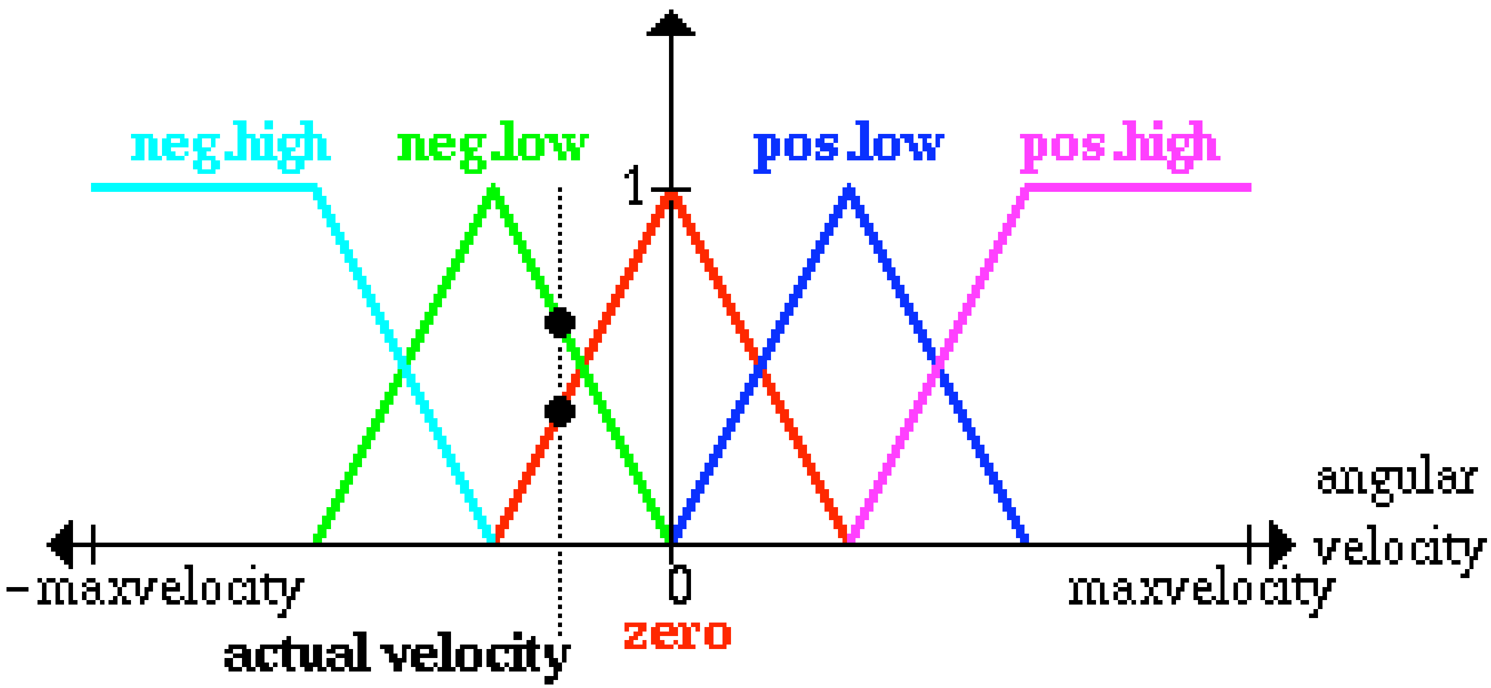
- Start with quantitative input data
- **Fuzzify** the data
- Derive conclusion based on fuzzy data
- **De-fuzzify** the conclusion to get quantitative output

Fuzzification



In this case, the actual ***angle*** is a **mixture** of zero and pos. low.

Fuzzification



Here the actual ***angular velocity*** is a **mixture** of zero and neg. low.

Multiple Applicable Rules:

- angle is a **mixture** of zero and pos. low.
- angular velocity is a **mixture** of zero and neg. low

	angle				
angular velocity	**neg. high**	**neg. low**	**zero**	**pos. low**	**pos. high**
neg. high			neg. high		
neg. low			**neg. low**		
zero	neg. high	neg. low	**zero**	**pos. low**	pos. high
pos. low		zero	low		
pos. hi		high			

Three entries in the rule base are applicable.
We must determine *how to combine them*.

Determine the **degree** to which each rule is applicable.

- Consider the rule

  ”If angle is zero and angular velocity is zero, the **speed** is zero".

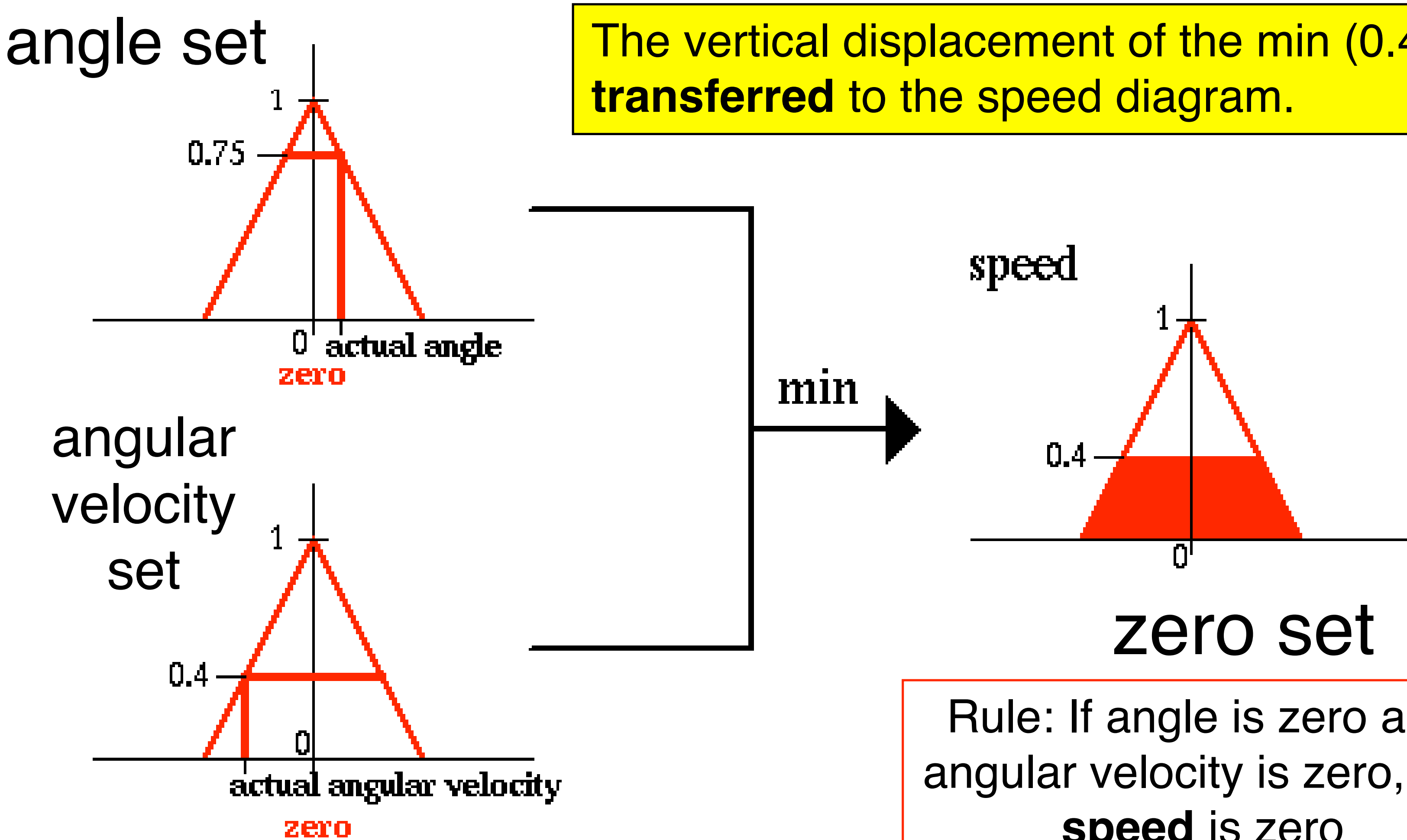
  Suppose the actual value belongs to the fuzzy set *zero* to a **degree** of 0.75 for "angle" **and** to a **degree** of 0.4 for "angular velocity".

- Since this is an **AND** operation, the **minimum** criterion is used. (For OR, the maximum would be used.)

- The fuzzy set **zero** of the variable "**speed**" is ***cut*** at the **min** 0.4 and the output patches are shaded up to that value.

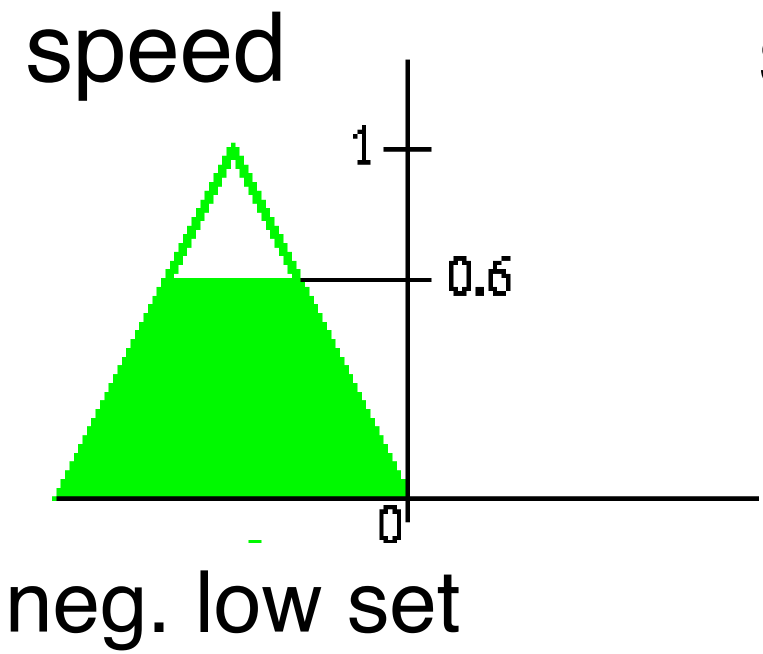
Using min combination for AND
(This is for ***one of three* speed rules**: zero.)

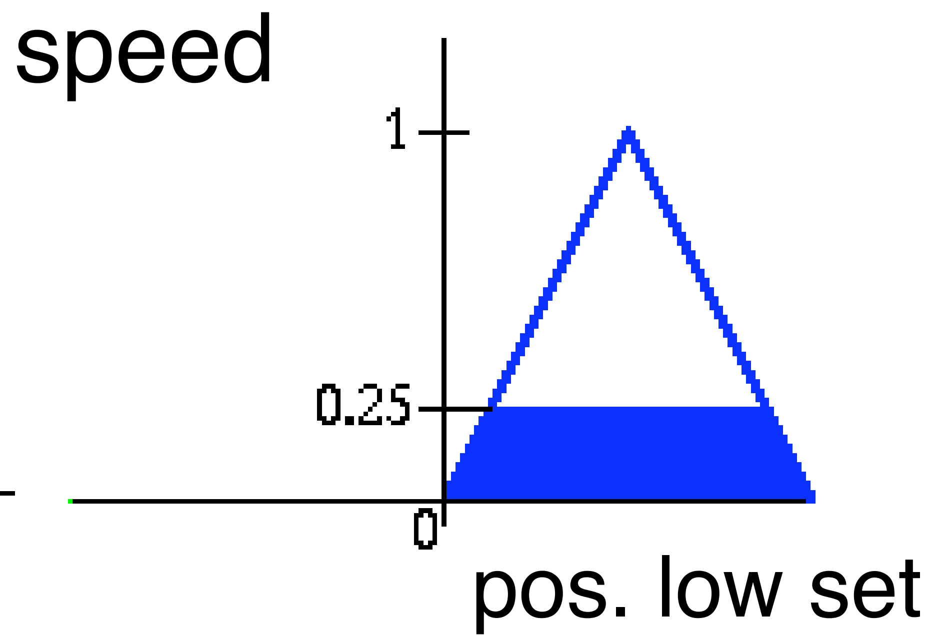
The vertical displacement of the min (0.4) is **transferred** to the speed diagram.



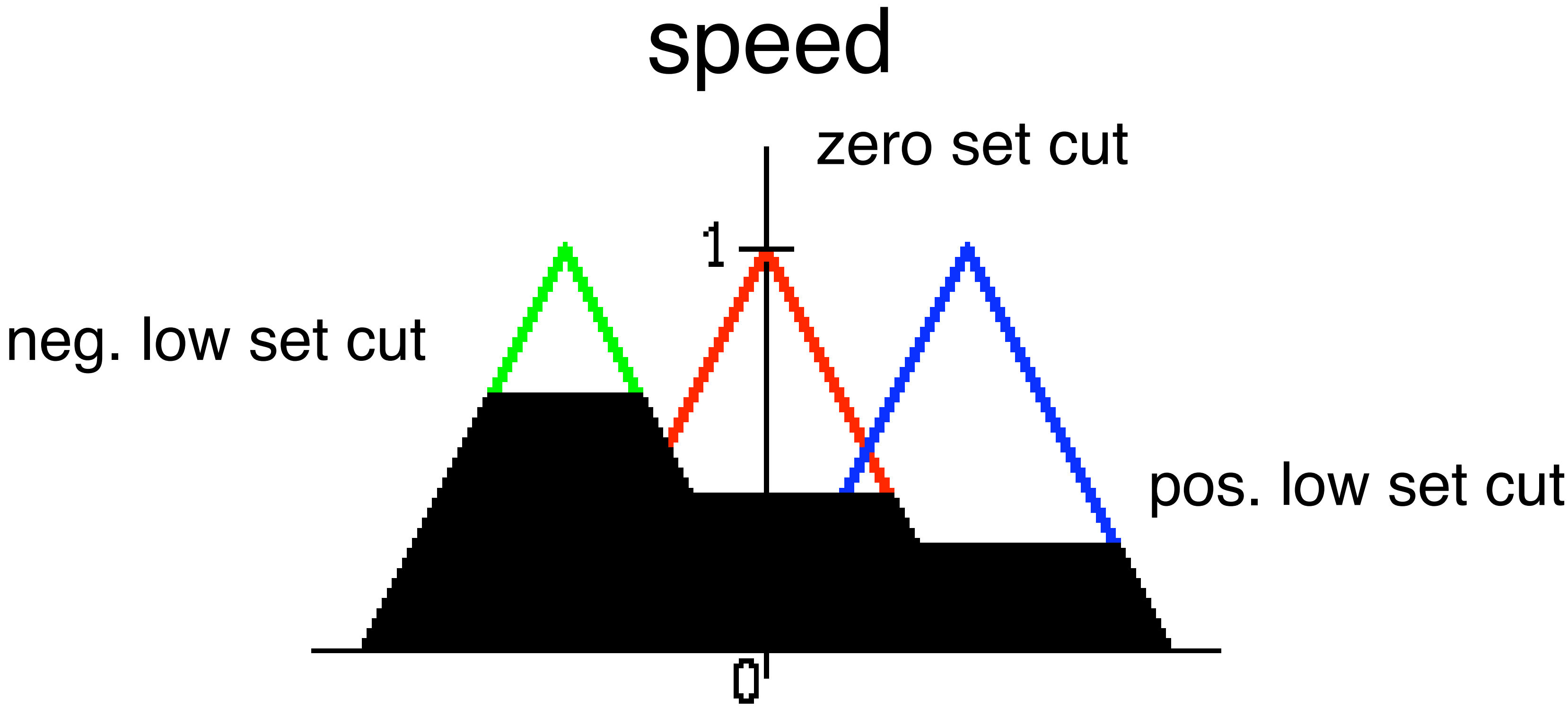
Rule: If angle is zero and angular velocity is zero, the **speed** is zero

Similarly, for *each* of the (3) applicable rules we get a cut for inferred speed.



