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Principal Component Analysis  
(PCA)

and

PCA Neural Networks:  
An Application of Unsupervised Hebbian Learning

and intro to  
Independent Components Analysis (ICA)

# What is PCA?

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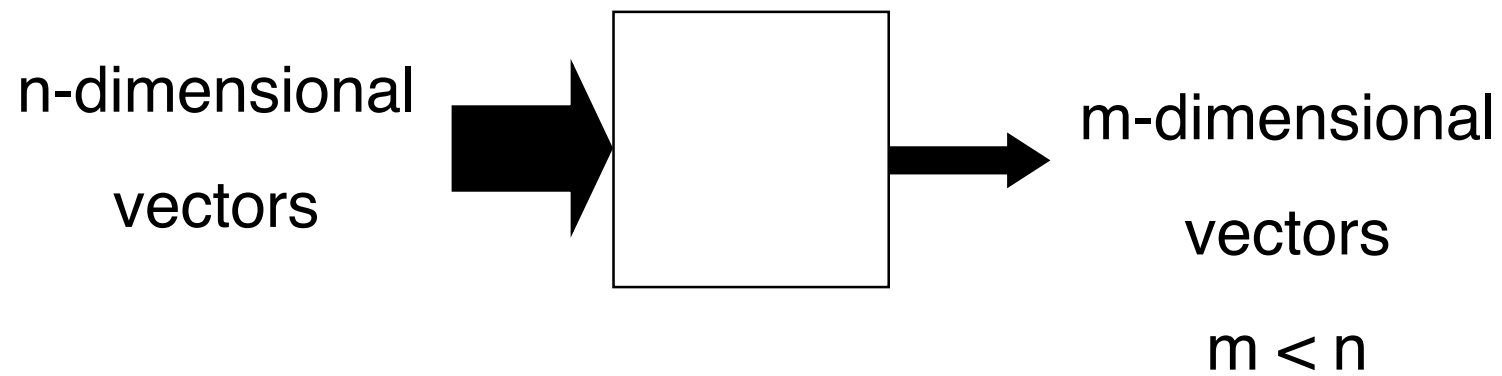
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- A standard statistical technique for reducing dimensionality of data (without using neural approach).
- Purpose: Better understanding or communication of data; used a lot in the sciences to select the most important features.
- In so reducing, we want to lose as little information as possible, given the before- and after- dimensions.
- Also known as **Karhunen-Loeve (K-L) transformation** (Watanabe, 1969).
- Could be used to **preprocess** input to, say, MLP.

# PCA

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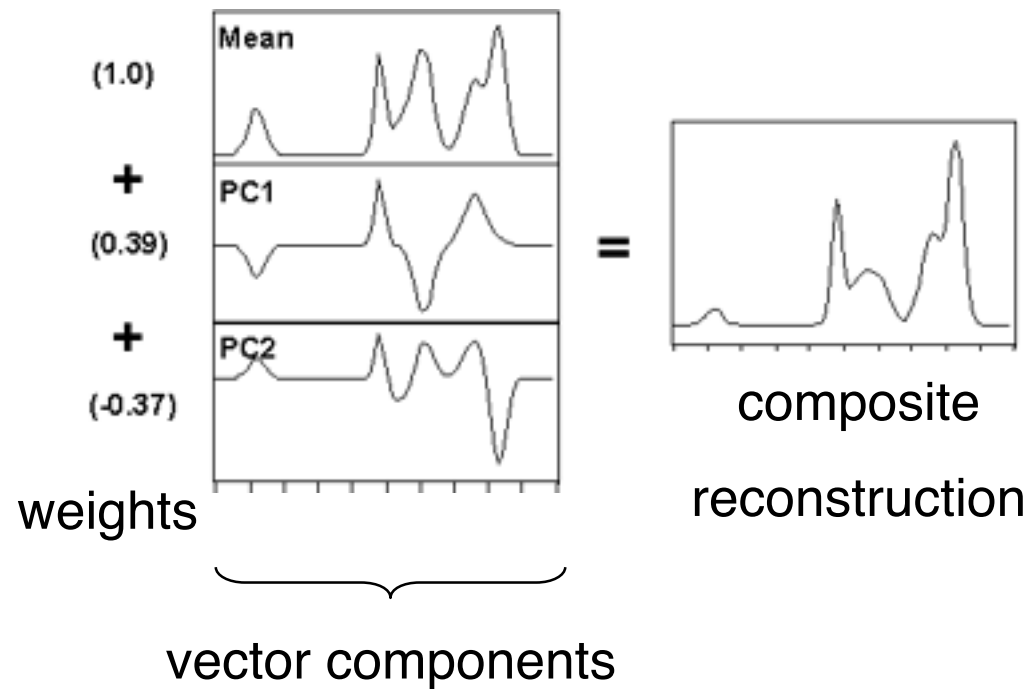
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# Reconstruction from Components

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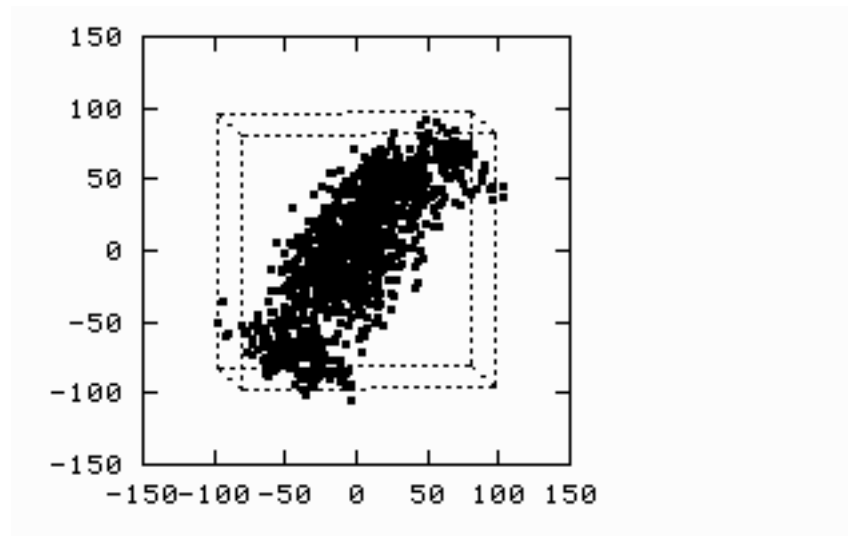
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# Scientific Uses

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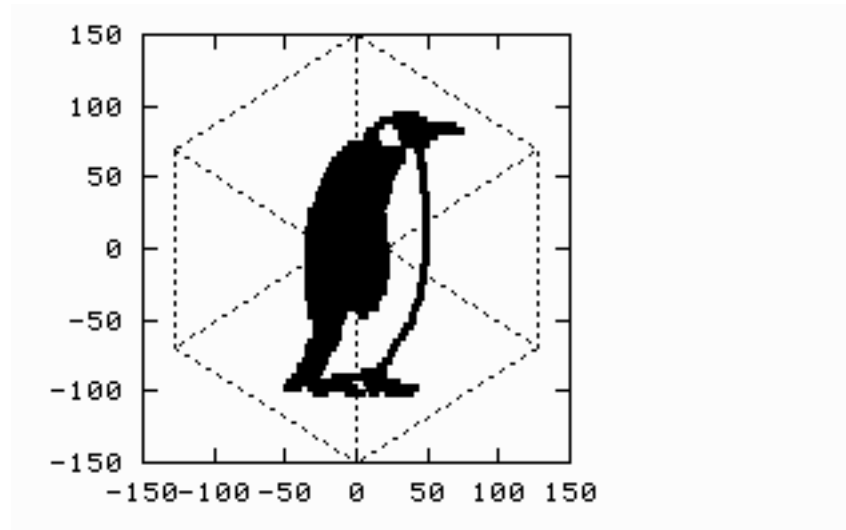


Transform coordinates to get a better understanding of the underlying phenomena.

# Scientific Uses

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Transform coordinates to get a better understanding of the underlying phenomena.

## Comparison with Linear Regression

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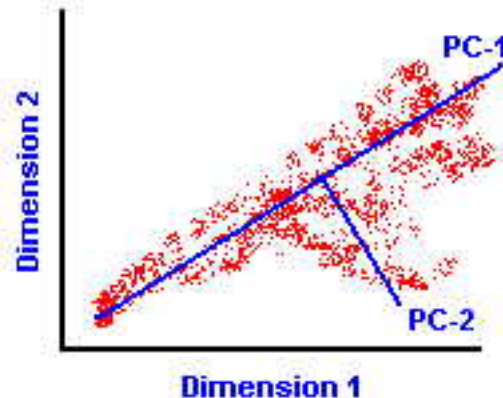
- Linear regression requires one to ***pre-identify*** dependent vs. independent variables.
- PCA does not.

# What is a “Principal Component”?

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- If we were to plot the data, the **first principal component** would be the **predominant direction** in the data.
- The second principal component would be the second-most predominant direction, etc. up to the number of **dimensions** of the data points.

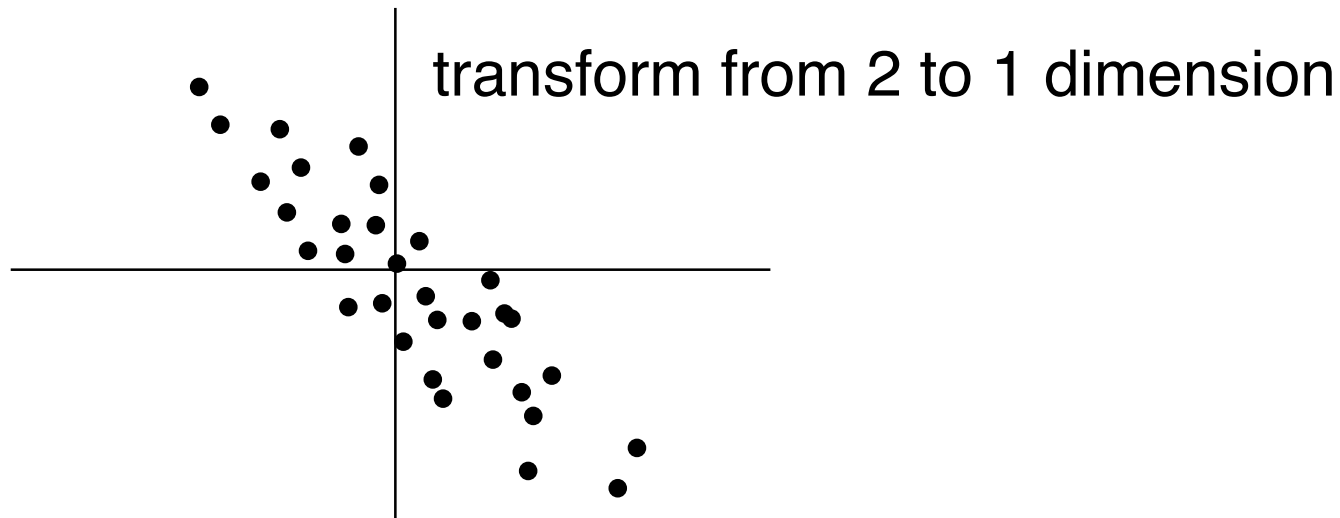


# The main idea

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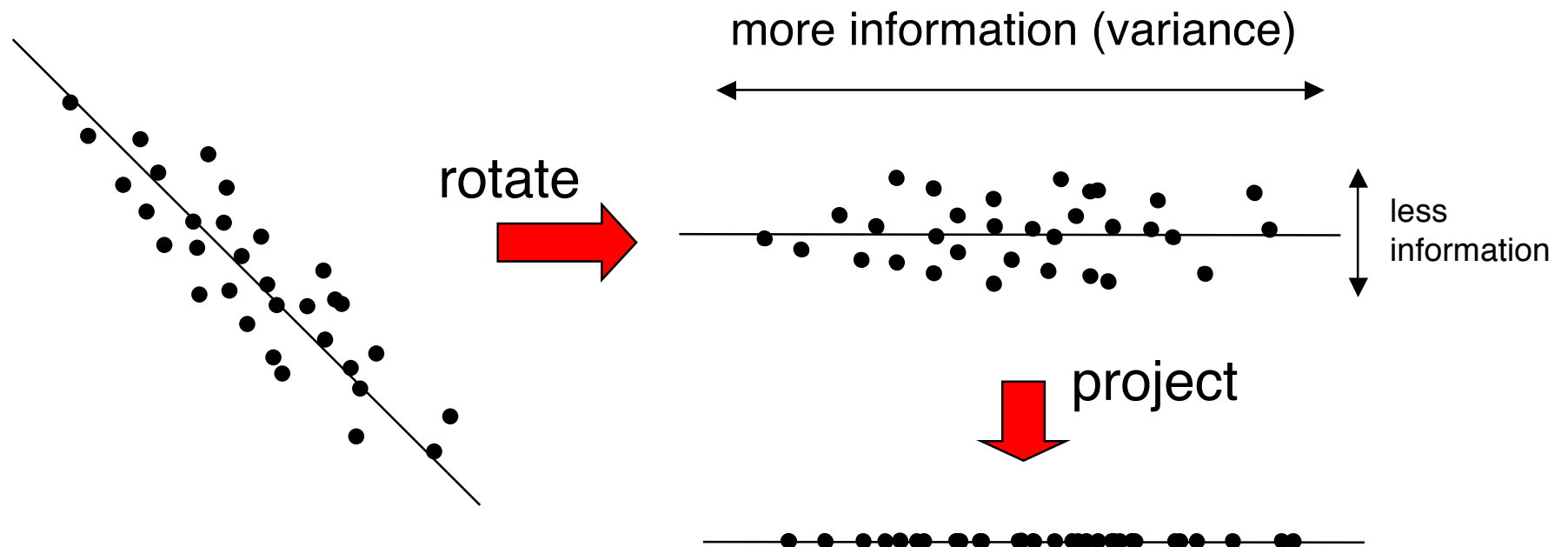
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- Transform the input data into fewer dimensions
- Preserve as much of the variance as possible



# Transformation

- Preserve as much of the variance as possible



# What is the Predominant Dimension?

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- This may seem subjective, but a mathematical definition can be provided.
- Let  $A$  be the matrix of data points (observations):
  - Each point is a column.
  - The rows correspond to the observed values of variables.

# Linear Transformation

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- The types of transformations shown are linear:
  - $B = WA$   
where
    - $A$  is the matrix of data (**each point as a column vector**), which is already mean-adjusted.
    - $W$  is an  $m \times n$  matrix,  $m \leq n$
    - $B$  is the transformed data matrix (called the “scores”)
- We want the columns of  $W$  to be **orthonormal** (i.e.  $WW^T = I$ ).
- The reconstruction  $A'$  of  $A$  from  $B$  will be obtained by:
  - $A' = W^TB$   
 $= W^T(WA) = (W^TW)A.$   
(Note:  $W^TW$  won't generally be  $I$ , although  $WW^T$  is.)

# Computing Principal Components

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- PCA tries to minimize the *expectation* of **reconstruction error**:

$$E[ \| A - A' \|^2 ] =$$

$$E[ \text{tr}( (A - A')(A-A')^T ) ] =$$

$$\text{tr}(E[AA']) - \text{tr}(W E[AA'] W^T) =$$

$$\text{tr}(R) - \text{tr}(WRW^T)$$

where

- $R = E[AA^T]$  is the *covariance matrix*
- $E[ ]$  is the expectation, i.e. average over the data points
- **tr** is the “trace” (sum of diagonal elements), which is equal to the sum of its eigenvalues.

## Note on Covariance vs. Correlation

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- Most PCA presentations are based on **covariance**, which assumes input variables have **commensurate** units.
- Can also base PCA on **correlation**, which doesn't.
- To use covariance in the general case, first **normalize** by dividing by each mean-adjusted variable by its variance.

## What are the Principal Components, really?

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- Want  $W$  such that **error**  $\text{tr}(R) - \text{tr}(WRW^T)$  is **minimized**.
- Equivalently,  $WRW^T$  (i.e. the **variance** of the transformed data) is **maximized**.
- The **eigenvectors** of the covariance matrix  $R = AA^T$ , **ordered** corresponding to largest to smallest eigenvalue, are the **principal components**.
- **PCA: Construct matrix  $W$**  as using the top so-many correspondingly-ordered eigenvectors as rows.

# Example (worked in Matlab)

## Turtle Shell Classification (D. Morrison)

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data = (columns are data points)

1.3000	1.4000	1.5000	1.2000	1.1000	(Height)
3.2000	2.8000	3.1000	2.9000	3.0000	(Width)
3.7000	4.1000	4.6000	4.8000	4.8000	(Length)

Can fewer than 3  
variables explain all?

means =

1.3000  
3.0000  
4.4000

```
[M,N] = size(data);
```

```
means = mean(data, 2);
```

```
mean_adjusted = data - repmat(means,1,N);
```

mean\_adjusted =

0	0.1000	0.2000	-0.1000	-0.2000
0.2000	-0.2000	0.1000	-0.1000	0
-0.7000	-0.3000	0.2000	0.4000	0.4000

# Example

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covariance =

```
0.0250  0.0025  -0.0275
0.0025  0.0250  -0.0250
-0.0275 -0.0250  0.2350
```

variances =

```
0.2415
0.0225
0.0210
```

explained\_variation =

```
0.8472
0.0791
0.0737
```

```
covariance = (mean_adjusted * mean_adjusted') / (N-1);
[eigenvectors, eigenvalues] = eig(covariance);
[junk, rindices] = sort(-1*eigenvalues);
variances = eigenvalues(rindices);
explained_variation = variances/sum(variances)
```

# Example: Principal Components

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PC = (column vectors)

```
-0.1265  0.5534 -0.8232
-0.1153 -0.8325 -0.5419
 0.9852 -0.0263 -0.1691
```

```
PC = eigenvectors(:, rindices);

% transform the original data set by column
scores = PC * mean_adjusted;
```

scores = (transformed data, columns)

```
-0.4656 -0.3703  0.1947  0.2866  0.3546
-0.5459 -0.0076  0.0021  0.3116  0.2398
-0.1236  0.0531  0.2282 -0.0283 -0.1294
```

# Example: Orthonormality Check

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orthogonality\_check =

1.0000	0.0000	0
0.0000	1.0000	0.0000
0	0.0000	1.0000

$$\text{orthogonality\_check} = \text{PC} * \text{PC}'$$

# Example: Reconstruction 1

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subtracted\_means =

1.3000	1.3000	1.3000	1.3000	1.3000
3.0000	3.0000	3.0000	3.0000	3.0000
4.4000	4.4000	4.4000	4.4000	4.4000

reconstruction\_using\_all\_components =

1.3000	1.4000	1.5000	1.2000	1.1000
3.2000	2.8000	3.1000	2.9000	3.0000
3.7000	4.1000	4.6000	4.8000	4.8000

reconstruction\_error =

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

reconstruction\_using\_all\_components = ...  
(PC' \* scores) + subtracted\_means

reconstruction\_error = reconstruction\_using\_all\_components - data

# Example: Reconstruction 2

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reconstruction\_using\_first\_two\_components =

```
1.4218  1.3477  1.2751  1.2278  1.2275
3.1967  2.8014  3.1060  2.8993  2.9966
3.7209  4.0910  4.5614  4.8048  4.8219
```

reconstruction\_error =

```
0.1218 -0.0523 -0.2249  0.0278  0.1275
-0.0033  0.0014  0.0060 -0.0007 -0.0034
0.0209 -0.0090 -0.0386  0.0048  0.0219
```

mse\_reconstruction =

0.0058

```
reconstruction_using_first_two_components = ...
    (components(1:2, :) * scores(1:2, :)) + subtracted_means

reconstruction_error = reconstruction_using_first_two_components - data

mse_reconstruction = mse(reconstruction_error)
```

# Example: Reconstruction 3

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reconstruction\_using\_first\_component\_only =

1.3589	1.3468	1.2754	1.2637	1.2551
2.7423	2.7951	3.1077	3.1586	3.1962
4.0167	4.0952	4.5603	4.6359	4.6919

reconstruction\_error =

0.0589	-0.0532	-0.2246	0.0637	0.1551
-0.4577	-0.0049	0.0077	0.2586	0.1962
0.3167	-0.0048	-0.0397	-0.1641	-0.1081

mse\_reconstruction =

0.0360

```
reconstruction_using_first_component_only = ...  
    (components(1:1, :)' * scores(1:1, :)) + subtracted_means  
  
reconstruction_error = reconstruction_using_first_component_only - data  
  
mse_reconstruction = mse(reconstruction_error)
```

# Matlab cov Function

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A = (mean adjusted, by row)

```
      0      0.2000     -0.7000
    0.1000     -0.2000     -0.3000
    0.2000      0.1000      0.2000
   -0.1000     -0.1000      0.4000
   -0.2000          0      0.4000
```

>> R = cov(A)

R =

```
    0.0250    0.0025   -0.0275
    0.0025    0.0250   -0.0250
   -0.0275   -0.0250    0.2350
```

# Matlab pcacov function

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```
>> [pc, variances, explained] = pcacov(R)
```

```
pc =
```

```
-0.1265    0.5534    0.8232  
-0.1153   -0.8325    0.5419  
 0.9852   -0.0263    0.1691
```