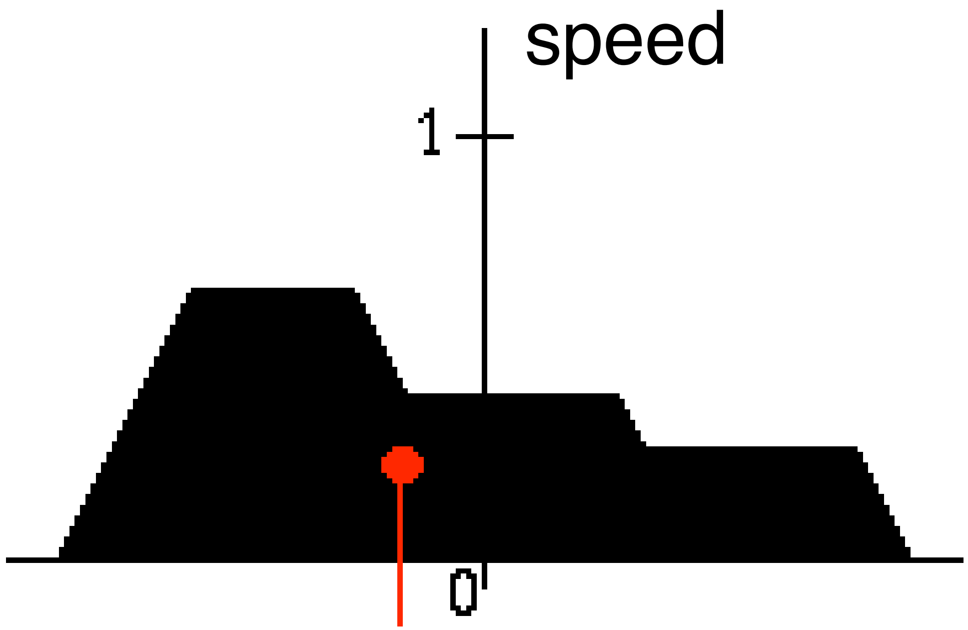


We then combine the results of the three rules to get an **output fuzzy set**.



Defuzzification

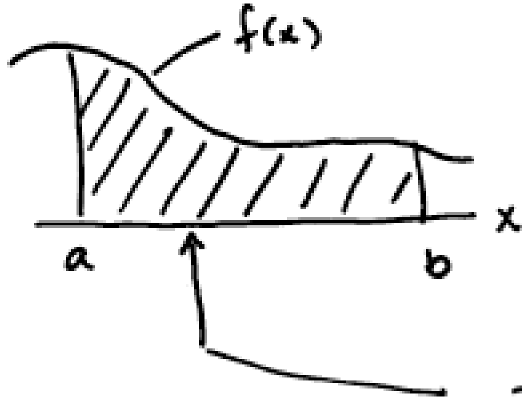
To actually ***set*** the speed, we need a ***number***, not a fuzzy set.
Various rules can be used to get the number.
The most common one is to use the **centroid** of the fuzzy set.



quantiative value of speed found at centroid

Centroid Review

Centroid (wrt x)



The point at which the areas on either side are equally balanced.

$$\frac{\int_a^b x f(x)\, dx}{\int_a^b f(x)\, dx}$$

An example worked in detail:

- Chapter 3 of “Fuzzy Logic for Just Plain Folks”: Let's Build a Fuzzy Logic Control System

- Includes code in Basic: http://www.fuzzy-logic.com/ch3.htm

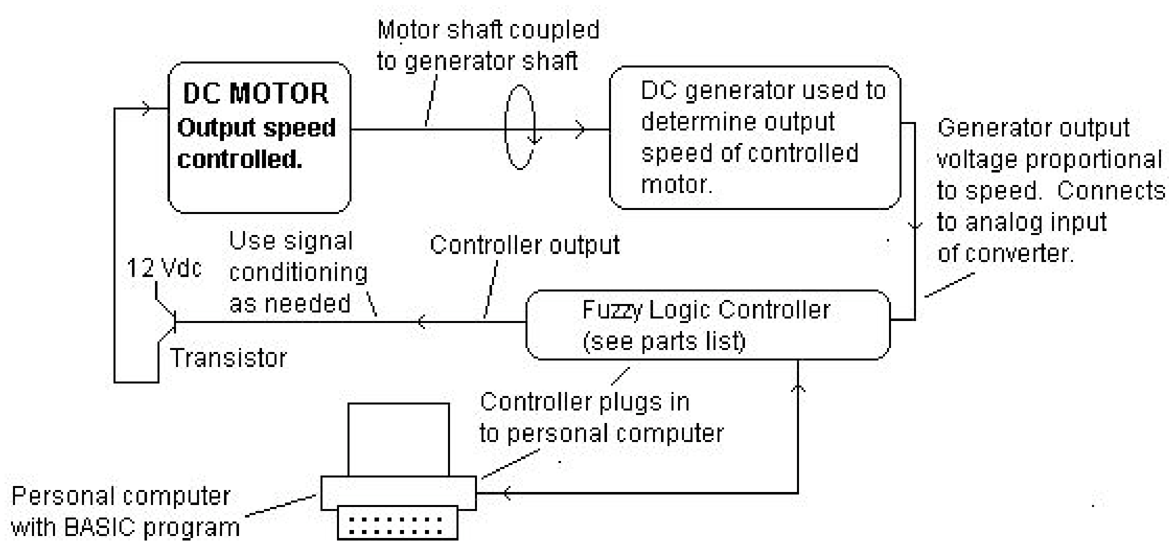


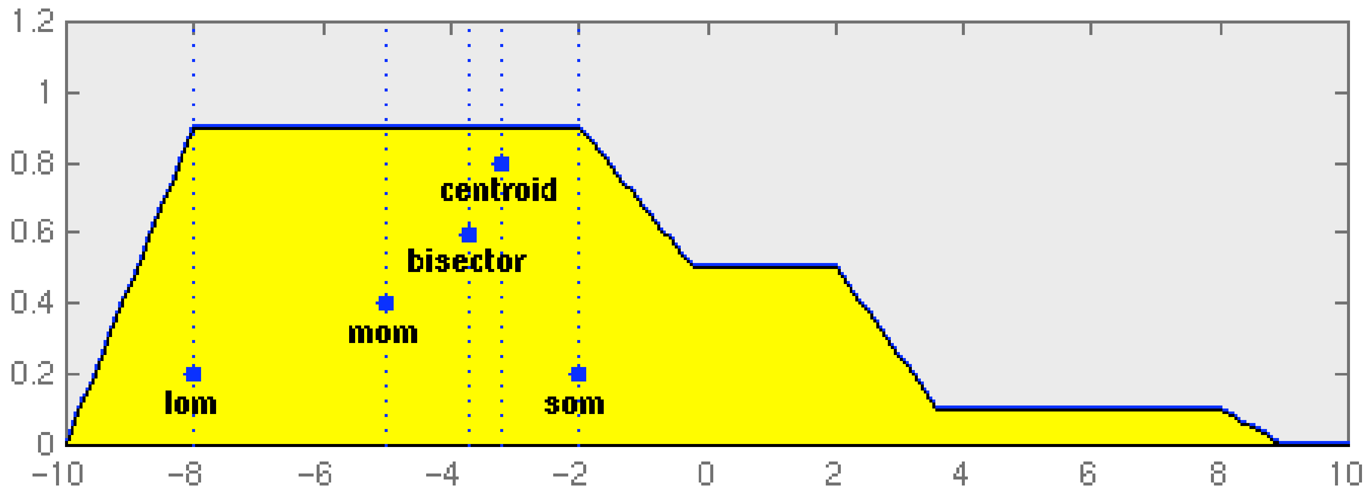
Figure 3. Motor Speed Control System

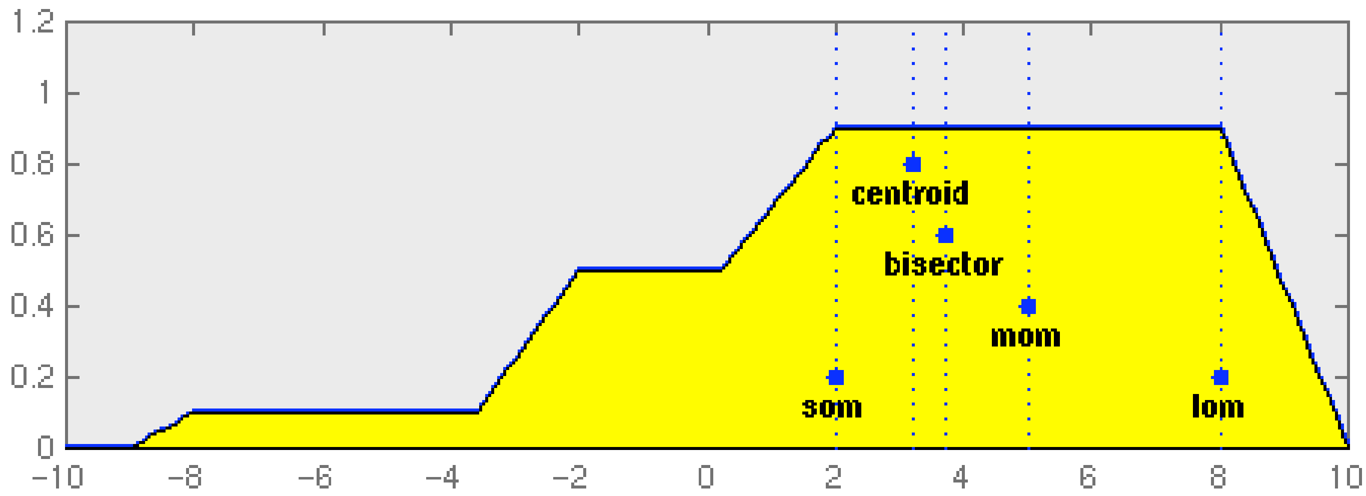
Examples: Matlab's Defuzzification Rules