```
variances =
```

```
 0.2415  
 0.0225  
 0.0210
```

Eigenvalues

```
explained =
```

```
84.7212  
 7.9114  
 7.3674
```

% Variance explained

# Singular Value Decomposition: Another approach to computing PCs

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- The singular-value decomposition (SVD) of A is:  
$$A = USV^T$$
- where:
  - The columns of U are the eigenvectors of  $AA^T$ .
  - The columns of V are the eigenvectors of  $A^T A$ , i.e. the PC's.
  - S is pseudo-diagonal (diagonal insofar as this is possible with a non-square matrix, the rest of the entries being 0).

- PC from SVD in Matlab:

$$Y = \text{mean\_adjusted} / \text{sqrt}(N-1)$$

$$[U, S, PC] = \text{svd}(Y')$$

$$[\text{Note: } Y' = U S PC']$$

# Planets Example

(<http://www.cs.mcgill.ca/~sqrt/dimr/dimreduction.html>)

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- Consider a 3-dimensional data set where the variables are the logarithms of
  - distance to the sun
  - equatorial diameter
  - density

# Data set (prior to taking logs)

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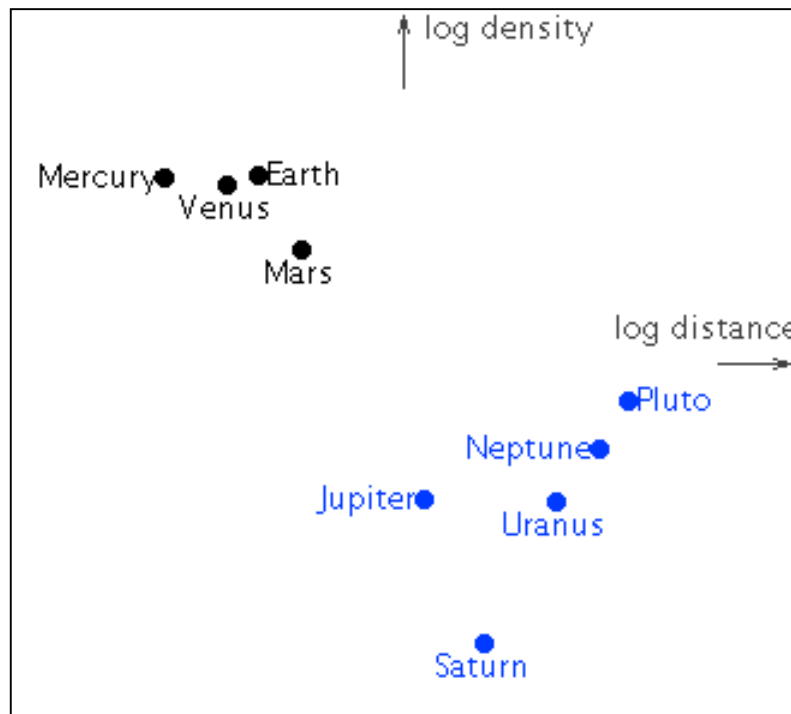
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<b>planet</b>	<b>distance</b>	<b>diameter</b>	<b>density</b>
Mercury	0.387	4878	5.42
Venus	0.723	12104	5.25
Earth	1.000	12756	5.52
Mars	1.524	6787	3.94
Jupiter	5.203	142800	1.314
Saturn	9.539	120660	0.69
Uranus	19.18	51118	1.29
Neptune	30.06	49528	1.64
Pluto	39.53	2300	2.03

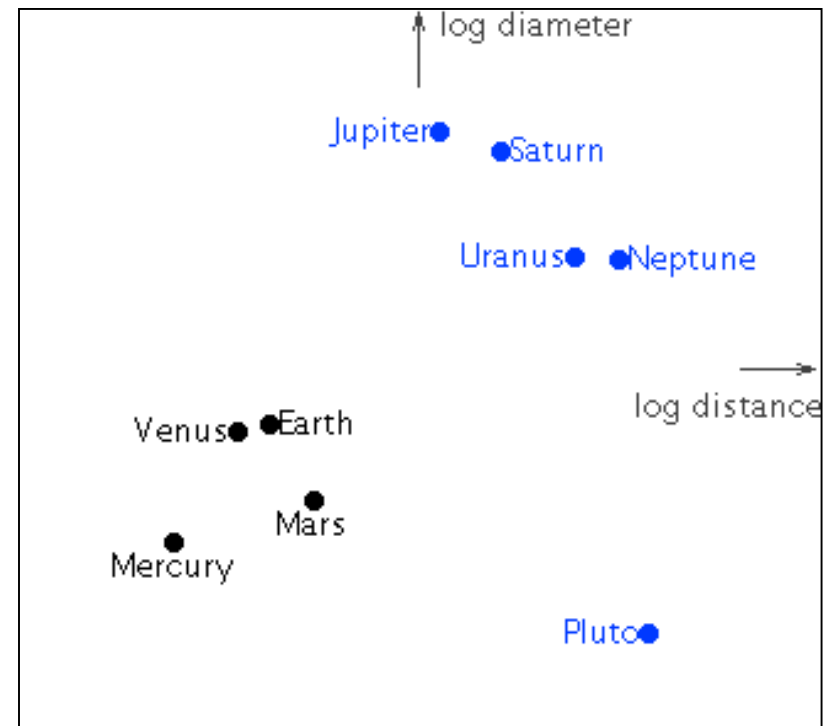
# Log Data Projections

(projections being a special case of linear transformation)

In **two dimensions**, we can plot any one variable against another, e.g.



or



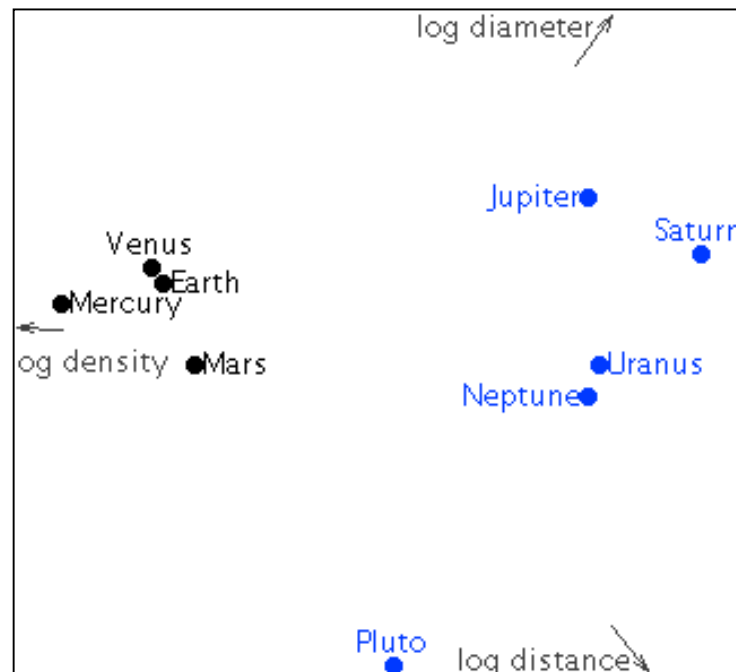
Of various possible plots, which show the widest variation among planets?

# Maximizing Variance

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- Transforming using first two principal components **preserves more of the variance** (summing variances in each dimension) in the projection than does projecting on any 2 variables:

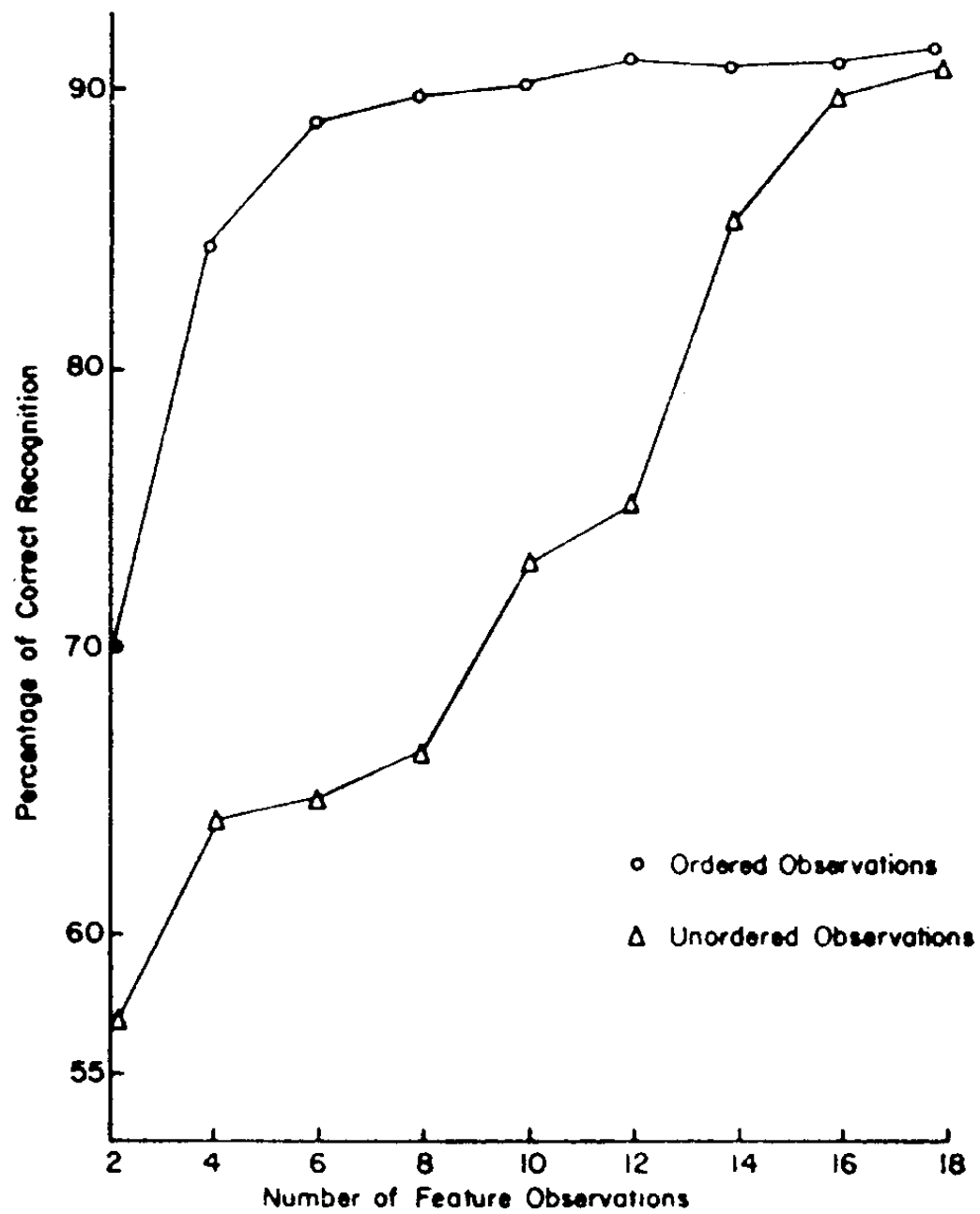


# Application: Handwritten Character recognition (K.S. Fu, 1968)

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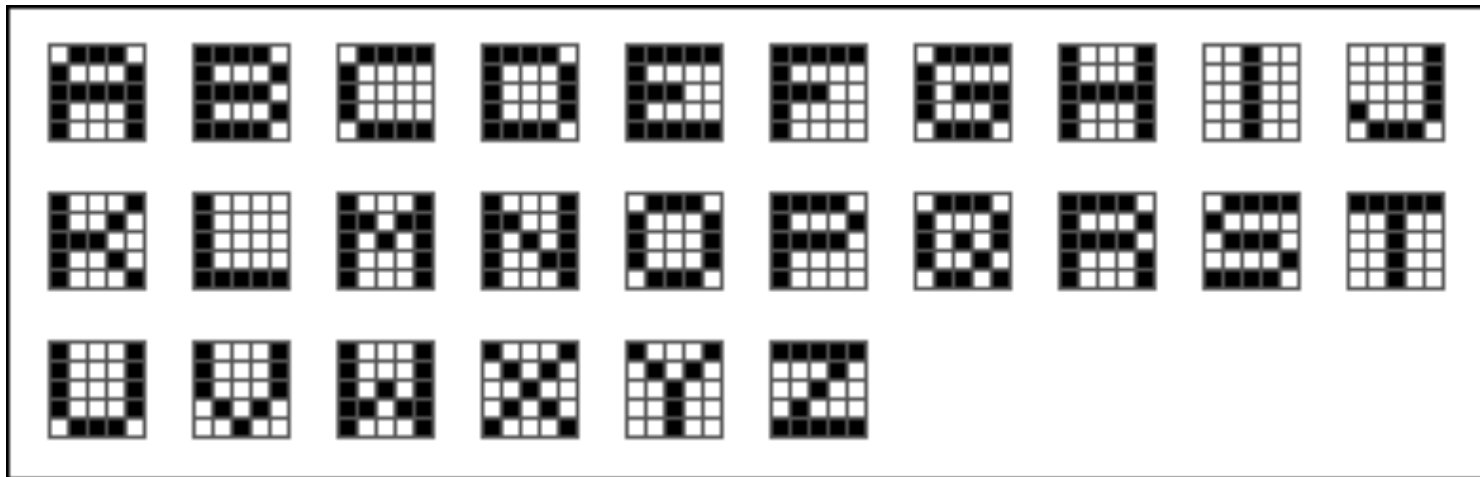
- Attempt recognition based upon **18 radial measurements**, spaced at 20 degree increments, of characters (240 samples).
- Recognition rate was computed vs. the number of measurements used (2, 4, 6, ... out of 18) in no particular order.
- The same computation was done with the measurements **ordered by principal components**.
- The next slide compares the two approaches.



# A Related Example

(<http://www.cs.mcgill.ca/~sqr/dimr/dimreduction.html>)

- A 25-dimensional data set:

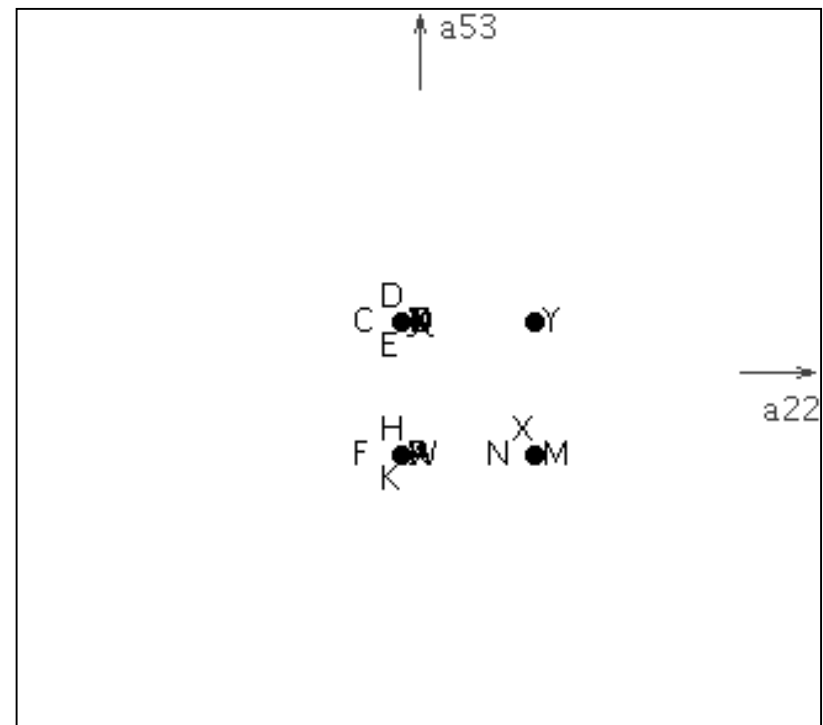


# Projected Data

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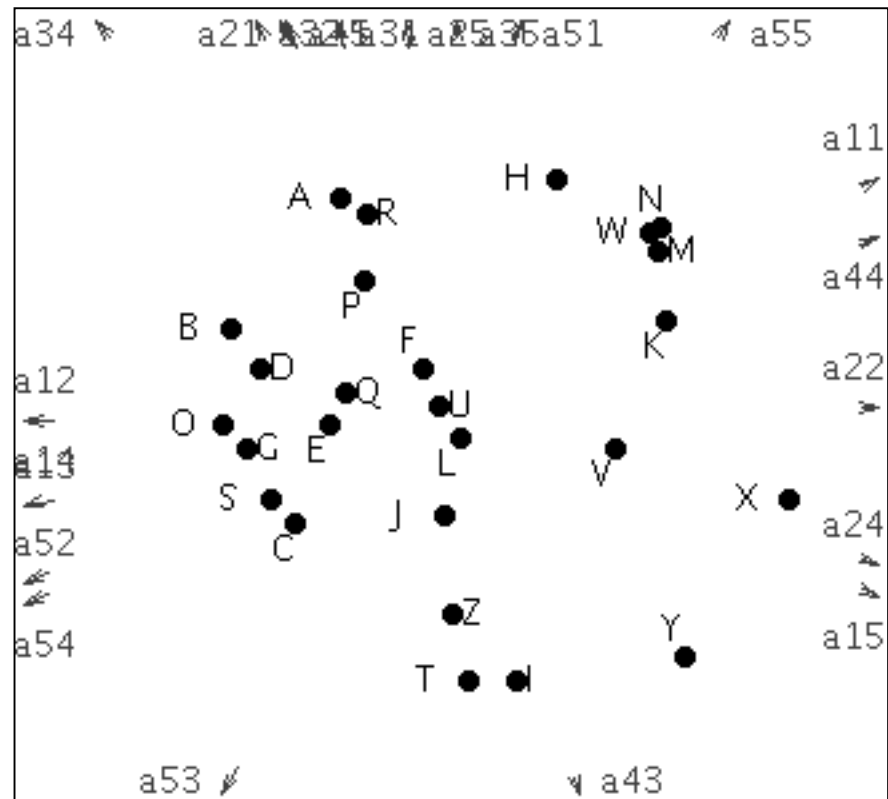
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- Projecting on obvious 2 variables does not help much in discriminating the points, e.g.



# Projected Data

- Projecting on the first two principal components achieves the maximum variation in two dimensions:



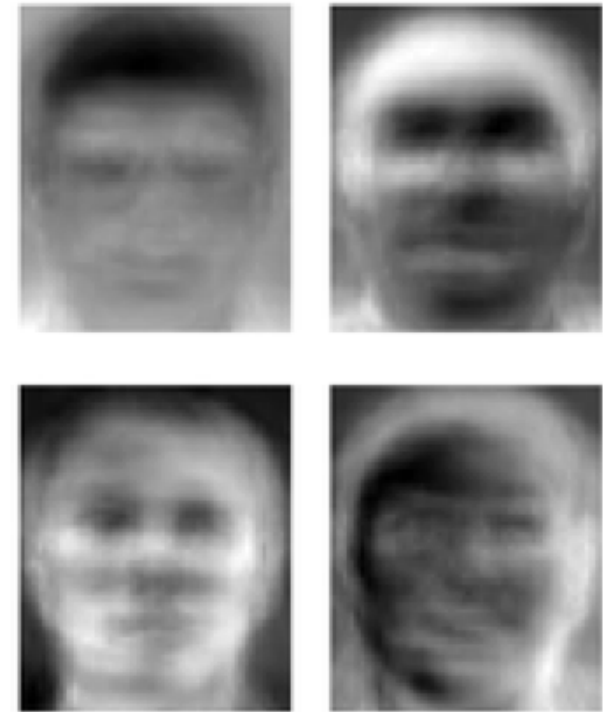
# Eigenfaces

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**Eigenfaces** are a set of **eigenvectors** used in the **computer vision** problem of human **face recognition**. The approach of using eigenfaces for **recognition** was developed by Sirovich and Kirby (1987) and used by **Matthew Turk** and **Alex Pentland** in face classification. It is considered the first successful example of facial recognition technology.

*[citation needed]* These **eigenvectors** are derived from the **covariance matrix** of the **probability distribution** of the **high-dimensional vector space** of *possible faces of human beings*.



Some eigenfaces from [AT&T Laboratories Cambridge](#).



# Eigenface Generation

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To generate a set of eigenfaces, a large set of digitized images of human faces, taken under the same lighting conditions, are normalized to line up the eyes and mouths. They are then all resampled at the same pixel resolution. Eigenfaces can be extracted out of the image data by PCA. Here are the steps involved in converting an image of a face into eigenfaces:

1. Prepare a training set. The faces constituting the training set  $T$  should be already prepared for processing.
2. Subtract the mean. The average matrix  $A$  has to be calculated and subtracted from the original in  $T$ . The results are stored in variable  $S$ .
3. Calculate the covariance matrix.
4. Calculate the eigenvectors and eigenvalues of this covariance matrix.
5. Choose the principal components.

# Eigenface Selection

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There will be a large number of eigenfaces created before step 5, and far fewer are really needed. Select from them those that have the highest eigenvalues. For instance, if we are working with a 100 x 100 image, then this system will create 10,000 eigenvectors. Since most individuals can be identified using a database with a size between 100 and 150, most of the 10,000 can be discarded, and only the most important should remain.

The eigenfaces that are created will appear as light and dark areas that are arranged in a specific pattern. This pattern is how different features of a face are singled out to be evaluated and scored. There will be a pattern to evaluate symmetry, if there is any style of facial hair, where the hairline is, or evaluate the size of the nose or mouth. Other eigenfaces have patterns that are less simple to identify, and the image of the eigenface may look very little like a face.

# Eigenface Recognition Algorithm

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For an algorithm in more detail, see:

<http://openbio.sourceforge.net/resources/eigenfaces/eigenfaces-html/facesOptions.html>

# Eigenfaces App: FERET Photobook

<http://vismod.media.mit.edu/vismod/demos/facerec/basic.html>

Database: bank

Display mode: image-orig

Search metric: norm-edge-nv

Working Set: 310

Left button to select  
Middle button to search  
Right button for info

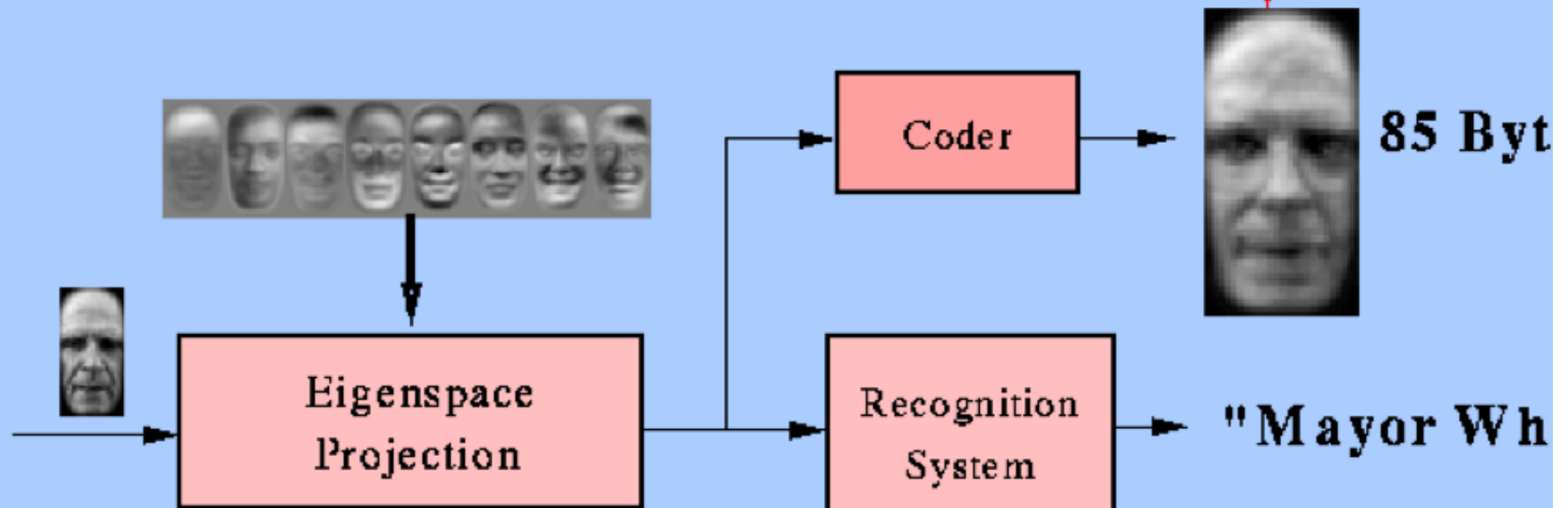
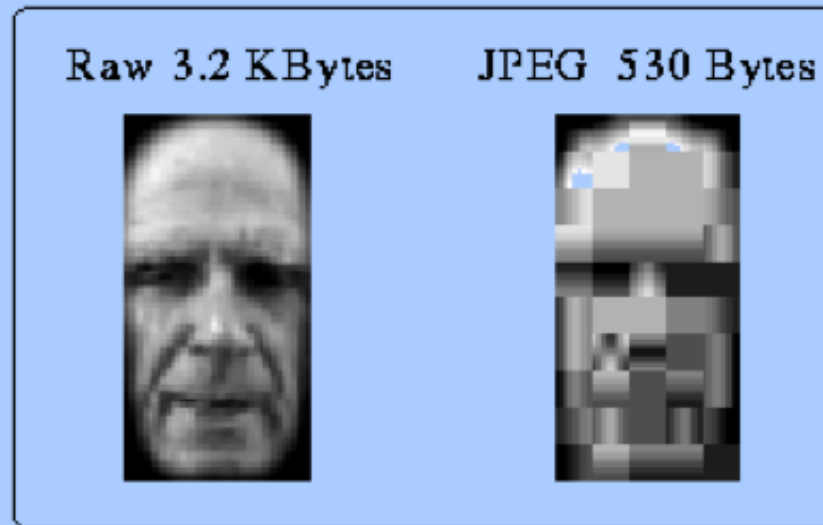
00287fa010,940422 00287fb010,940422 00287fa010,940422 00287fb010,940422

00477fa010,940519 00285fa010,940422 00285fb010,940422 00288fa010,940422

00288fb010,940422 00474fa010,940519 00289fa010,940422 00277fa010,940422

Initialize  
Shuffle  
Load Query  
Save Query  