mom = mean of max

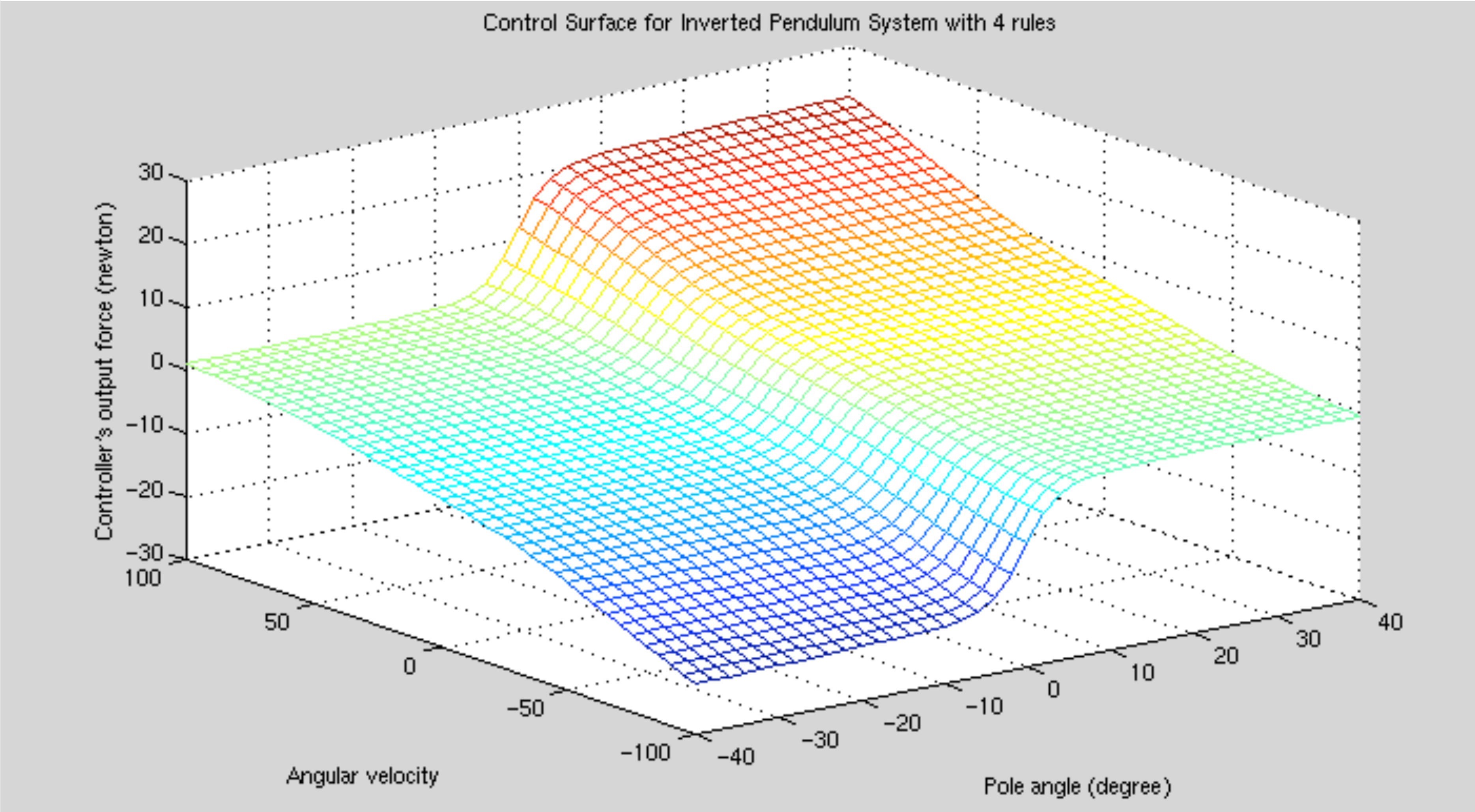
lom = largest of max

som = smallest of max

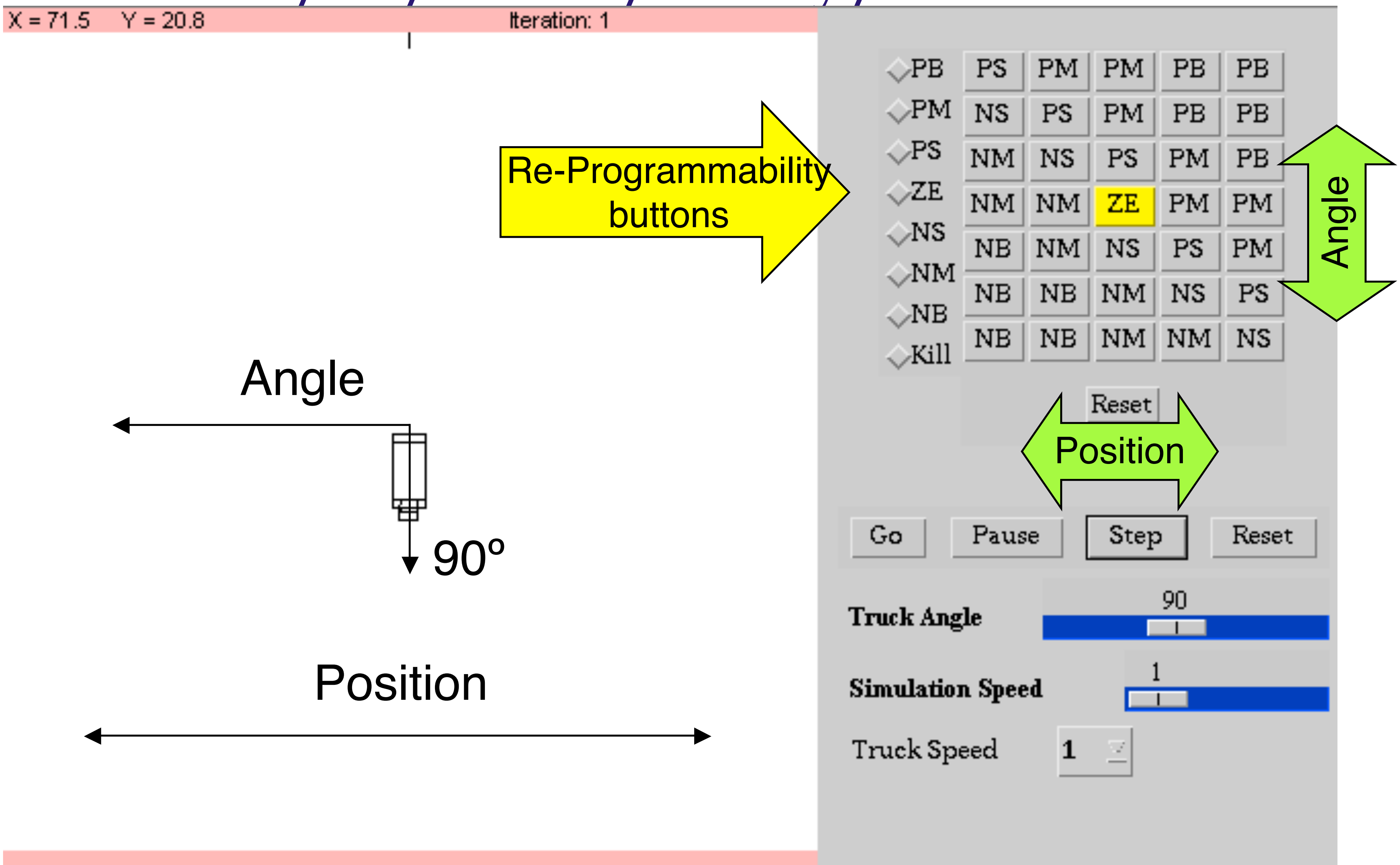




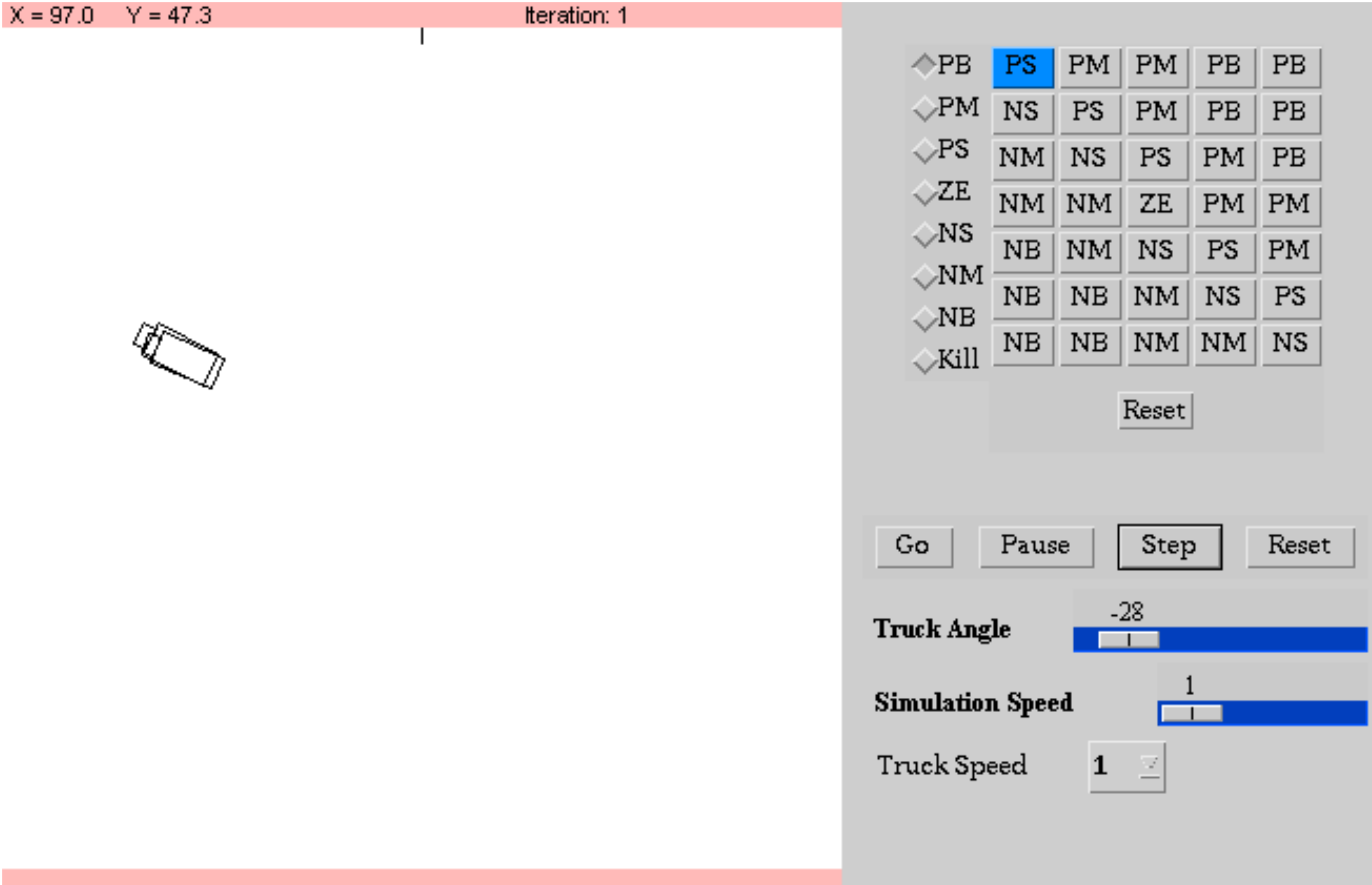
Control Surface for an Inverted Pendulum: Force = f(Angle, AngularVelocity)

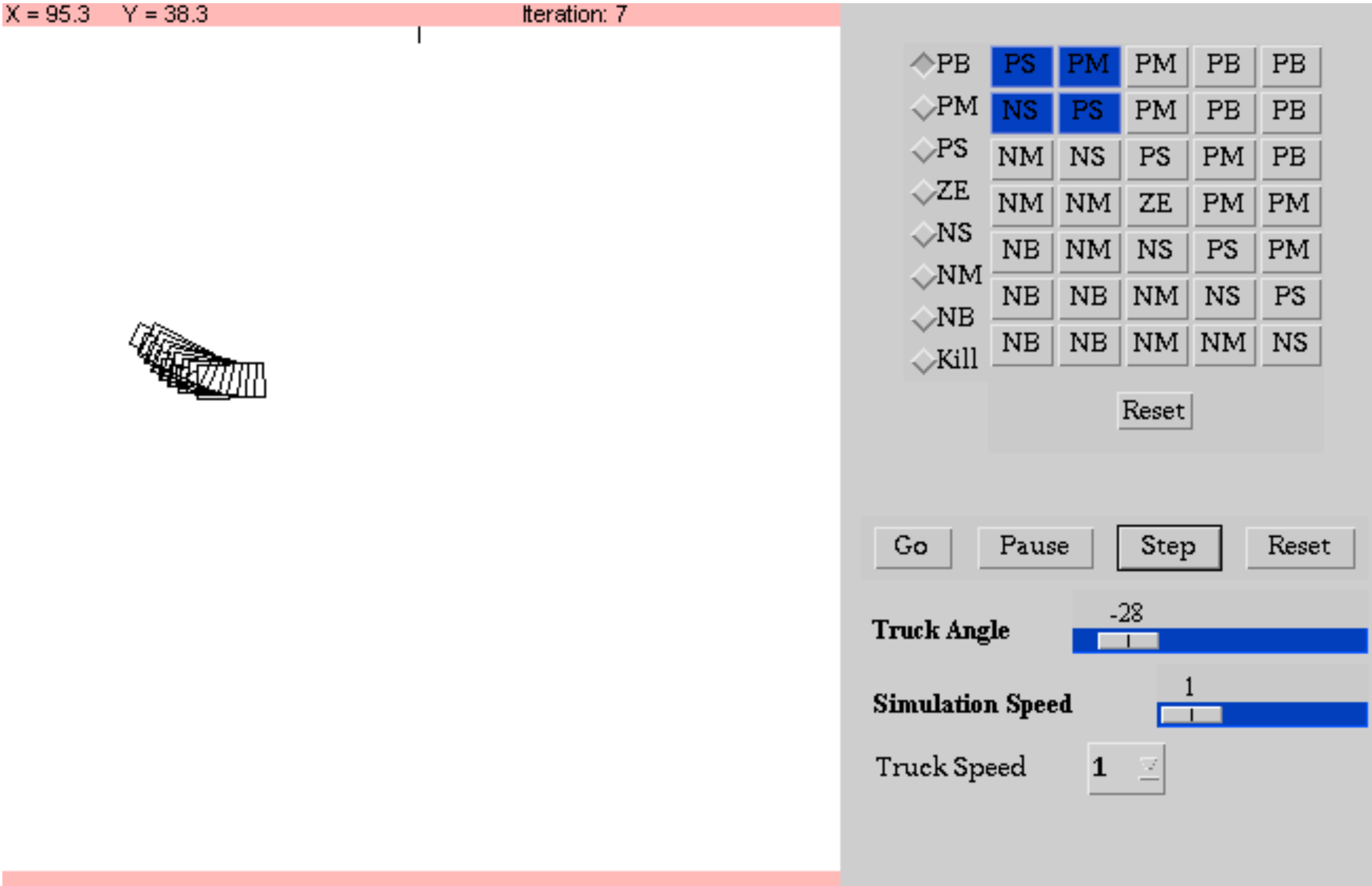


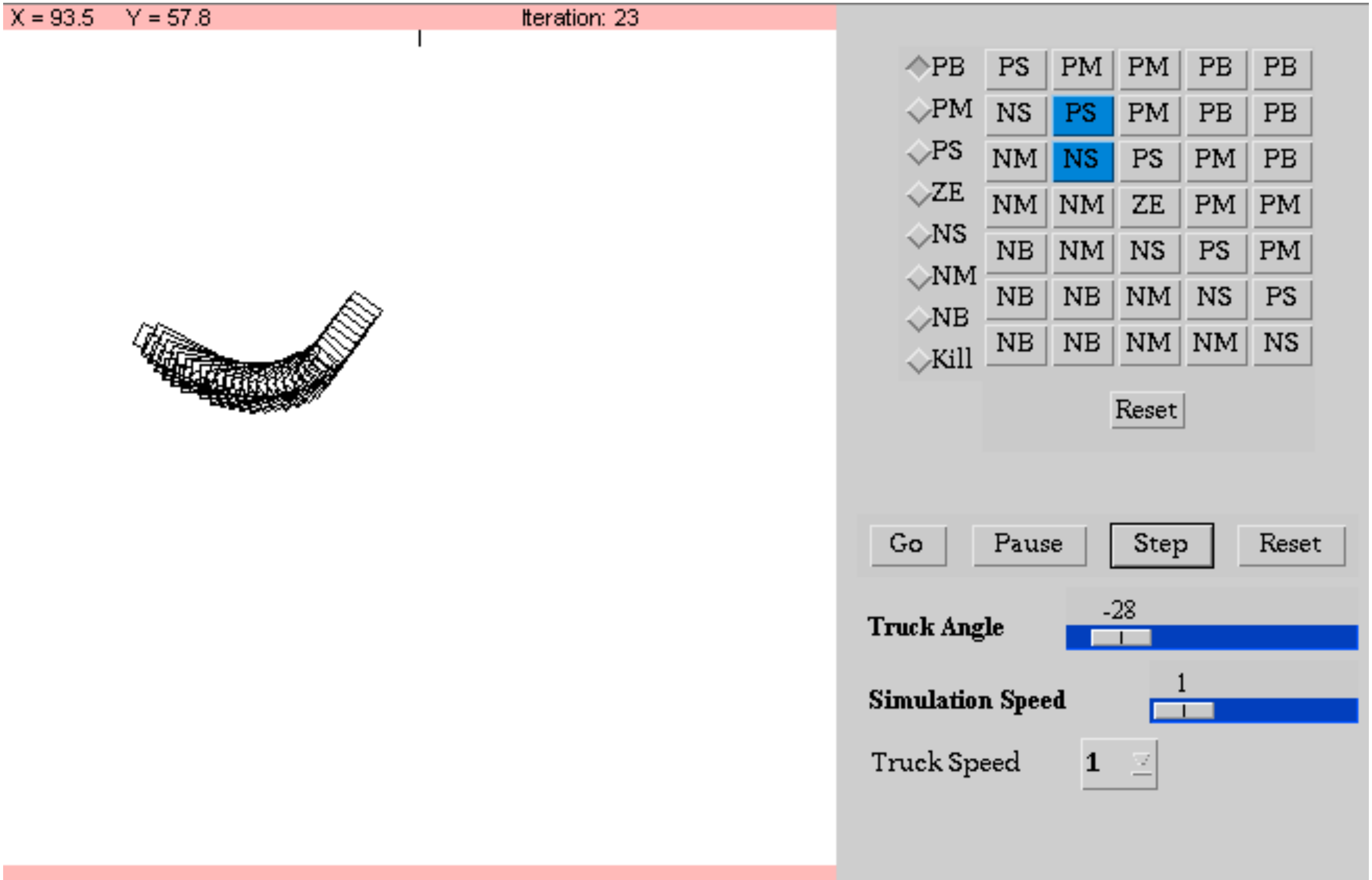
Fuzzy Truck-Backer Applet
/cs/cs152/fuzzy/truck