Text...  
Symbols...  
Label...  
Hook...  
G Label...  
Resize  
Refresh Cache  
Page Up/Down  
Page 1 of 26  
Jump to page  
Jump to item  
Quit Photobook

# Recognition and Coding



# Recognition and Coding

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Once the image is suitably normalized with respect to individual geometry and contrast, it is projected onto a set of normalized eigenfaces. The figure above shows the first few eigenfaces obtained from a KL expansion on an ensemble of 500 normalized faces. In our system, the projection coefficients are used to index through a database to perform identity verification and recognition using a nearest-neighbor search.



Here, the geometrically aligned and normalized image is projected onto a custom set of eigenfaces to obtain a feature vector which is used for recognition purposes as well as facial image coding.

# See for Enhancements:

<http://vismod.media.mit.edu/vismod/demos/facerec/basic.html>

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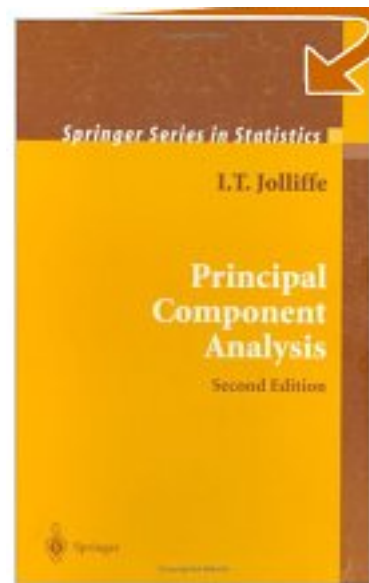
For the algorithm, see:

<http://openbio.sourceforge.net/resources/eigenfaces/eigenfaces-html/facesOptions.html>

# PCA Source

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L.T. Jolliffe

# What is a PCA Neural Network?

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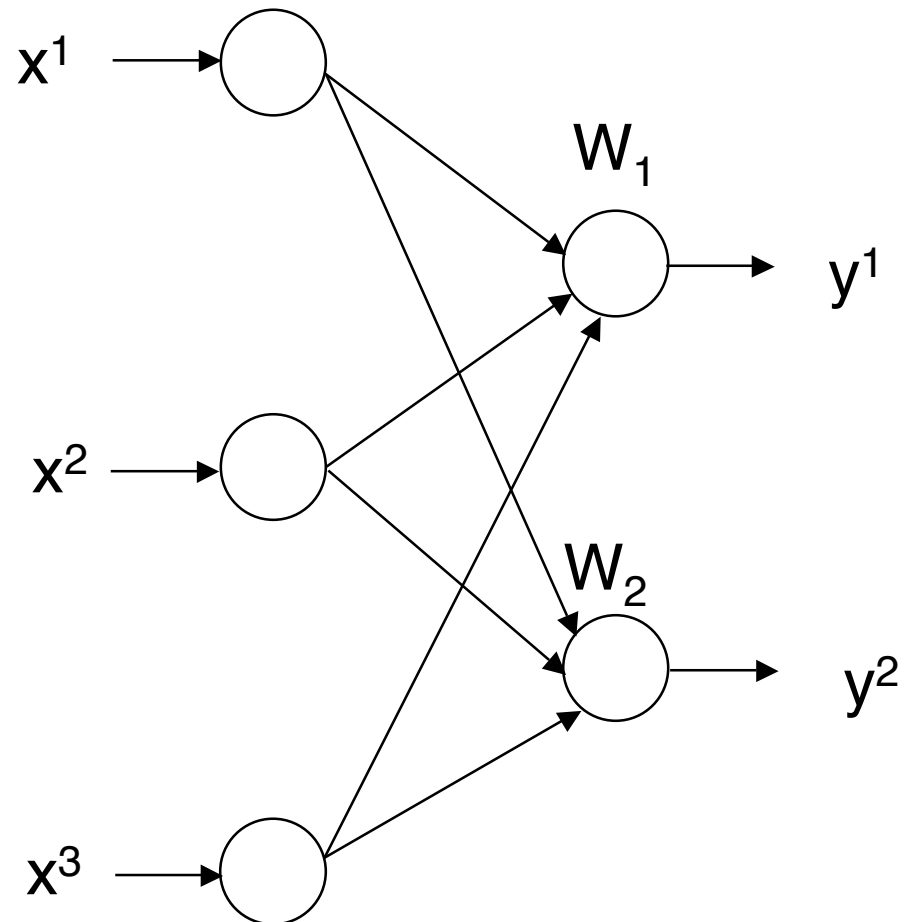
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- The matrix  $W$  of PCA can be interpreted as **weights** of a 1-layer neural network, also known as an **autoencoder**.
- Training is by variants on Hebbian learning.
- The user pre-selects the number of output variables.
- The network effectively learns PCs by outputting the corresponding scores.
- The weights are the learned transformation matrix, however they will not necessarily be orthogonal.

# 3→2 PCA Network

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# PCA Hebbian-Learning Rules

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- $\Delta W = \eta y_i x_i^T - K_i W$ , where
  - $x_i$  is an input vector
  - $y_i$  is the corresponding output vector ( $= W x_i$ )
  - $y_i x_i^T$  is the **Hebbian** component
  - $K_i$  is **one of the following weight decay** components:
    - 0 pure Hebbian (unbounded)
    - $y_i y_i^T$  (Williams' rule, 1985)
    - $3D(y_i y_i^T) + 2L(y_i y_i^T)$  (Oja and Karhunen rule, 1985)
    - $L(y_i y_i^T)$  (Sanger's rule, 1989)

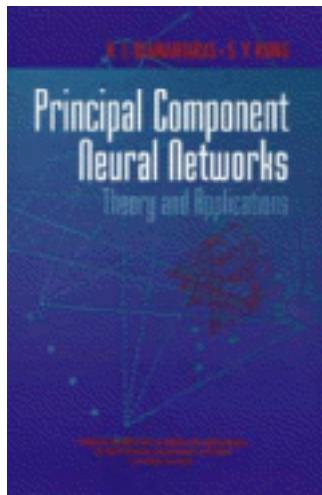
where  $D(M)$  maps the **diagonal** entries of  $M$  to themselves, and other entries to 0, and

- $L(M)$  maps entries **below** the main diagonal to themselves, and other entries to 0.

# Other Variations Exist: Source for PCA NN

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Diamantaras, K.I. / Kung, S.Y.

Principal Component Neural  
Networks  
Theory and Applications

1996. 256 pages.

ISBN 0-471-05436-4 John Wiley & Sons

The coverage in this book is extensive. It gives several additional learning algorithms, including ones with **lateral** connections as well as feed-forward ones.

# Other means of learning for dimensionality reduction

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- Self-Organizing Map  
(# of dimensions in superstructure = reduced dimension)
- LVQ  
(# of neurons = reduced dimension)
- Projection pursuit, another statistical technique, effectively learns PC's sequentially

# PCA Shortcomings

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- The PCA user must select **how many** target dimensions to use.
- PCA only finds **linear** sub-spaces.
- PCA works best if the individual components are **Gaussian-**distributed.
- There are many variations, some of which partially resolve some of these issues.
- Related area: “Factor Analysis”

# Independent Component Analysis (ICA)

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- A technique for BSS (Blind Source Separation).
- Proposed for neuro-mimetic hardware in 1983 by Herault and Jutten.
- ICA seeks to extract components that are **statistically independent**.
- Two variables  $x, y$  are *statistically independent* iff
$$P(x, y) = P(x)P(y).$$
Equivalently,
$$E\{g(x)h(y)\} - E\{g(x)\} E\{h(y)\} = 0$$
where  $g$  and  $h$  are **any** functions.

# PCA vs. ICA

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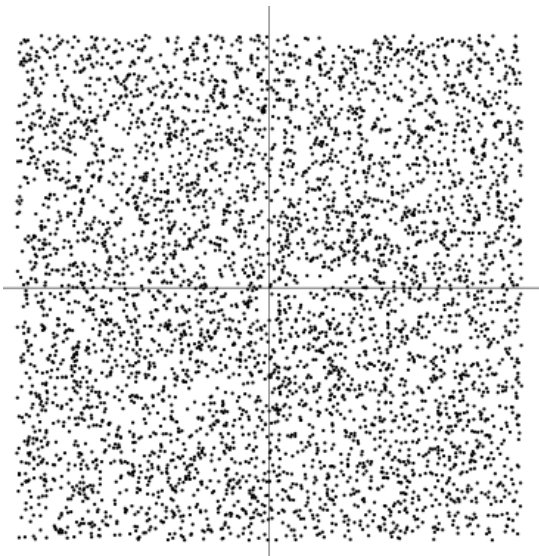
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- PCA finds **uncorrelated** components.
- ICA finds **independent** components.
- Independent implies uncorrelated, but not necessarily conversely.
- The input signals to ICA, being mixtures, will usually be correlated.
- The outputs, being independent, will not be.

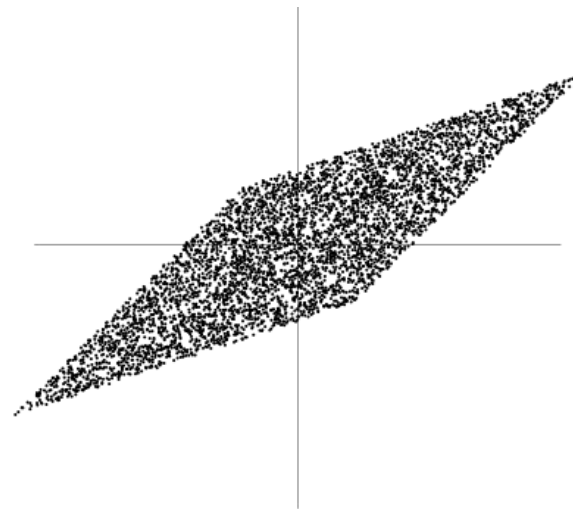
# Signals vs. Mixtures

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Joint distribution of two independent signals



Joint distribution of a mixture of the same signals

# ICA

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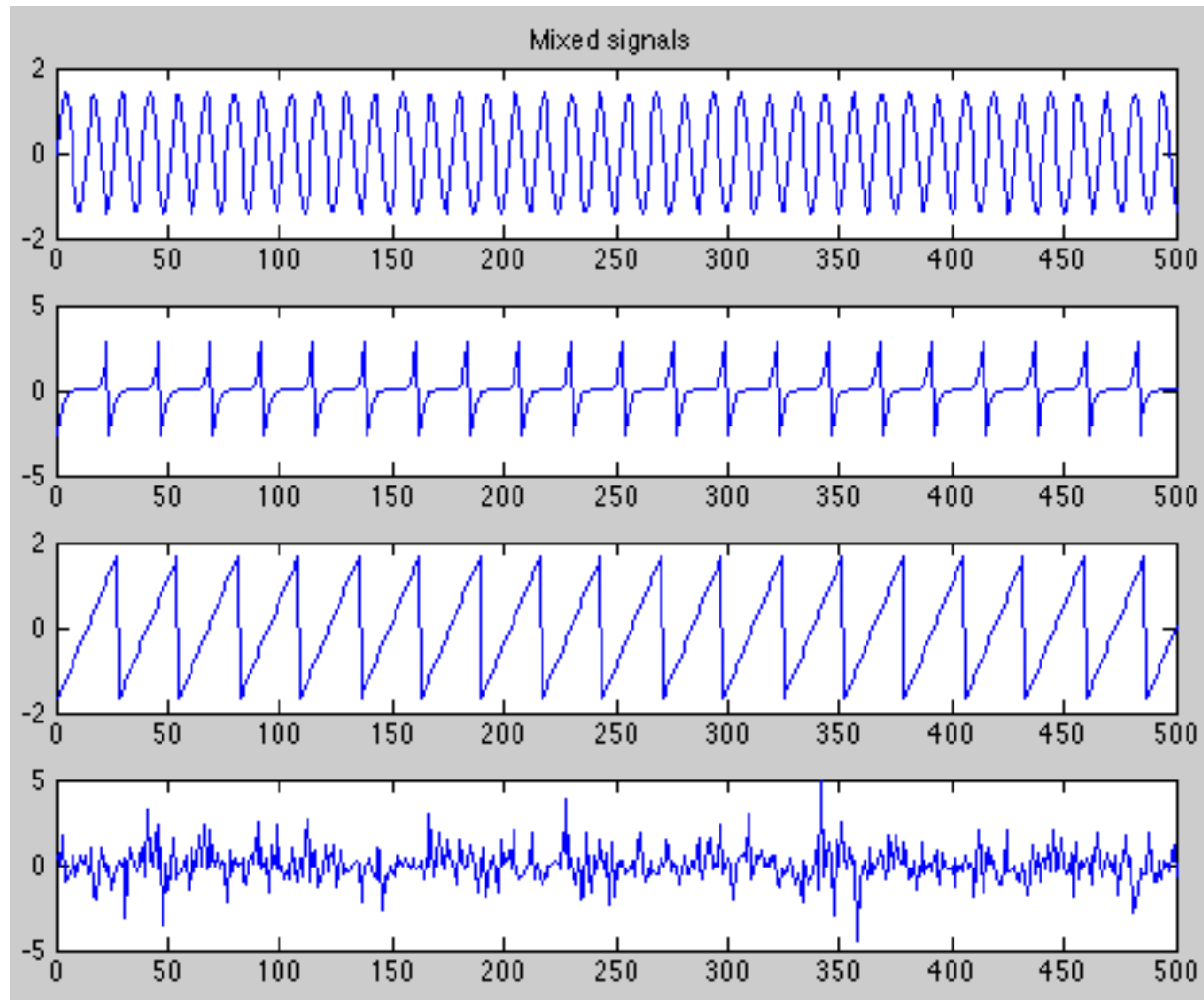
- Suppose we have  $n$  distinct mixtures of  $n$  statistically-independent signals.
- We wish to transform the mixtures to (signals as close as possible to) the original signals.

# Fast-ICA Demo

## Signals to be Mixed

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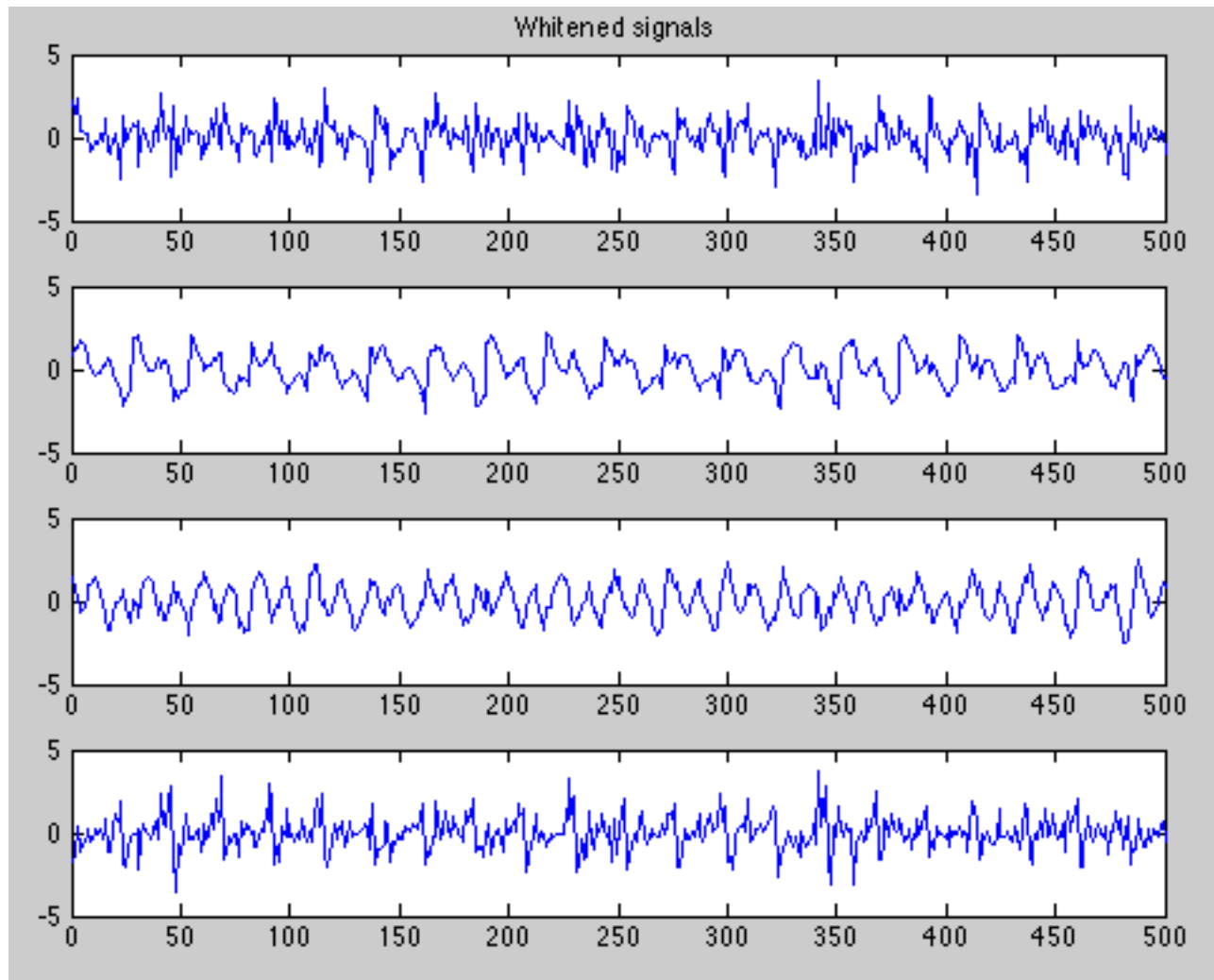
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# Whitened Signals after Mixing

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# Whitening Transformation

## (essentially a form of PCA)

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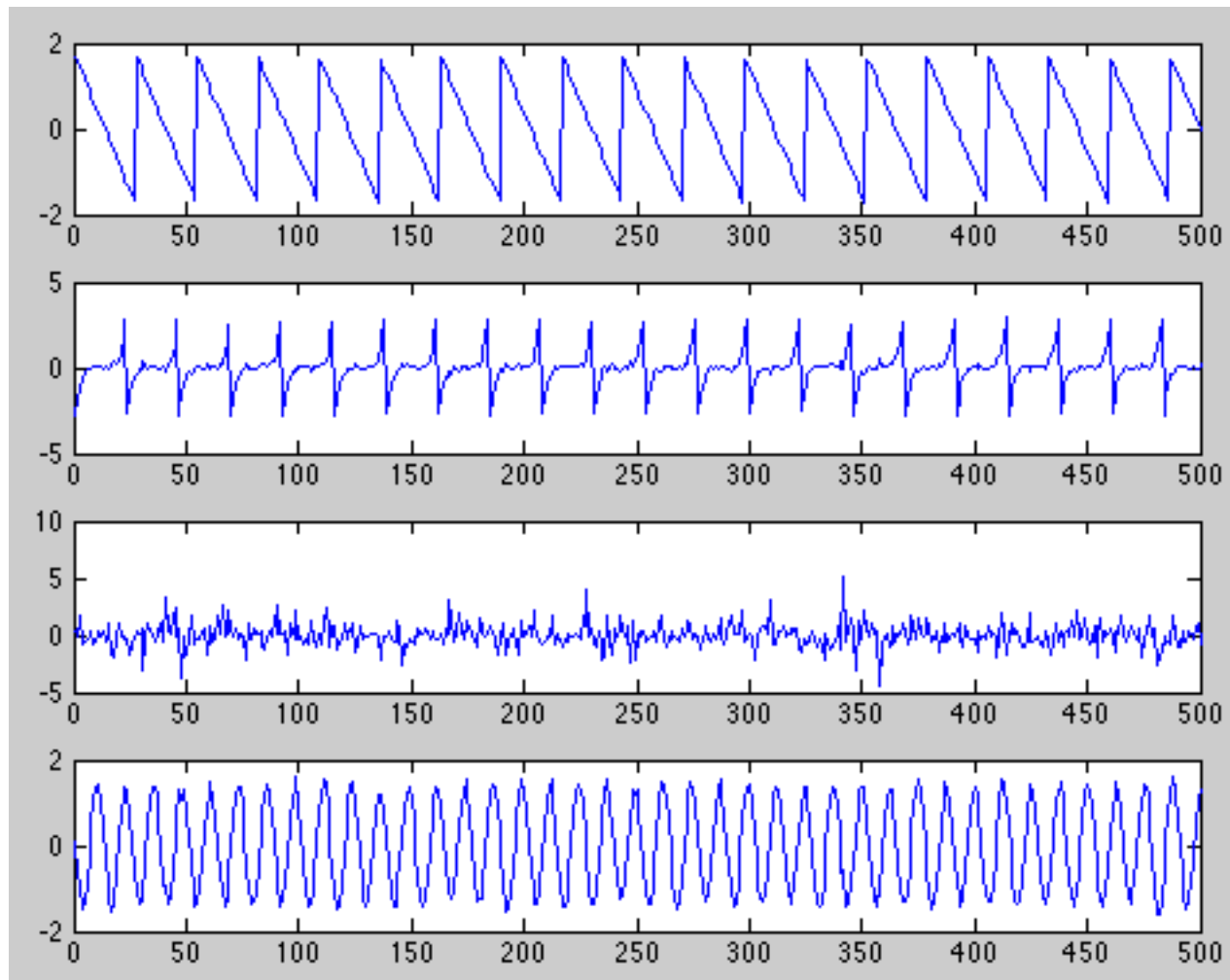
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- Seek a linear transformation  $V$  such that when  $y = Vx$  we get  $E\{yy'\} = I$ .
- Set  $V = C^{-1/2}$ , where  $C = E\{xx'\}$  is the correlation matrix of the data.
- Then  $E\{yy'\} = E\{Vxx'V'\} = C^{-1/2}CC^{-1/2} = I$ .
- For computational aspects, see:
- [http://en.wikipedia.org/wiki/Square\\_root\\_of\\_a\\_matrix](http://en.wikipedia.org/wiki/Square_root_of_a_matrix)
- [http://en.wikipedia.org/wiki/White\\_noise](http://en.wikipedia.org/wiki/White_noise)

# Extracted Independent Components

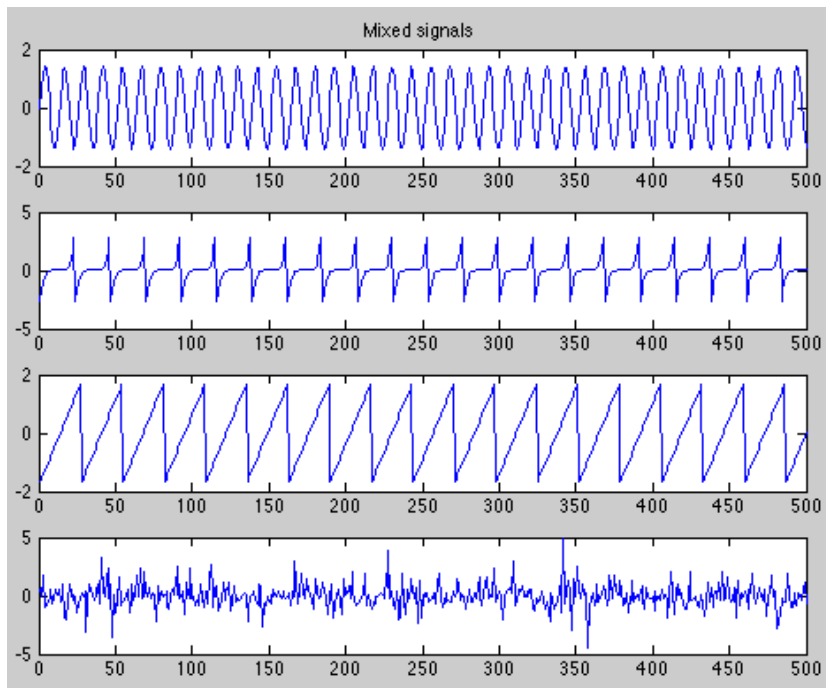
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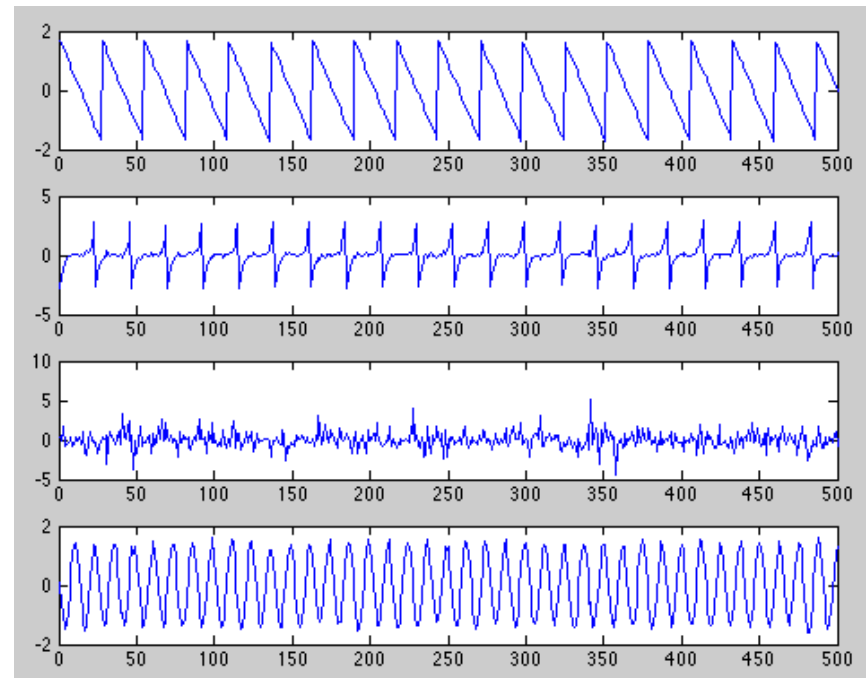


# Comparison

## Originals



## Extractions from mixtures



# ICA Demos

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- [http://www.cnl.salk.edu/~tewon/Blind/blind\\_audio.html](http://www.cnl.salk.edu/~tewon/Blind/blind_audio.html)