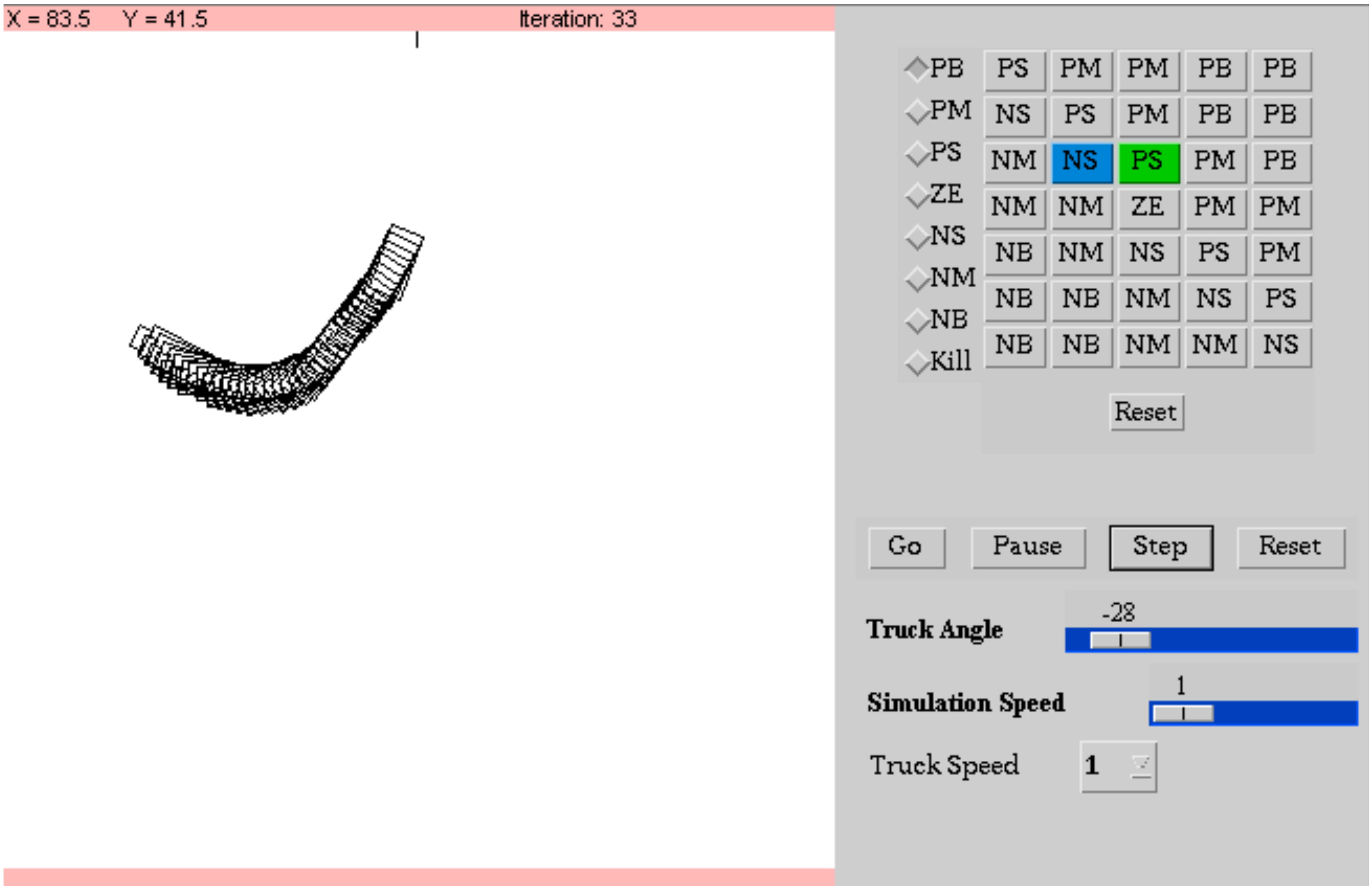


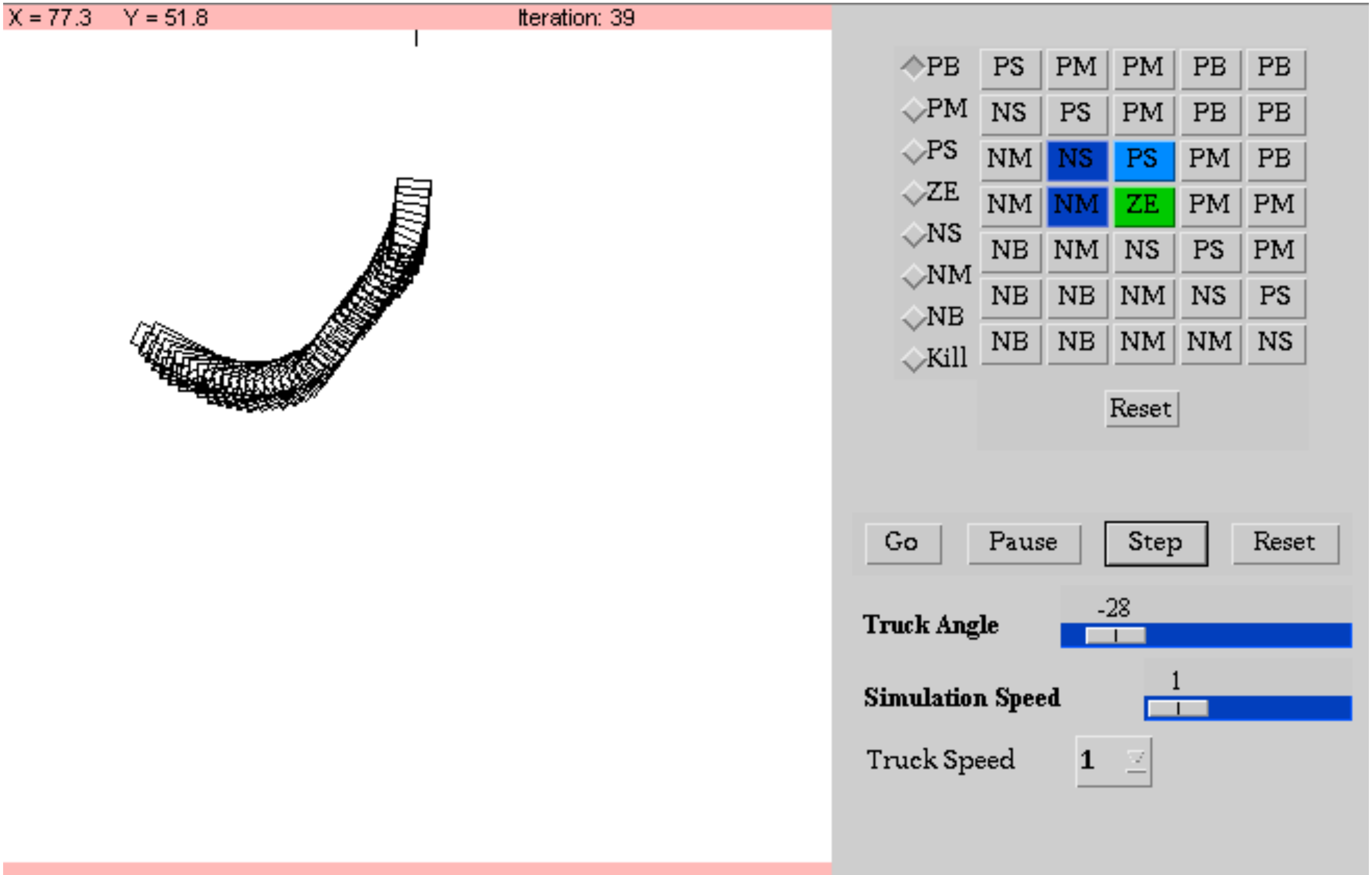
An example simulation

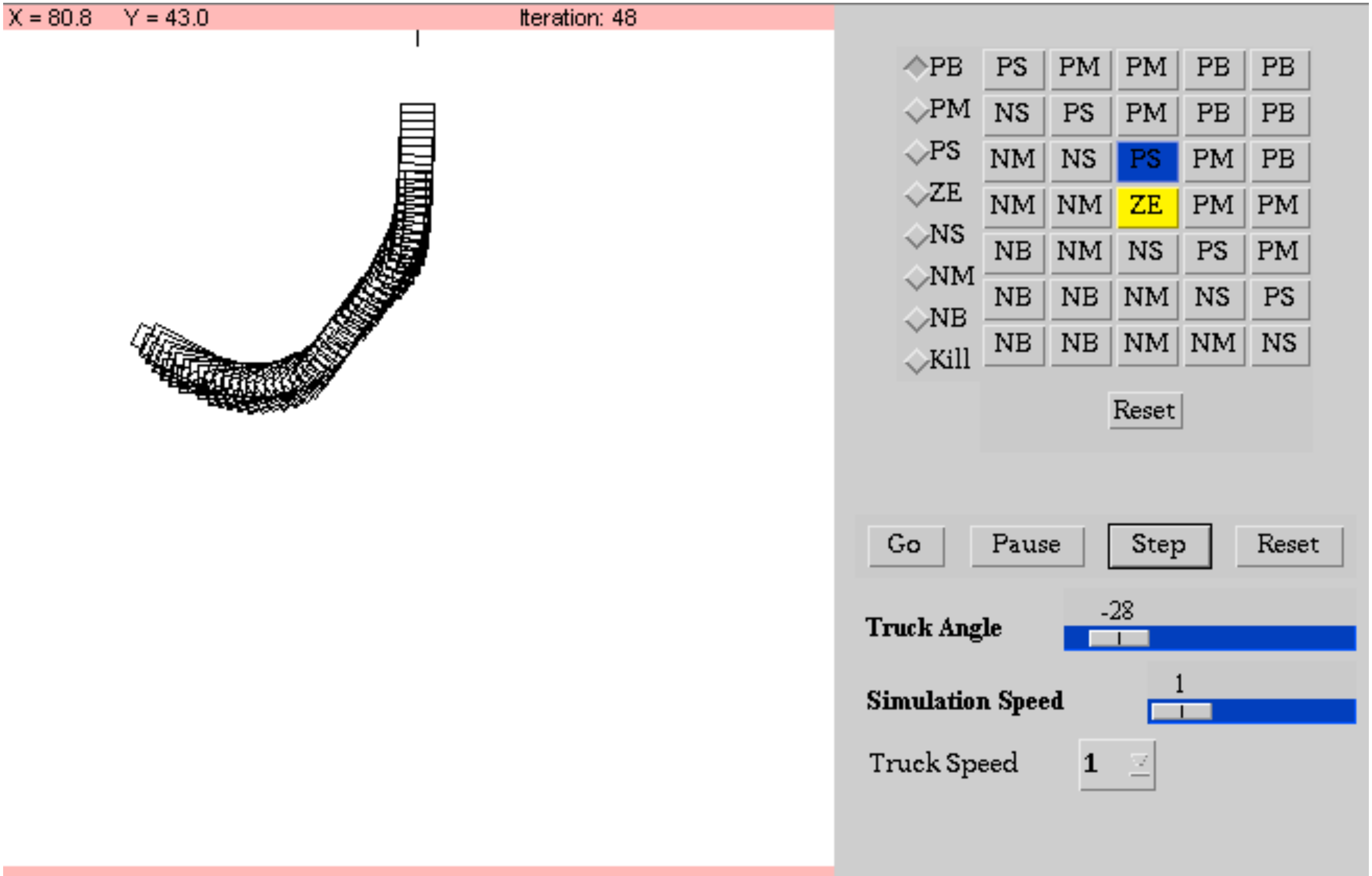


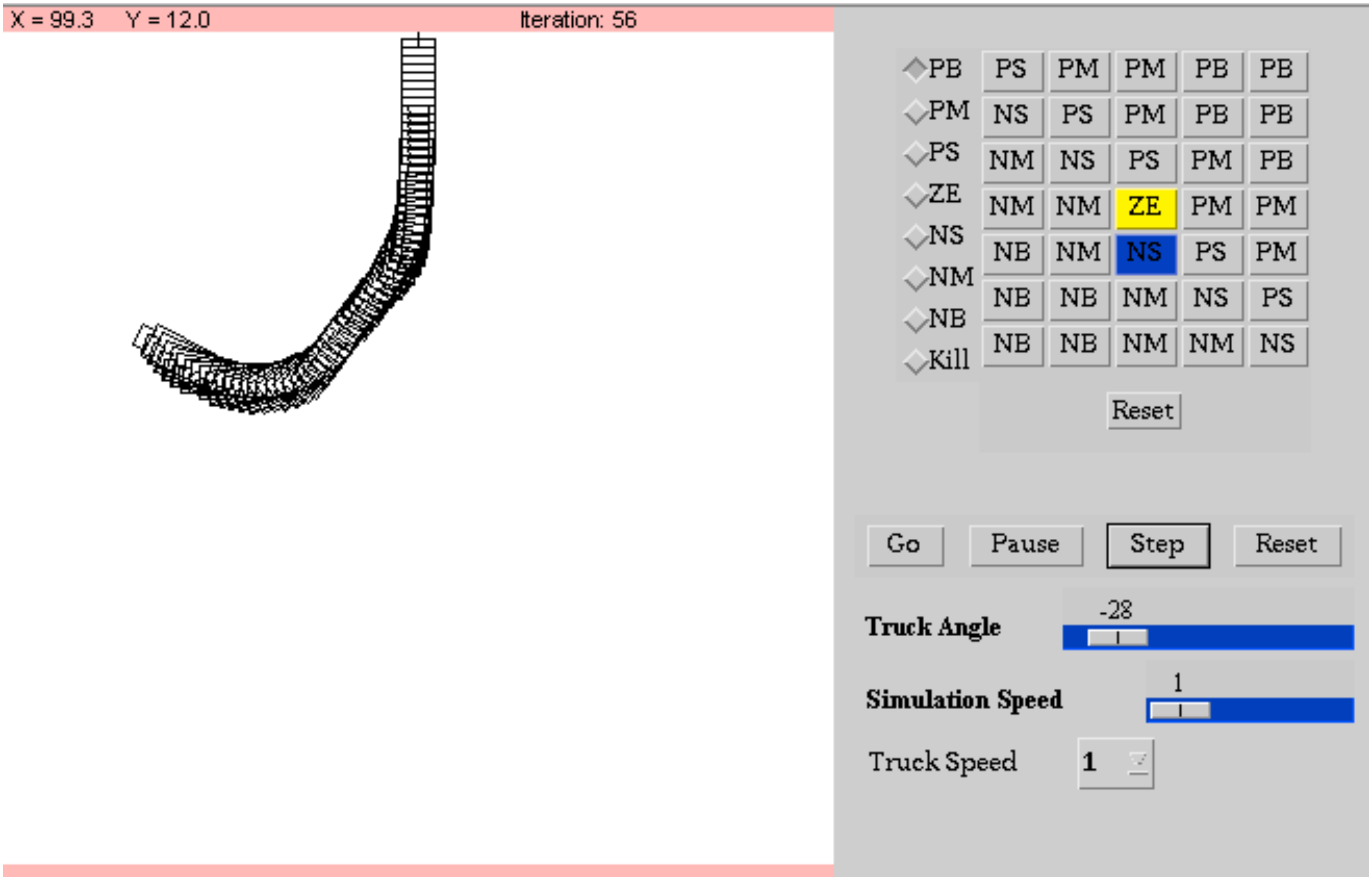

X = 95.3 Y = 38.3
Iteration: 7
PB
PM
PS
ZE
NS
NM
NB
Kill
PS PM PM PB PB
NS PS PM PB PB
NM NS PS PM PB
NM NM ZE PM PM
NB NM NS PS PM
NB NB NM NS PS
NB NB NM NM NS
Reset
Go
Pause
Step
Reset
Truck Angle
-28
Simulation Speed
1
Truck Speed
1


X = 93.5
Y = 57.8
Iteration: 23
PB
PM
PS
ZE
NS
NM
NB
Kill
PS PM PM PB PB
NS PS PM PB PB
NM NS PS PM PB
NM NM ZE PM PM
NB NM NS PS PM
NB NB NM NS PS
NB NB NM NM NS
Reset
Go
Pause
Step
Reset
Truck Angle
-28
Simulation Speed
1
Truck Speed
1


X = 83.5 Y = 41.5
Iteration: 33
PB
PM
PS
ZE
NS
NM
NB
Kill
PS PM PM PB PB
NS PS PM PB PB
NM NS PS PM PB
NM NM ZE PM PM
NB NM NS PS PM
NB NB NM NS PS
NB NB NM NM NS
Reset
Go
Pause
Step
Reset
Truck Angle
-28
Simulation Speed
1
Truck Speed
1

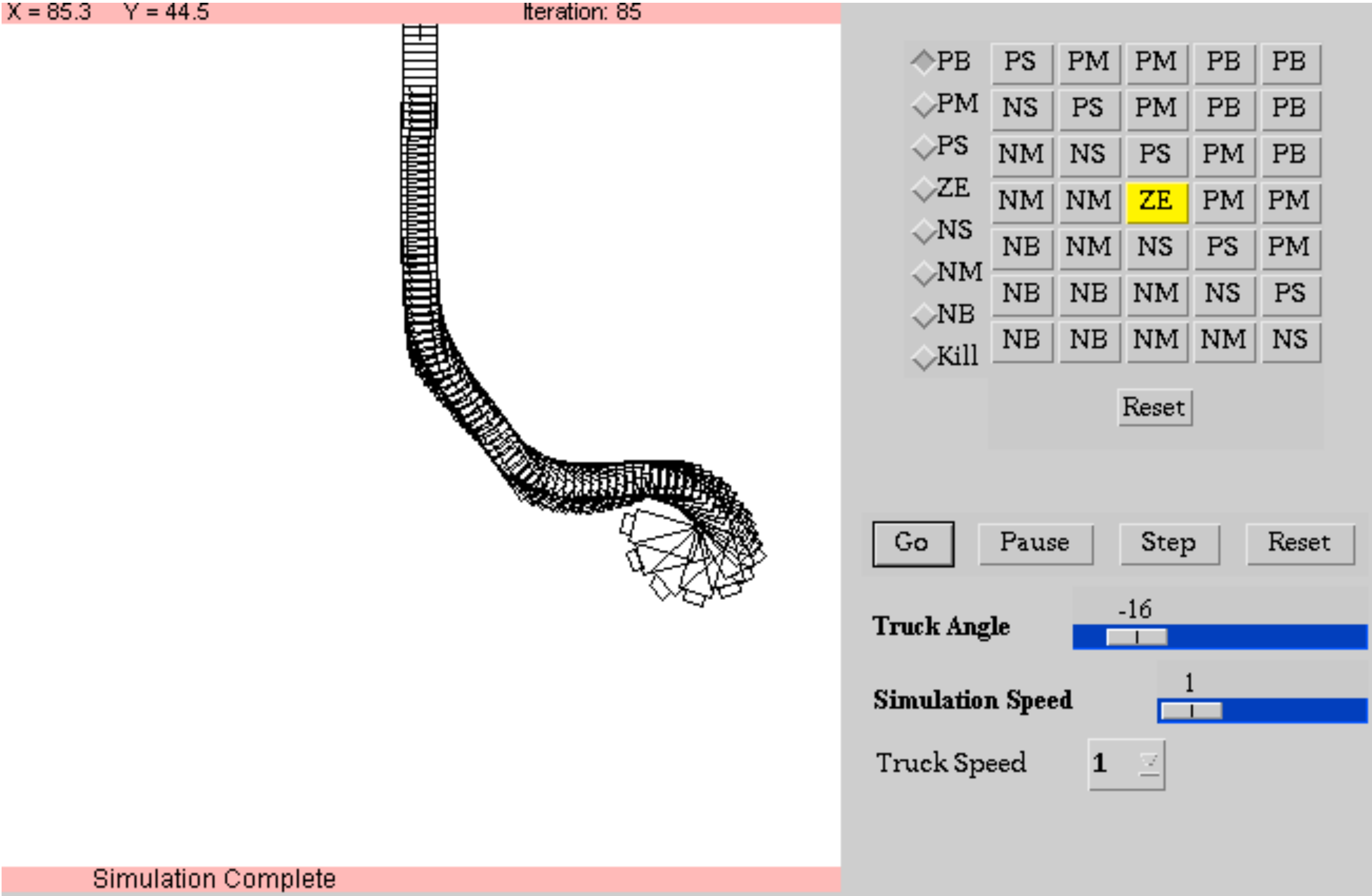

X = 77.3 Y = 51.8
Iteration: 39
PB
PM
PS
ZE
NS
NM
NB
Kill
PS PM PM PB PB
NS PS PM PB PB
NM NS PS PM PB
NM NM ZE PM PM
NB NM NS PS PM
NB NB NM NS PS
NB NB NM NM NS
Reset
Go
Pause
Step
Reset
Truck Angle
-28
Simulation Speed
1
Truck Speed
1


X = 80.8 Y = 43.0
Iteration: 48
PB
PM
PS
ZE
NS
NM
NB
Kill
PS PM PM PB PB
NS PS PM PB PB
NM NS PS PM PB
NM NM ZE PM PM
NB NM NS PS PM
NB NB NM NS PS
NB NB NM NM NS
Reset
Go
Pause
Step
Reset
-28
Truck Angle
1
Simulation Speed
Truck Speed
1

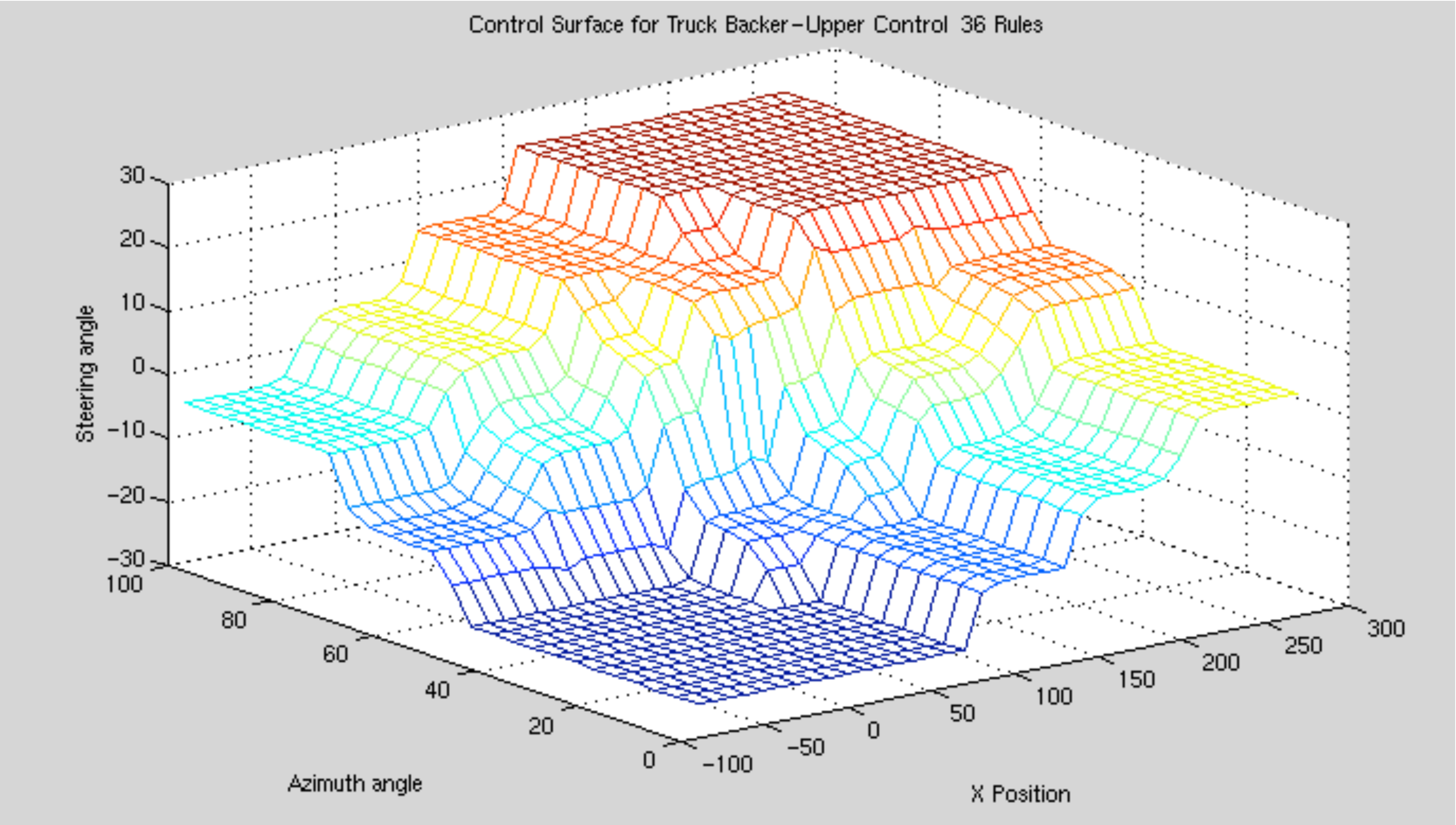


X = 99.3 Y = 12.0
Iteration: 56
PB
PM
PS
ZE
NS
NM
NB
Kill
PS PM PM PB PB
NS PS PM PB PB
NM NS PS PM PB
NM NM ZE PM PM
NB NM NS PS PM
NB NB NM NS PS
NB NB NM NM NS
Reset
Go
Pause
Step
Reset
Truck Angle
-28
Simulation Speed
1
Truck Speed
1

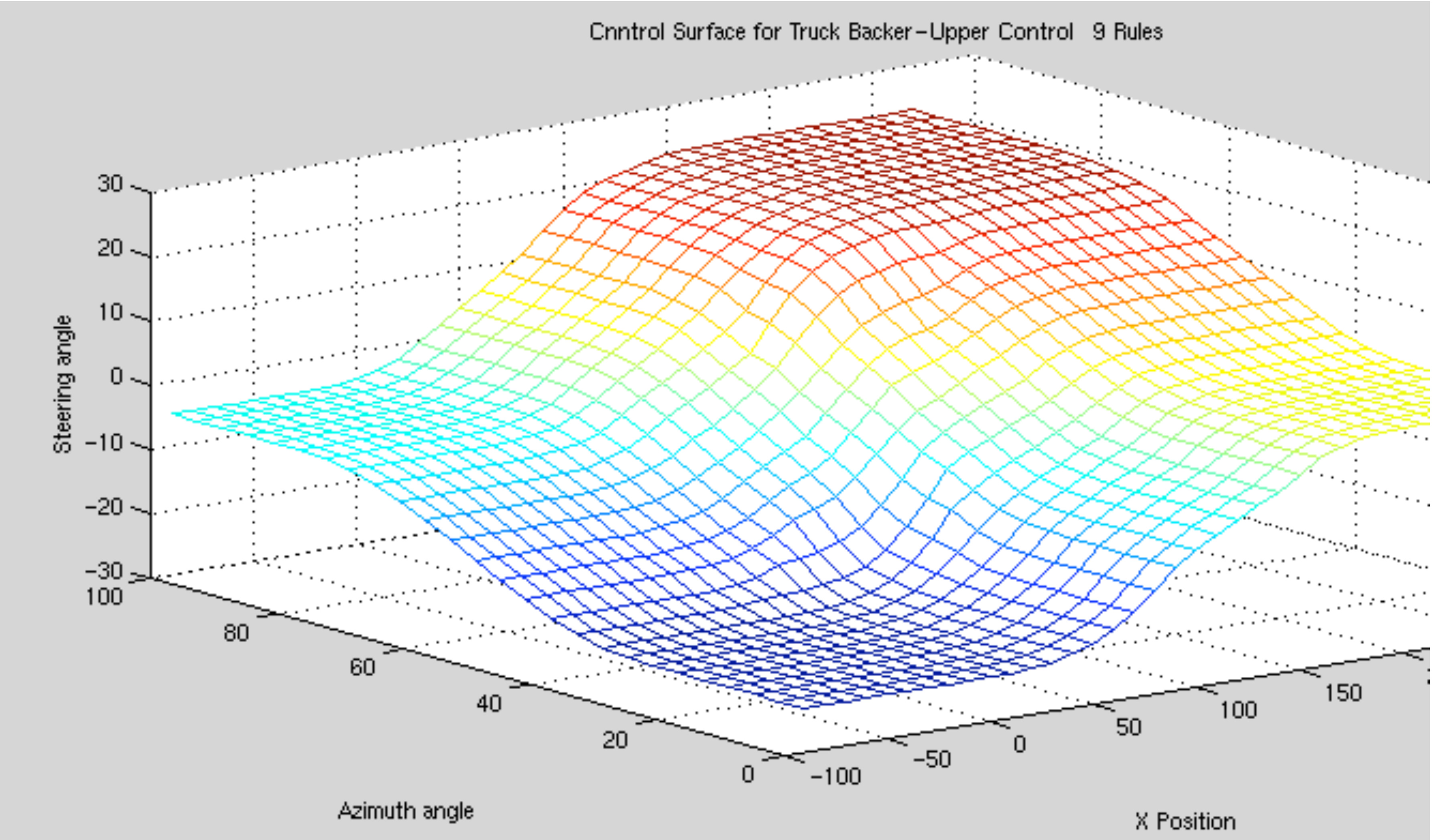
A different example simulation



Control Surface for another Truck-Backer

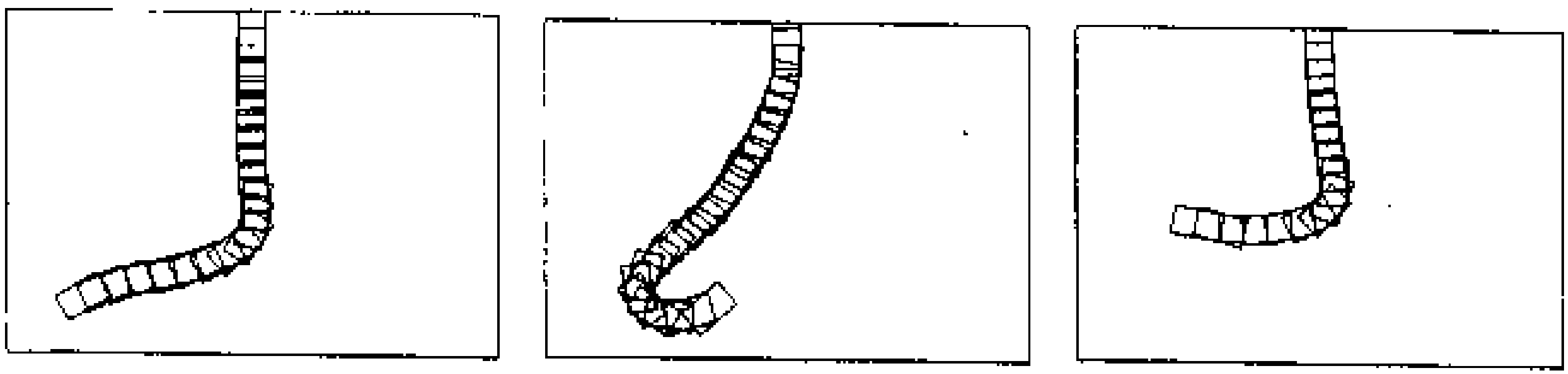


Another Truck Backer using only 9 Rules (/cs/cs152/fuzzy/fismat/fisdemo)