- [http://www.cis.hut.fi/projects/ica/cocktail/cocktail\\_en.cgi](http://www.cis.hut.fi/projects/ica/cocktail/cocktail_en.cgi)

# How can/does ICA work?

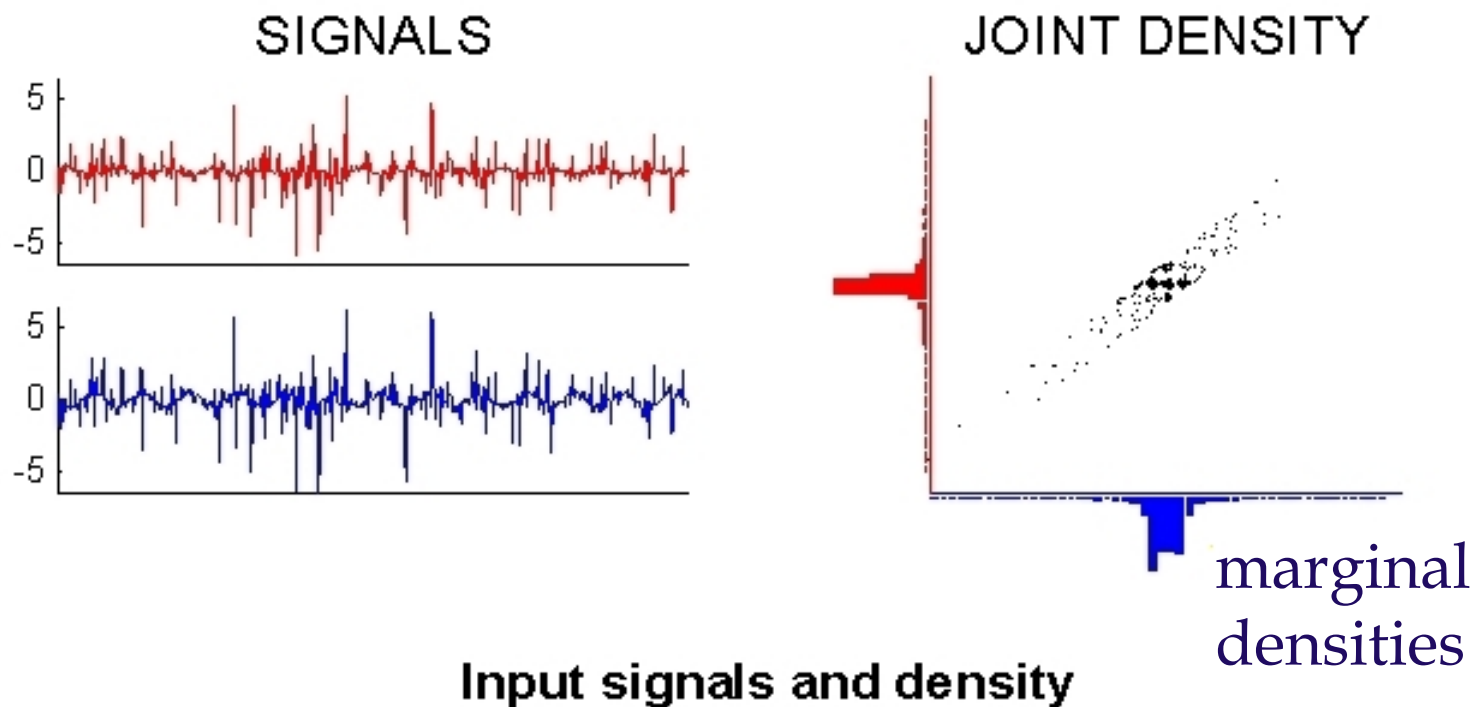
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- One version: “Fast ICA”
- Throughout, signals are assumed to be mean-adjusted.

# Correlation within Signal Mixture

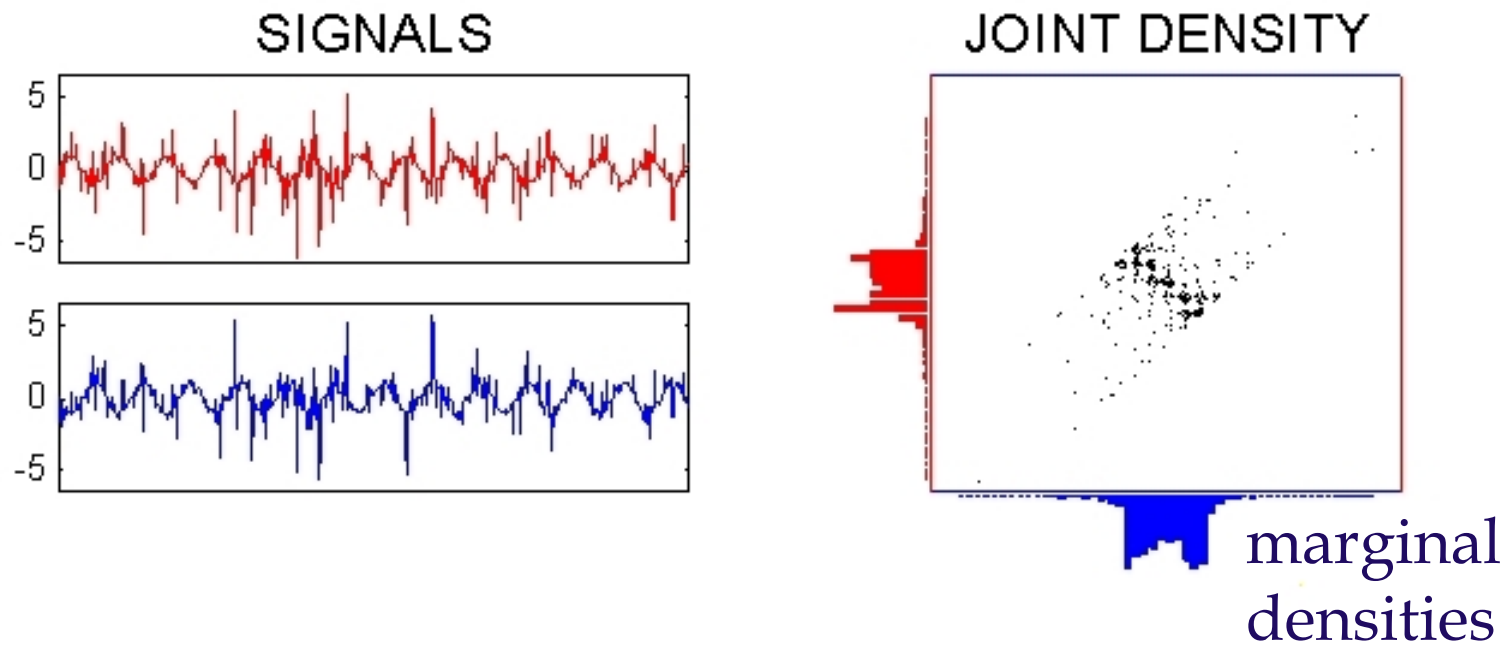
(from <http://www.cis.hut.fi/projects/ica/icademo/>)



# Effect of whitening : removing correlations in mixed signals

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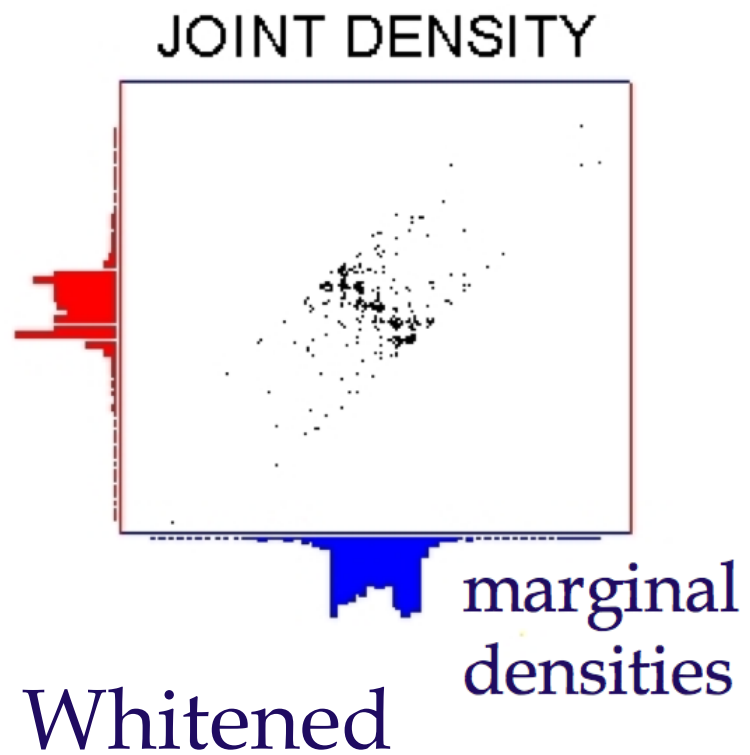
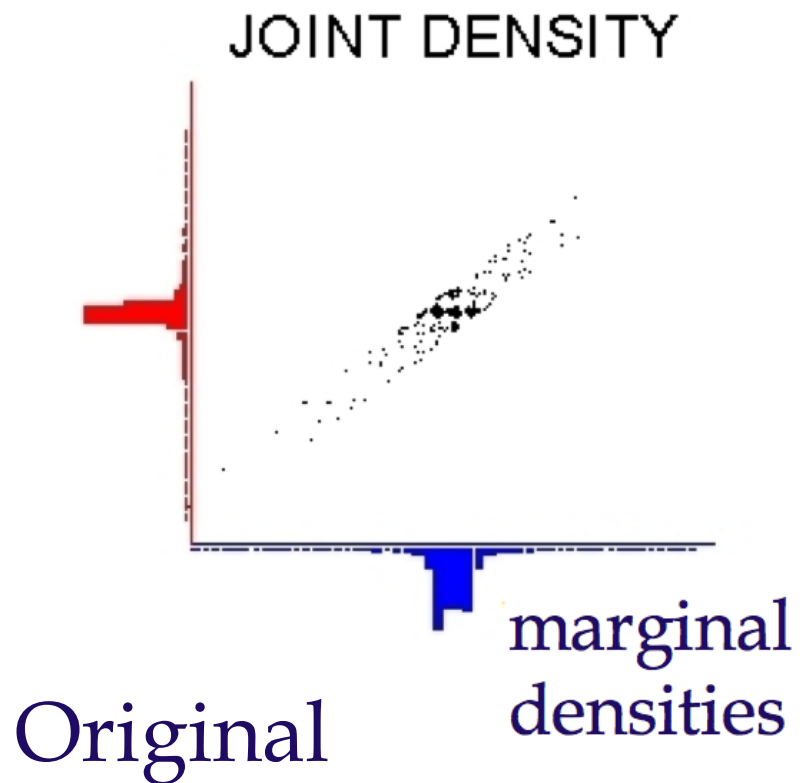


**Whitened signals and density**

# Original vs. whitened probability densities

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# Rotating the Whitened Signals

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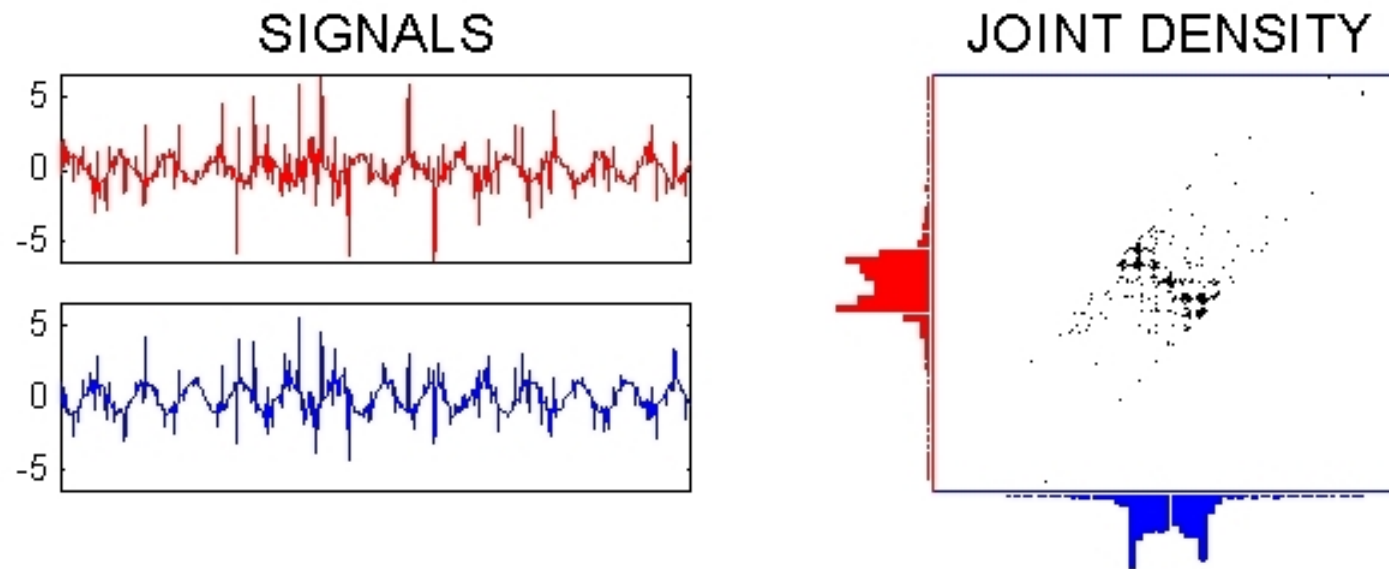
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- The independent components are achieved by an appropriate **rotation** of the whitened signals.
- This involves **maximizing** the non-normal aspects of the marginal densities, since
- A linear mixture of independent random variables is necessarily “more Gaussian” than the original variables
- Maximization can be done incrementally.

# Fast ICA Intermediate Rotation Steps

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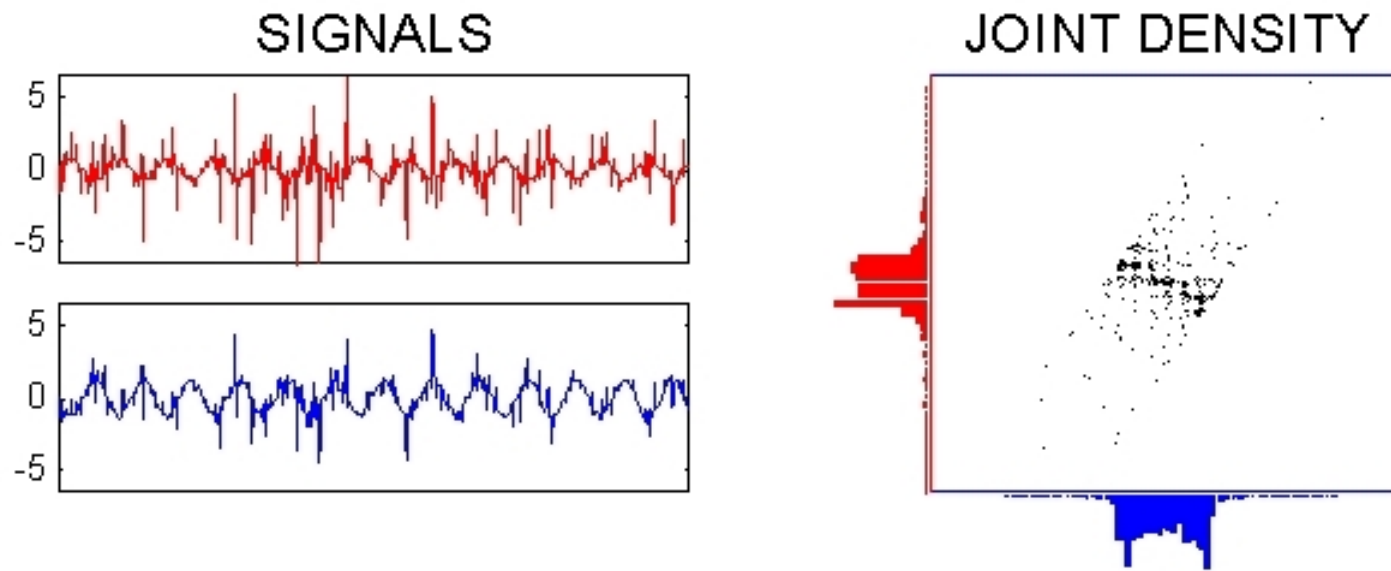


**Separated signals after 1 step of FastICA**

# Fast ICA Intermediate Rotation Steps

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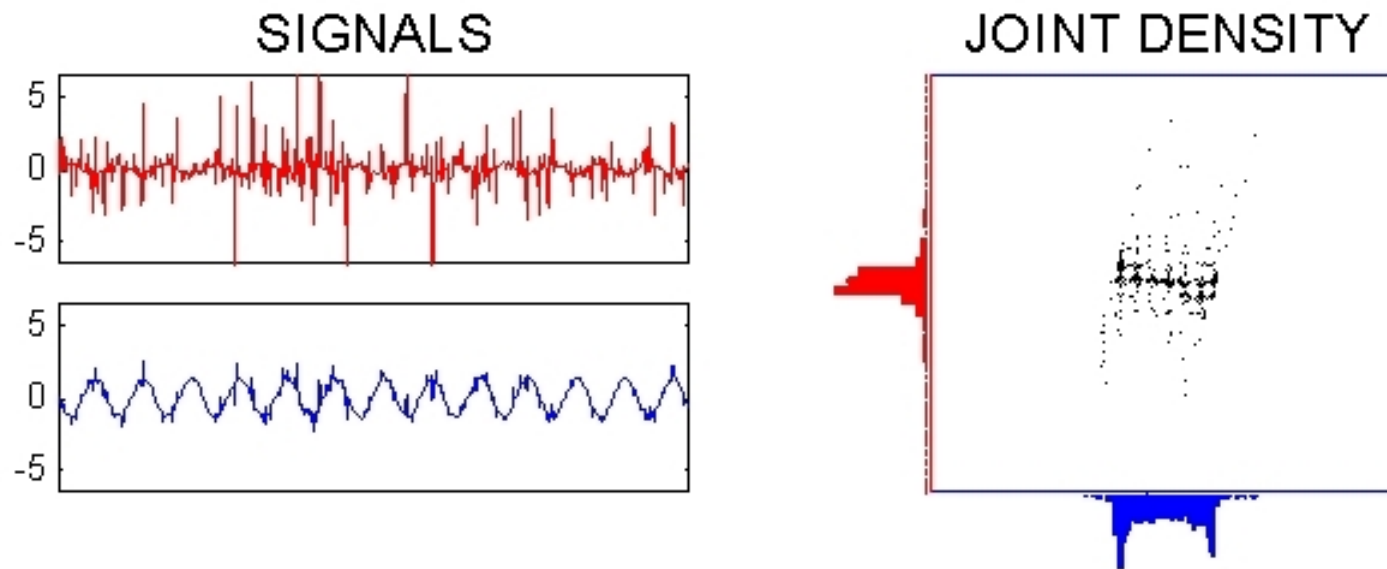


**Separated signals after 2 steps of FastICA**

# Fast ICA Intermediate Rotation Steps

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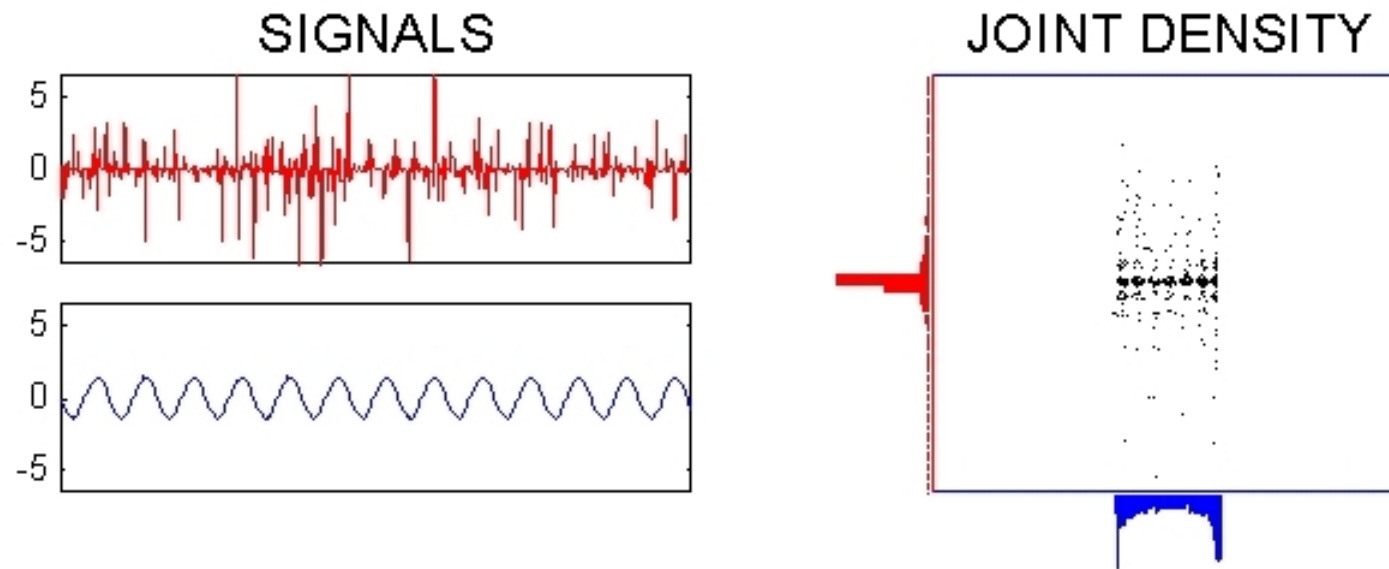


**Separated signals after 3 steps of FastICA**

# Fast ICA Intermediate Rotation Steps

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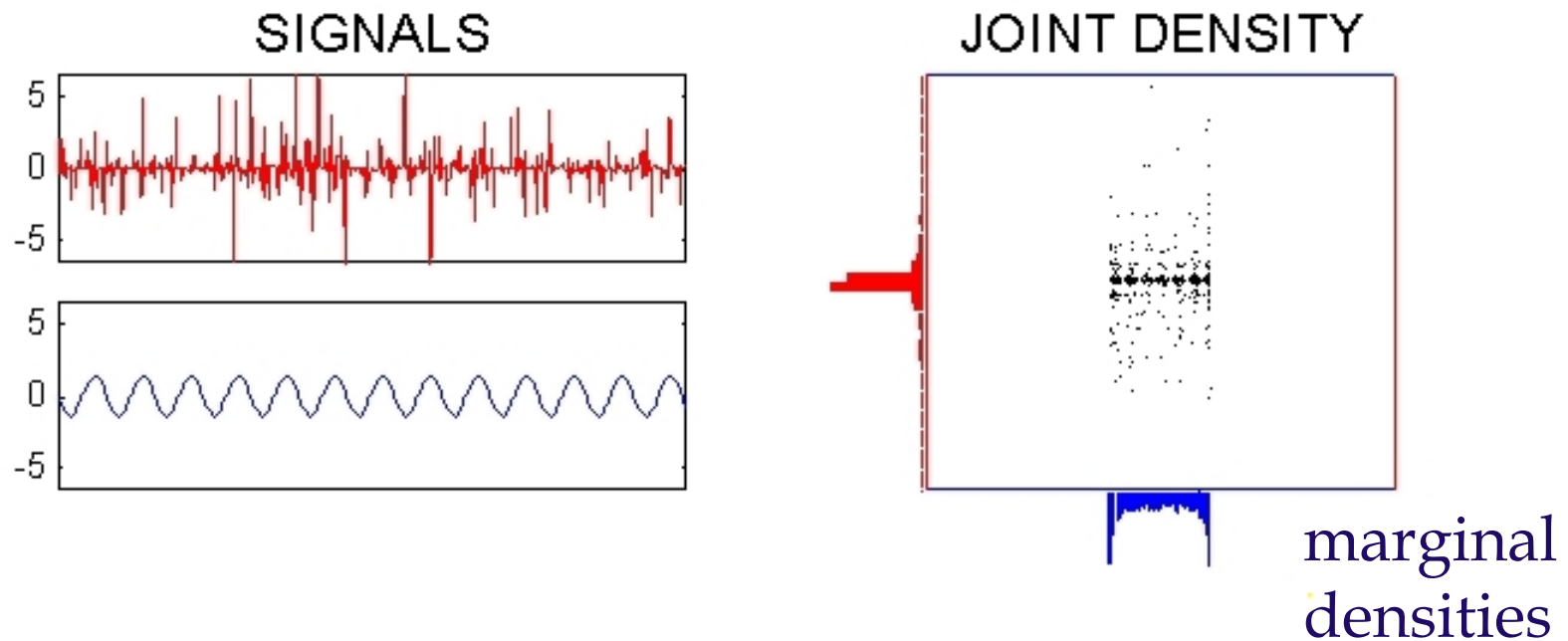


**Separated signals after 4 steps of FastICA**

# Separated Signals

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**Separated signals after 5 steps of FastICA**

# Neural ICA Algorithms

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- Bell and Sejnowski: “infomax” method, minimizing mutual information, using an iterative formulation, similar to PCA NN:

$$W_{k+1} = W_k + \beta_k [W_k^{-T} + z_k x_k^T]$$

$$z(i) = \partial/\partial u(i) \partial u(i)/\partial y(i)$$

$$u = f(Wx)$$

e.g.  $f = \text{tansig}$

- Also related to “maximum likelihood estimation”. Subsequent Bayesian derivation by Kevin Knuth

# Matlab Core Code based on Bell-Sejnowski

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```
% Begin gradient ascent on h ...
for iter=1:maxiter
    % Get estimated source signals, y.

    y = x*W; % wt vec in col of W.

    % Get estimated maximum entropy signals Y=cdf(y).

    Y = tanh(y);

    % Find value of function h.
    % h = log(abs(det(W))) + sum( log(eps+1-Y(:).^2) )/N;

    detW = abs(det(W));
    h = ( (1/N)*sum(sum(Y)) + 0.5*log(detW) );

    % Find matrix of gradients @h/@W_ji ...
    g = inv(W') - (2/N)*x'*Y;

    % Update W to increase h ...

    W = W + eta*g;

    % Record h and magnitude of gradient ...

    hs(iter)=h; gs(iter)=norm(g(:));
end;
```