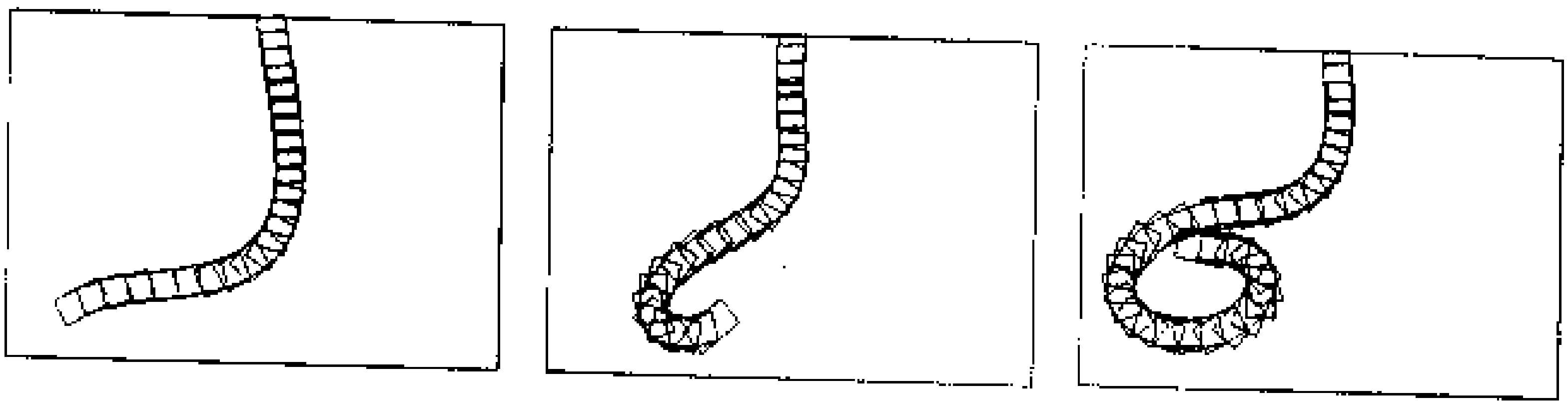
Kong & Kosko compared Fuzzy vs. Neural Controllers

Fuzzy



(a) (b) (c)

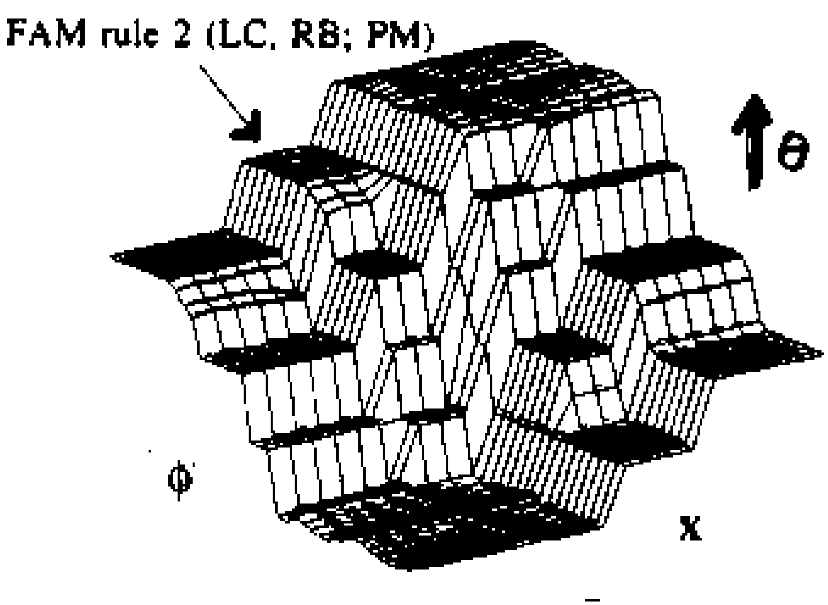
Neural



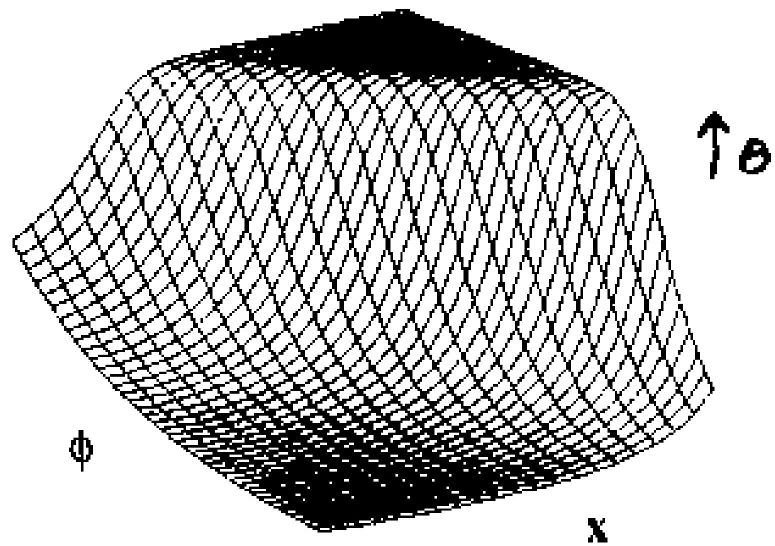
(a) (b) (c)

Kong & Kosko
Derived Controller Functions

Fuzzy



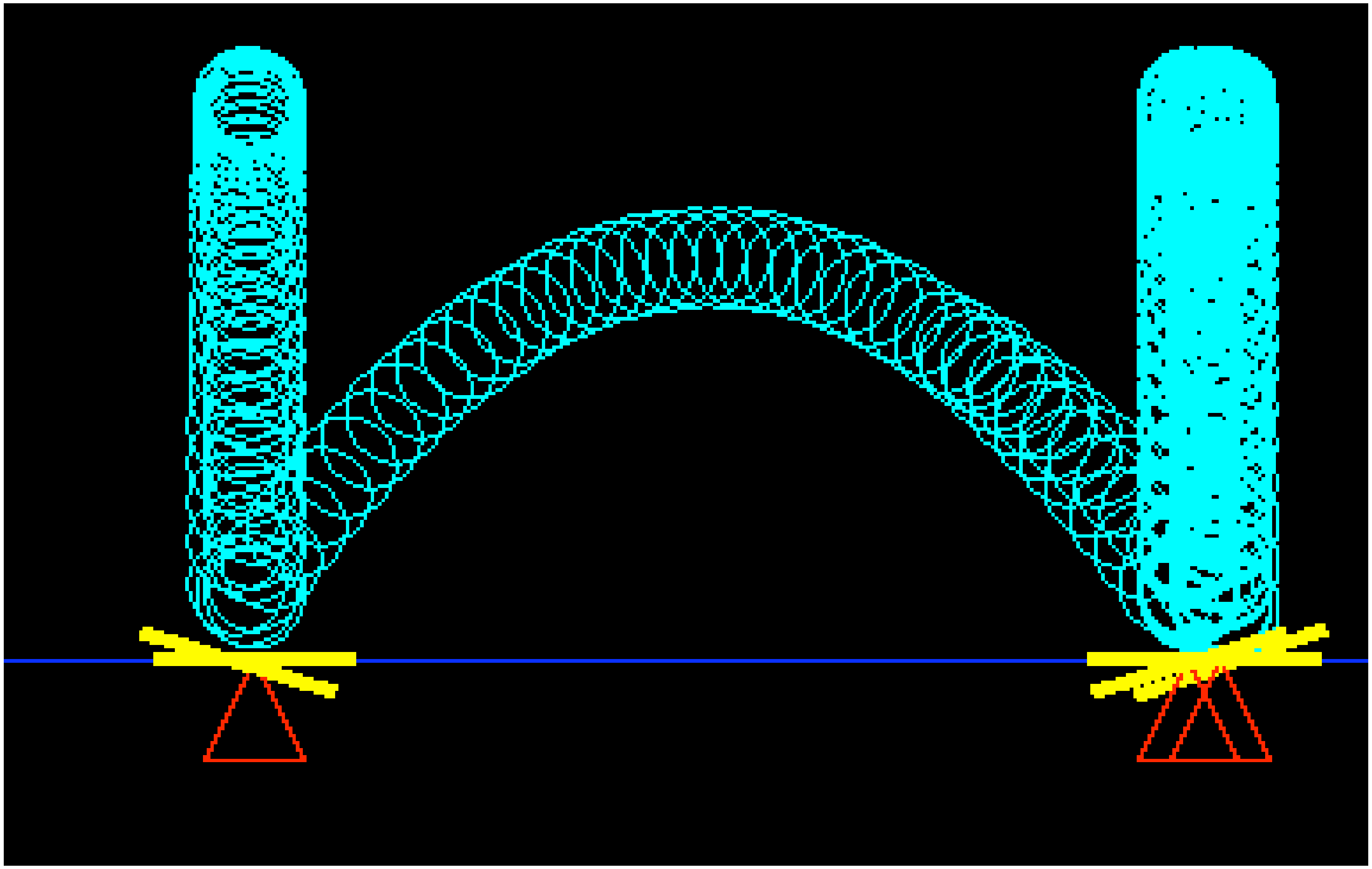
Neural



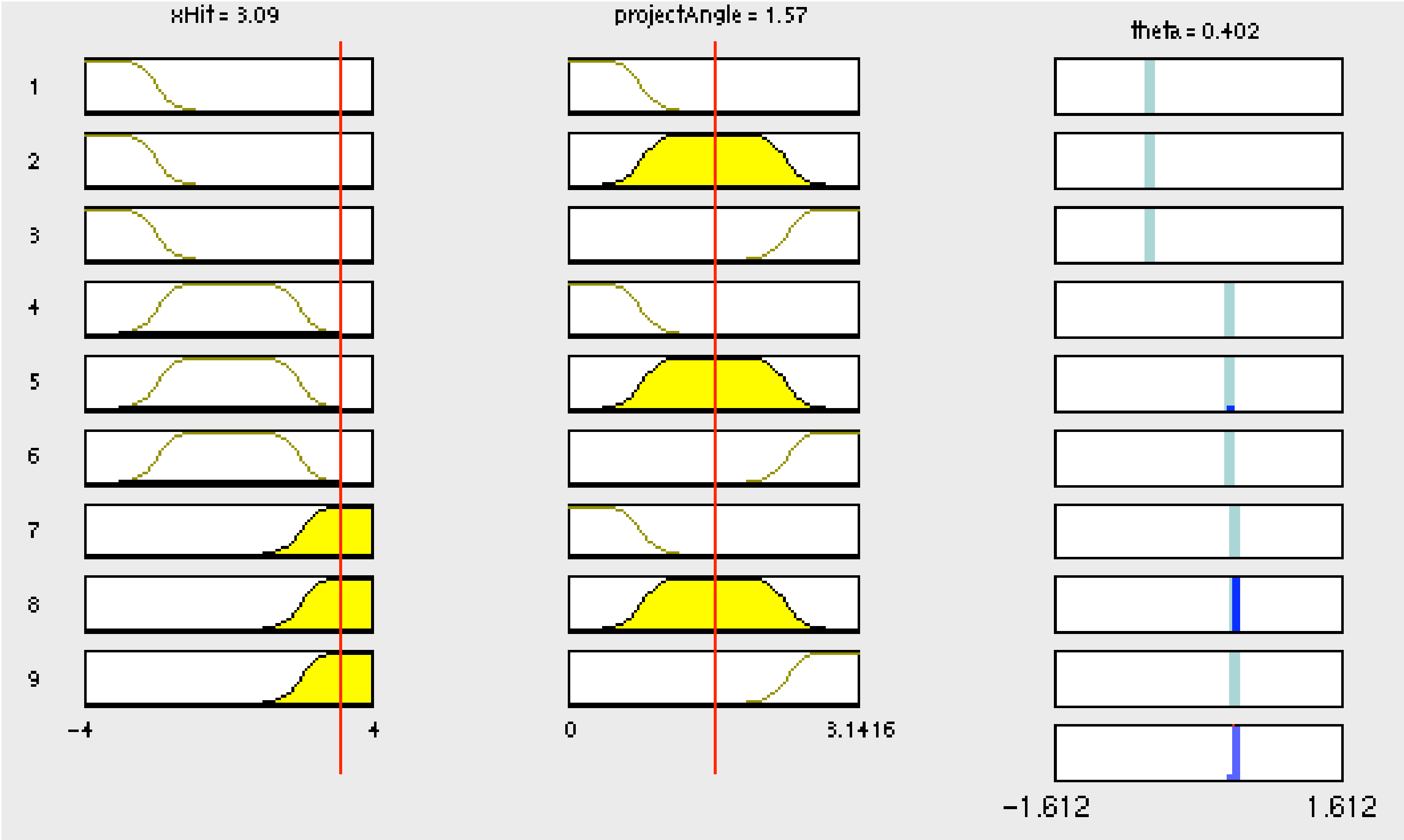
Kong & Kosko Konclusions

- Fuzzy
  - Regular path followed
  - “Trained” by common-sense hand-coded rules
  - Light-weight controller (comparisons and additions only)
- Neural
  - Sometimes followed irregular path
  - Training time-consuming
  - Controller computationally intensive

Matlab Fuzzy Ball-Juggler Demo



Ball-Juggler Rule View



Application to Game Programming

(cf. http://www.byondscape.com/ascape.dmb/Gazoot.2004-0315/)

A typical rule based on crisp logic:

```
DetermineAction()
        if(health < 10)
                if(ammo < 5) return ESCAPE
                if(ammo >= 5 && ammo < 20) return CAUTIOUS
                else return NORMAL
        else if(health >= 10 && health < 20)
                if(ammo < 5) return CAUTIOUS
                if(ammo >= 5 && ammo < 20) return NORMAL
                else return ATTACK
        else
                if(ammo < 5) return NORMAL
                if(ammo >= 5 && ammo < 20) return ATTACK
                else return FULL_ATTACK
```

Application to Game Programming

Fuzzy rule-base table (example):

Ammo	1.0	Low Then Escape	Medium Then Defence	High Then Attack
Health	1.2	Low Then Escape	Medium Then Defence	High Then Attack
Enemies in sight (I)	1.0	Many Then Escape	Some Then Defence	Few Then Attack

Possible Fuzzy-NN Connections

(from http://en.wikipedia.org/wiki/Neuro-fuzzy)

- Fuzzy logic based tuning of neural network training parameters.
- Fuzzy logic criteria for increasing a network size.
- Representing fuzzification, fuzzy inference and defuzzification through multi-layers feed-forward connectionist networks.
- Realizing fuzzy membership through clustering algorithms in unsupervised learning in SOMs and neural networks.
- Deriving fuzzy rules from trained RBF networks.

Applications of Fuzzy Logic to Training MLP Neural Networks

- Control of **variable learning rate in backpropagation** (Choi, et al.):
- Let **CE** denote Change of Error.
- Let **CCE** denote Change of CE.
- Fuzzy Rules:
  - If CE is small with no recent sign changes, then increase LR.
  - If CE has recent sign changes, then decrease LR.
  - If CE is small, and CCE is small with no recent sign changes, then increase LR and momentum.

Application of Neural Networks to Fuzzy Logic

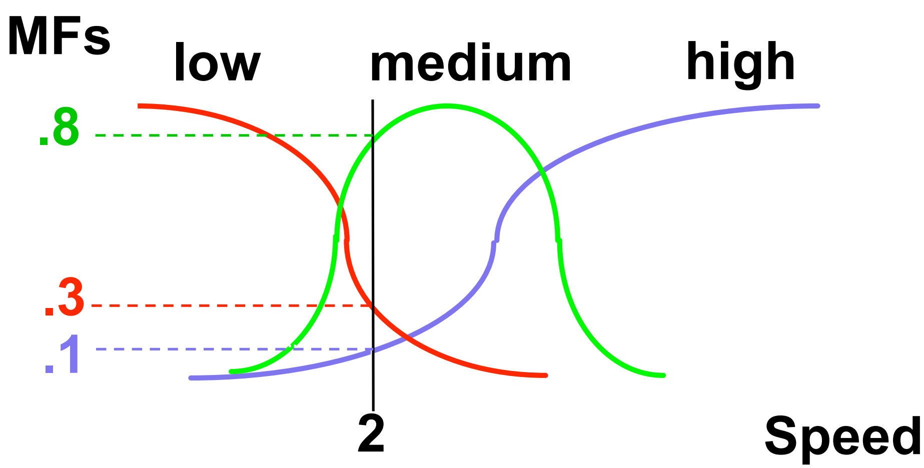
- N. K. Kasabov. Learning Fuzzy Rules through Neural Networks. Proc. of the First New Zealand Intl. Conf. on Artificial Neural Networks and Expert Systems, pages 137--139, 1993.
- ***Learned*** fuzzy rules for **forecasting gas demand**.
- Used backpropagation network with five layers.
- The input layer contains two nodes - temperature and the month and the output layer is the gas demand figure.
- **Fuzzy rules were then extracted from the net**.

Sugeno Control Model

- Extends Mamdani model
- Fuzzy part is still in antecedent of rules, which are used for selection
- Consequent of rules is more complex: some function (e.g. polynomial) of input variables

Fuzzy Inference System Using Sugeno-Style Rules

If speed is low then resistance = 2
If speed is medium then resistance = 4*speed
If speed is high then resistance = 8*speed



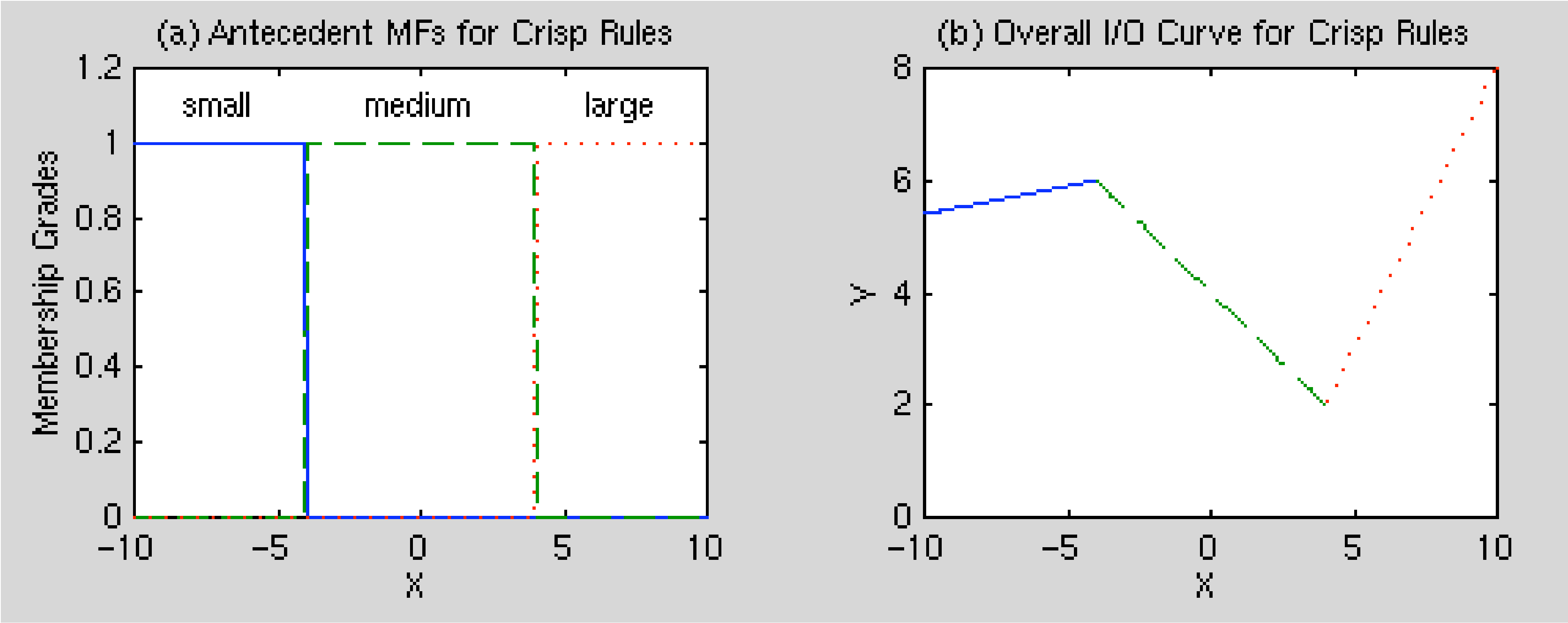
Rule 1: w1 = .3; r1 = 2
Rule 2: w2 = .8; r2 = 4*2
Rule 3: w3 = .1; r3 = 8*2

Resistance = Σ(wi*ri) / Σwi
= 7.12

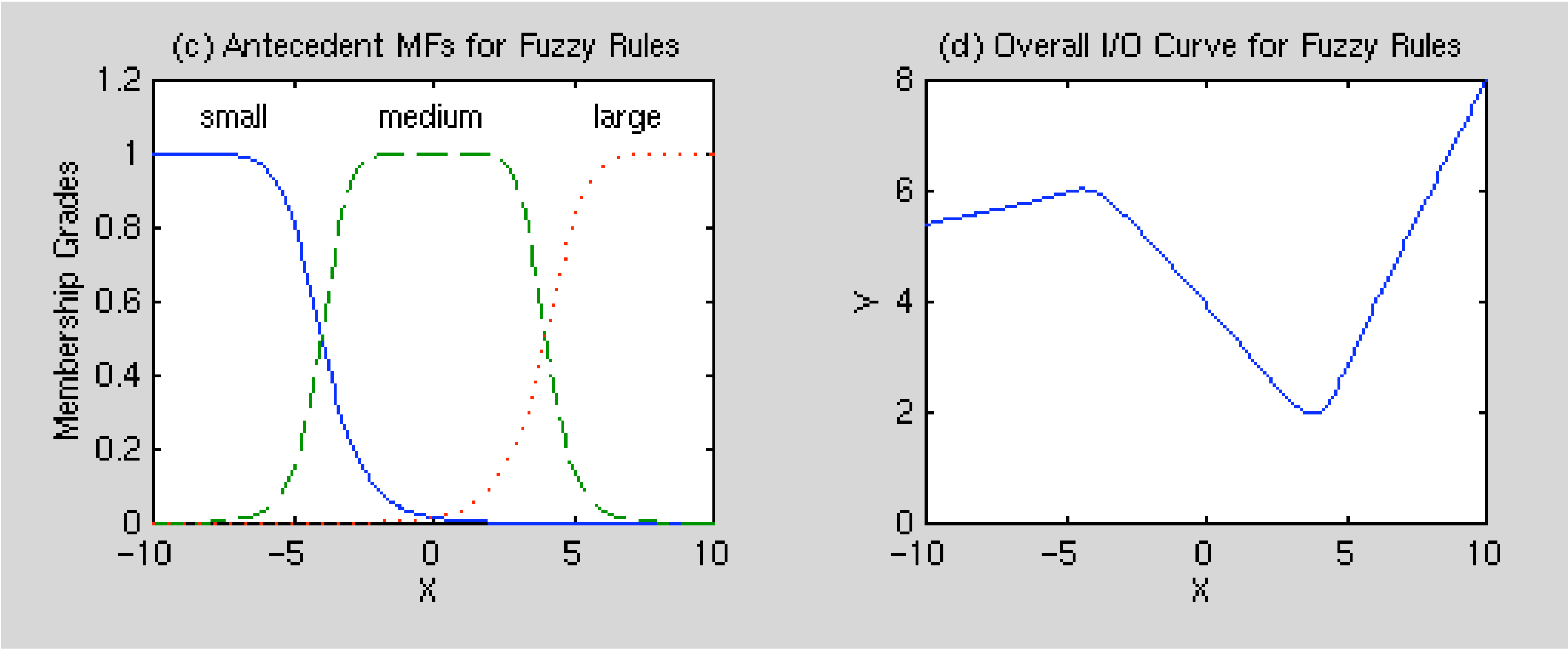
Example: 1-input Sugeno

- Rules:
  - If X is small, then Y = 0.1X + 6.4.
  - If X is medium, then Y = -0.5X + 4.
  - If X is large, then Y = X-2.
- The following 2 slides indicate the results of combining these rules using crisp **vs**. fuzzy logic.

demo sug1 (crisp)
(in /cs/cs152/matlab/soft)



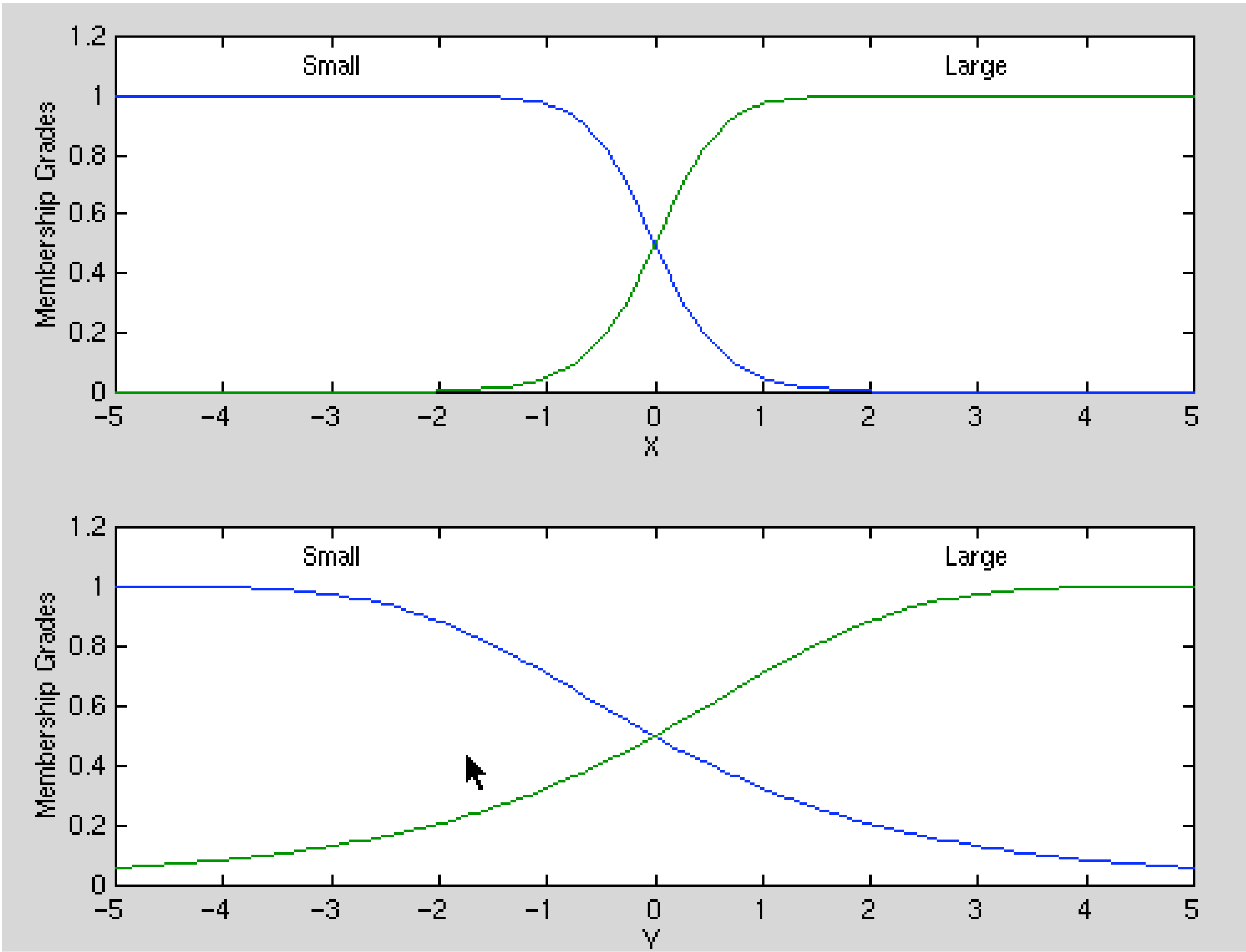
demo sug1 (fuzzy)



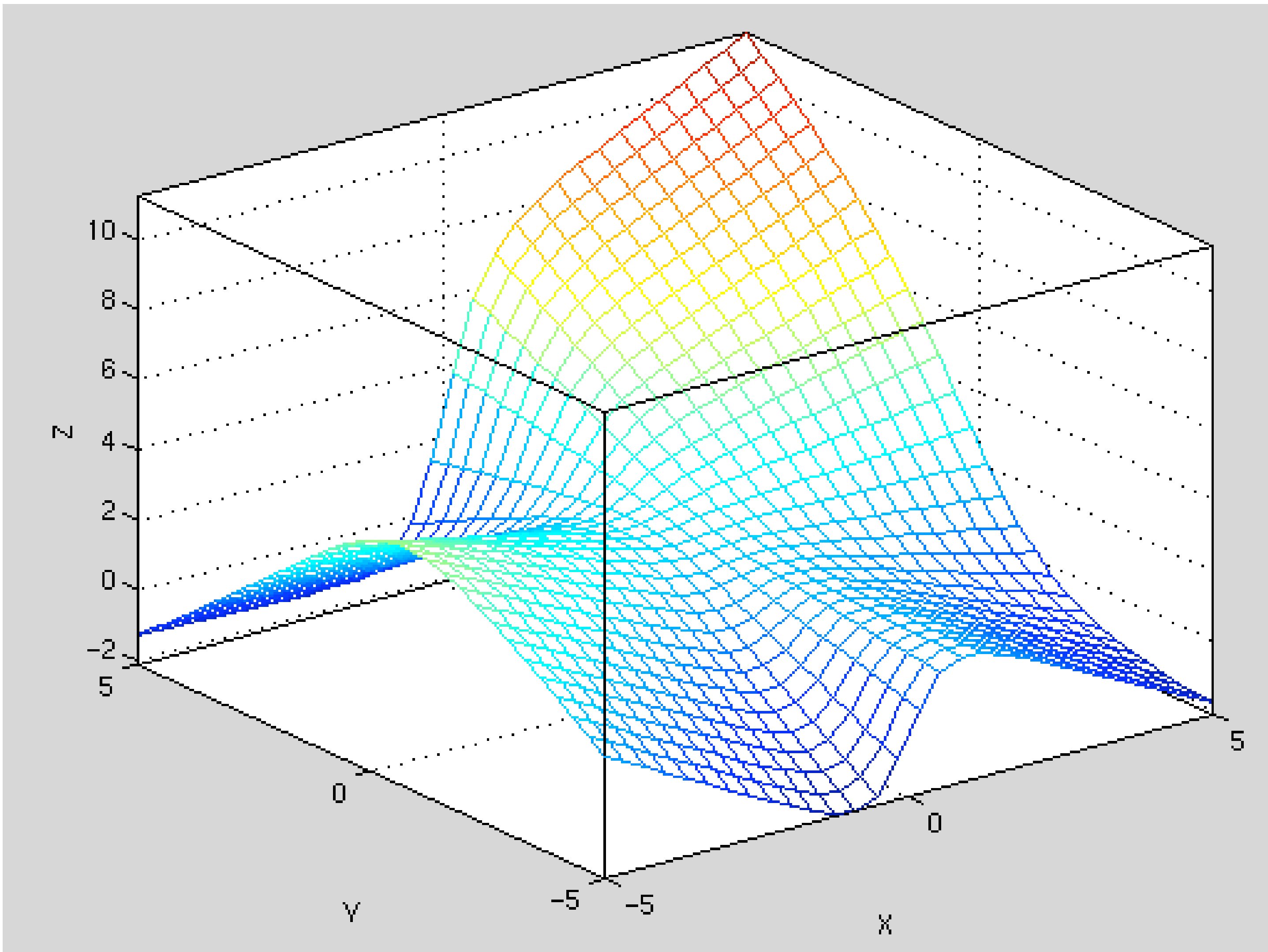
Example: 2-input Sugeno

- Rules:
  - If X is small and Y is small, then Z = -X+Y+1.
  - If X is small and Y is large, then Z = -Y+3.
  - If X is large and Y is small, then Z = -X+3.
  - If X is large and Y is large, then Z = X+Y+2.

MFs for demo sug2 (2-input)



Control surface for sug2



Tsukamoto model

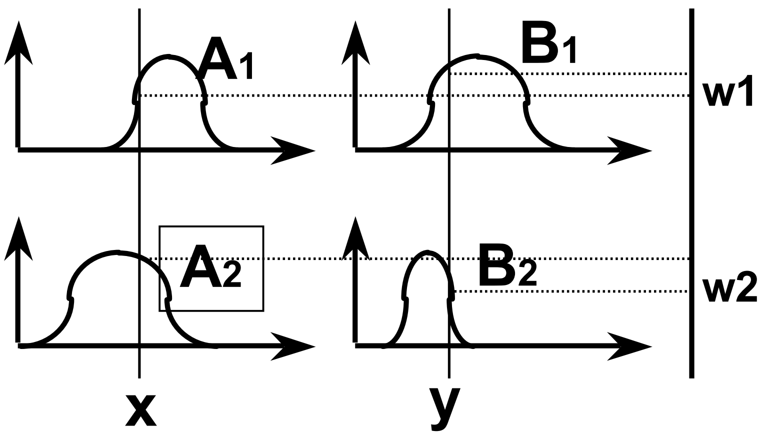
- Aggregate rule outputs by a **weighted average**, rather than by defuzzification.

Example of Hybrids: ANFIS
(Adaptive Neuro-Fuzzy Inference System)

- Developed by J.-S. R. Jang
- Uses Sugeno or Tsukamoto models
- Similar to Radial Basis Function network

ANFIS

- **Fuzzy reasoning**

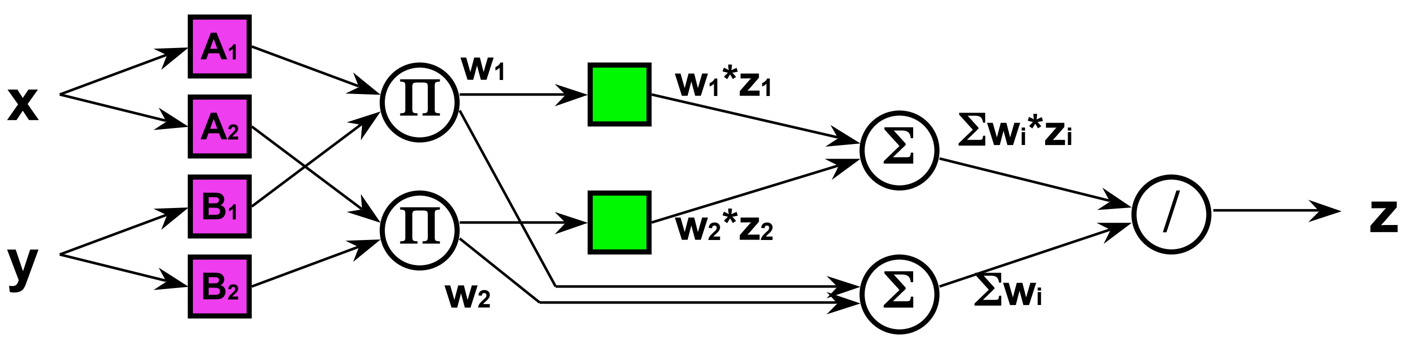


$$z_1 = p_1 * x + q_1 * y + r_1$$

$$z_2 = p_2 * x + q_2 * y + r_2$$

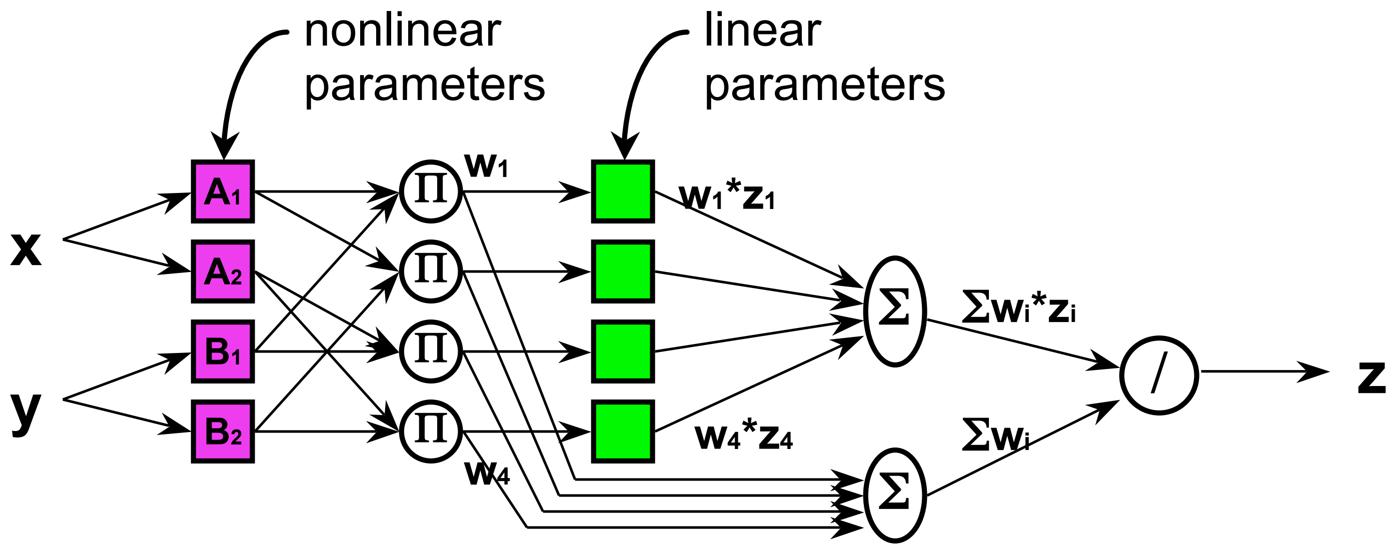
$$z = \frac{w_1 * z_1 + w_2 * z_2}{w_1 + w_2}$$

- **ANFIS (Adaptive Neuro-Fuzzy Inference System)**



ANFIS: Parameter ID

- Hybrid training method



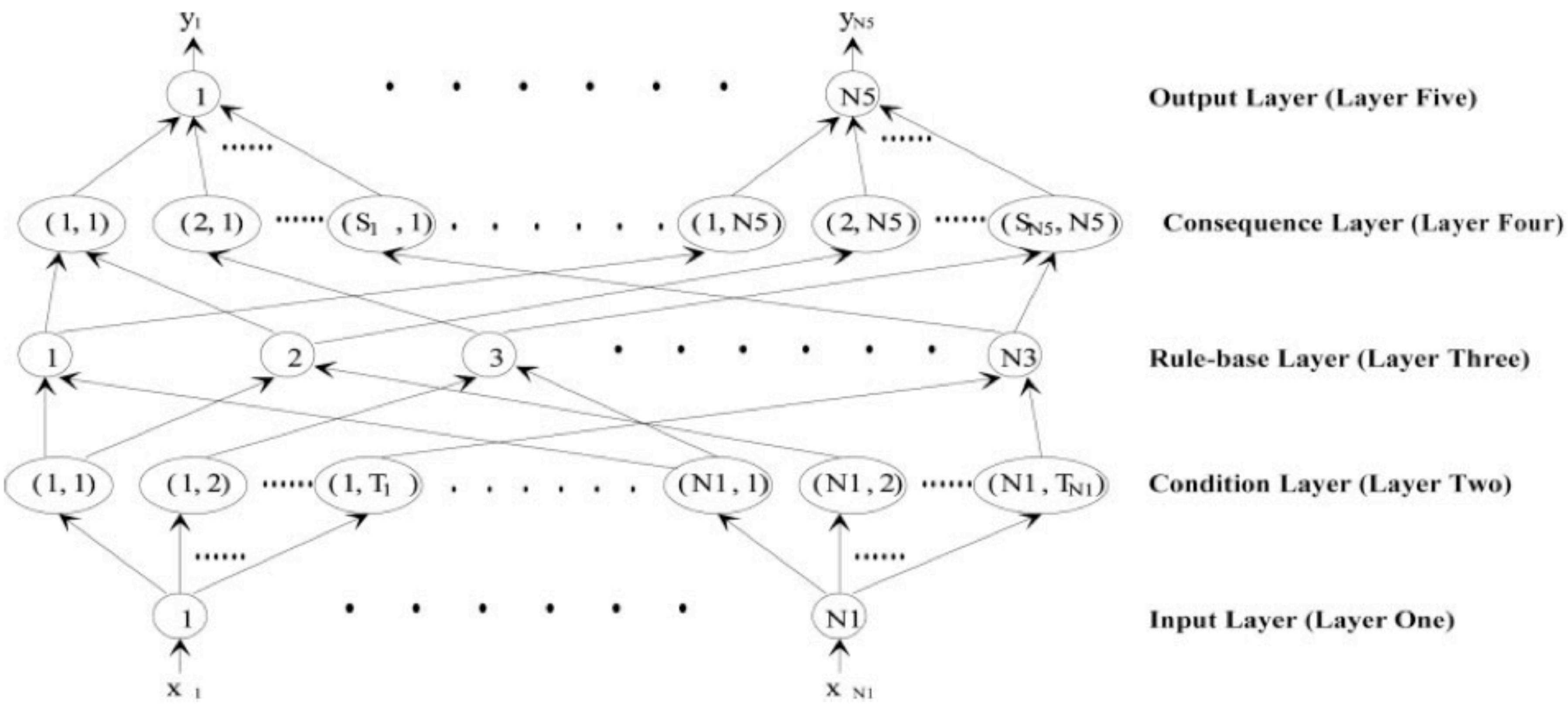
	forward pass	backward pass
MF param. (nonlinear)	fixed	steepest descent
Coef. param. (linear)	least-squares	fixed

Pseudo-outer product based fuzzy neural network (POPFNN)

Applied to Fingerprint Classification by

Zhou & Quek, 1966

POPFNN Structure



Layers in POPFNN

- Layer 1: Linguistic nodes: Inputs are non-fuzzy linguistic variables.
- Layer 2: Conditional layer: Convert inputs to fuzzy values.
- Layer 3: Rule nodes (fuzzy rule base)
- Layer 4: Consequence layer: Combine rule outputs.
- Layer 5: Output classification/confidence.

3 Learning Phases in POPFNN

- Self-organization: Initialize membership functions by determining widths and centroids.
- POP learning: Identify relevant fuzzy rules, based on Hebbian principle.
- Supervised learning: Fine-tuning using backpropagation.

Examples of POPFFN Rules for Fingerprint Authentication

Rule 1: if x_0 is large and x_1 is large, then 'fingerprint is authentic' is 1.000000 true;
Rule 2: if x_0 is median and x_1 is median, then 'fingerprint is authentic' is 0.959801 true; and
Rule 3: if x_0 is small and x_1 is small, then 'fingerprint is authentic' is 0.618143 true.

Critiques of Fuzzy Logic

- Google on the title for several examples.
- One of the harder ones follows (next slide).
- You should read the full critique before going gung-ho Fuzzy Logic for control systems.
- Yet FL will no doubt have its uses in information processing.

Contrary Opinion

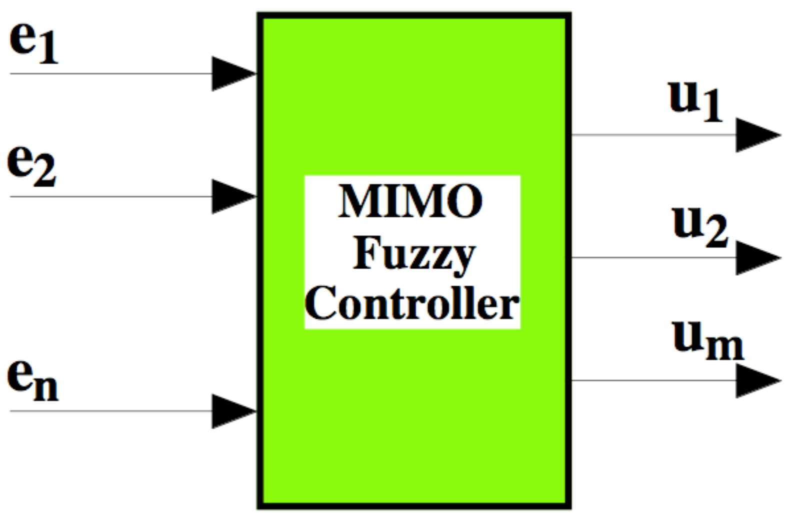
WHY I DESPISE FUZZY FEEDBACK CONTROL,
by Michael Athans (EECS Prof. at MIT)

http://fuzzy.iau.dtu.dk/download/athans99/athans99.ppt
also http://fuzzy.iau.dtu.dk/download/athans99/index.htm

Asserts, among other things, that fuzzy control only shown applicable to trivial (SISO) systems, not more complex (MIMO) systems.

Athans' Asserted Shortcomings of Fuzzy Controllers

- Fuzzy rules just generate nonlinear static functions.
- Performance specifications "vague" or nonexistent.
- Cannot generate "differential equation" controller rules.
- Not easy to differentiate noisy sensor signals by finite differencing, as almost always done in fuzzy applications.
- No utilization of dynamic filtering (e.g. Kalman filtering).
- Athans "has never seen a multiple-input multiple-output (MIMO) fuzzy control application".
- Combinatorial complexity for high-order and multivariable applications.



$$u_1 = h_1(e_1, e_2, \ldots, e_n)$$

$$u_2 = h_2(e_1, e_2, \ldots, e_n)$$

....

$$u_m = h_m(e_1, e_2, \ldots, e_n)$$

Prof. Zadeh responds

(http://www.lavoisier.fr/notice/gbEKO23RXA2OSSLO.html)

"Zadeh said that Athans had painted quite a picture, but one in which he could not recognize fuzzy control, and that the picture was not connected to reality. He immediately noted the large number of books on fuzzy control and stated that if the audience members took it upon themselves to read these books, they would see that many of the issues Athans had brought up were resolved by these texts. In the question-and-answer period that followed, the fuzzy control panel members asked questions of Athans and the conventional control panel members asked questions of Zadeh. As time remained, each participant fielded one additional question. Following the question-and-answer period, the participants made their 3-minute closing statements. The debate then ended with a handshake between the two old friends."