# Neural ICA Algorithms

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- Bell-Sejnowski was similar to an earlier formulation by Herault-Jutten :

$$W = (I+S)^{-1}$$

$$S_{k+1} = S_k + \beta_k g(y_k) h(y_k^T)$$

$g = t, h = t^3; g = \text{hardlim}, h = \text{tansig}$

# Neural ICA Algorithms

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- EASI (Equivariant Adaptive Separation via Independence): Cardoso et al:

$$W_{k+1} = W_k - \beta_k [y_k y_k^T - I + g(y_k) h(y_k)^T - (y_k) g(y_k)^T] W_k$$

$g=t, h=tansig$

# ICA Image Separation Demo (David Gleich '03)

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Original 3  
images

Mixture

# ICA Image Separation Example (David Gleich Gleich '03)

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Bell and Sejnowski  
separation

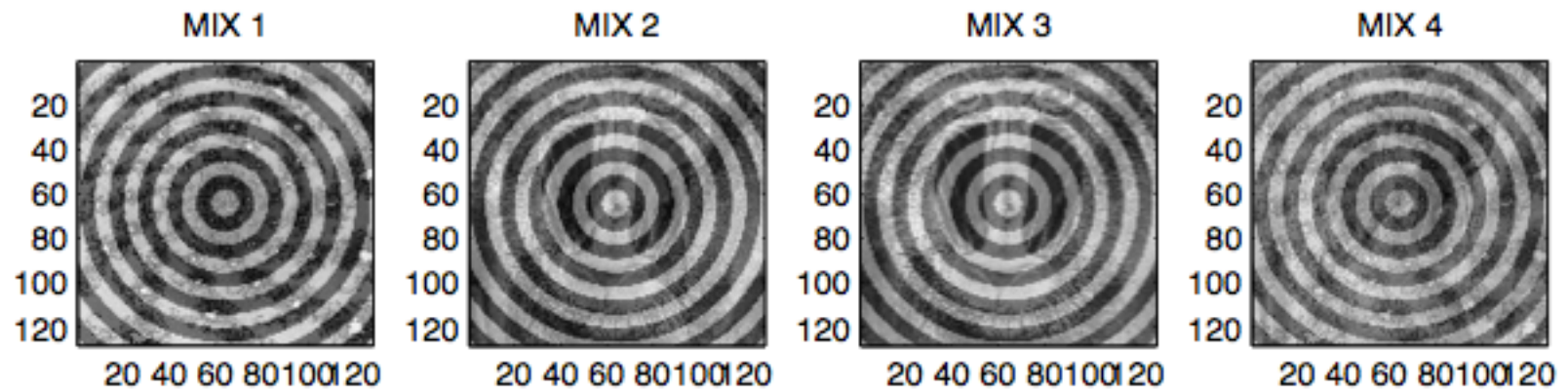
EASI Algorithm  
separation

Original images

# 4-source mixture

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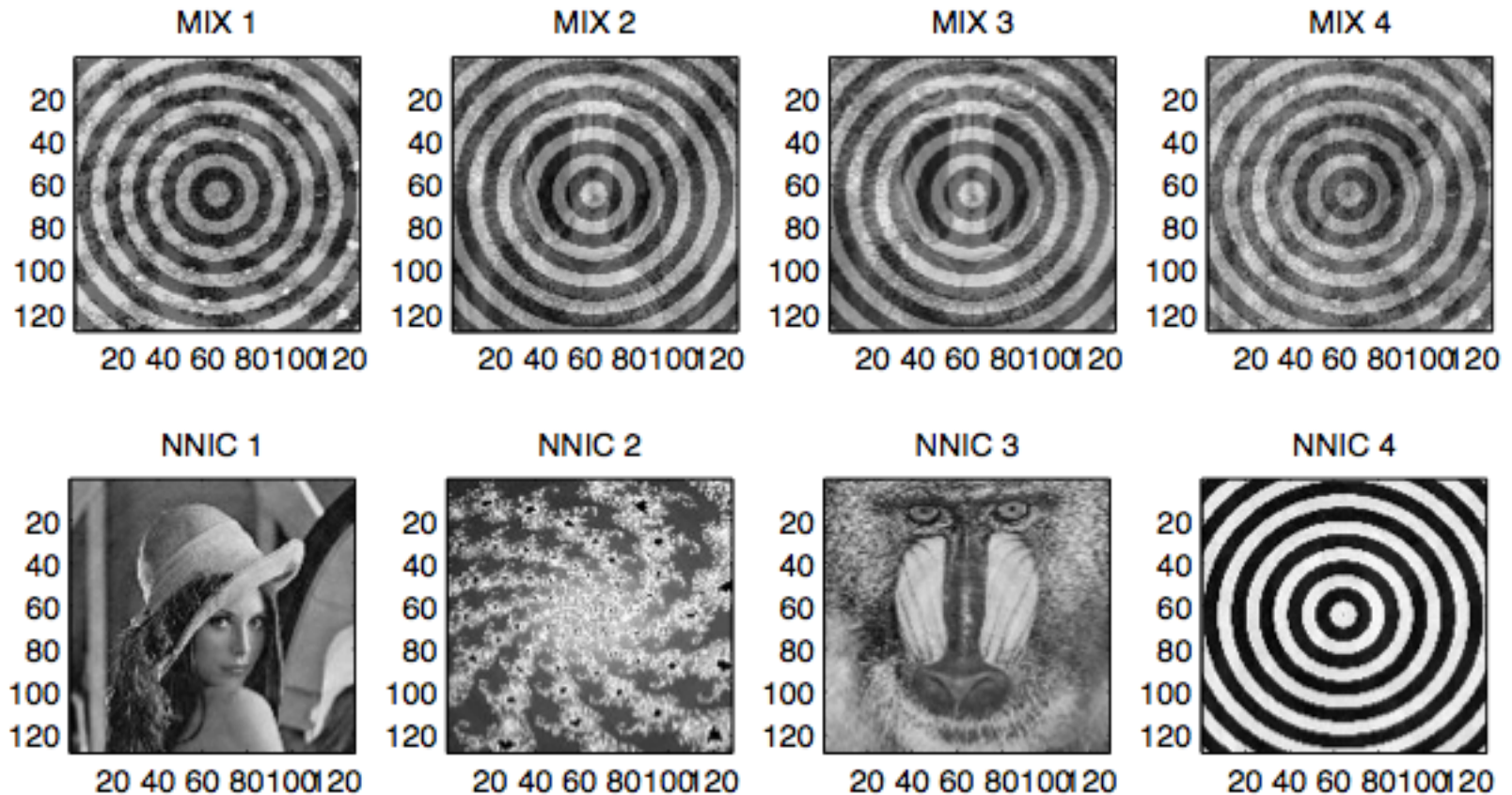


# 4-source separation example

Simone Fiori, Journal of Machine Learning Research 6 (2005) 743-781

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# Image Noise Removal Using ICA

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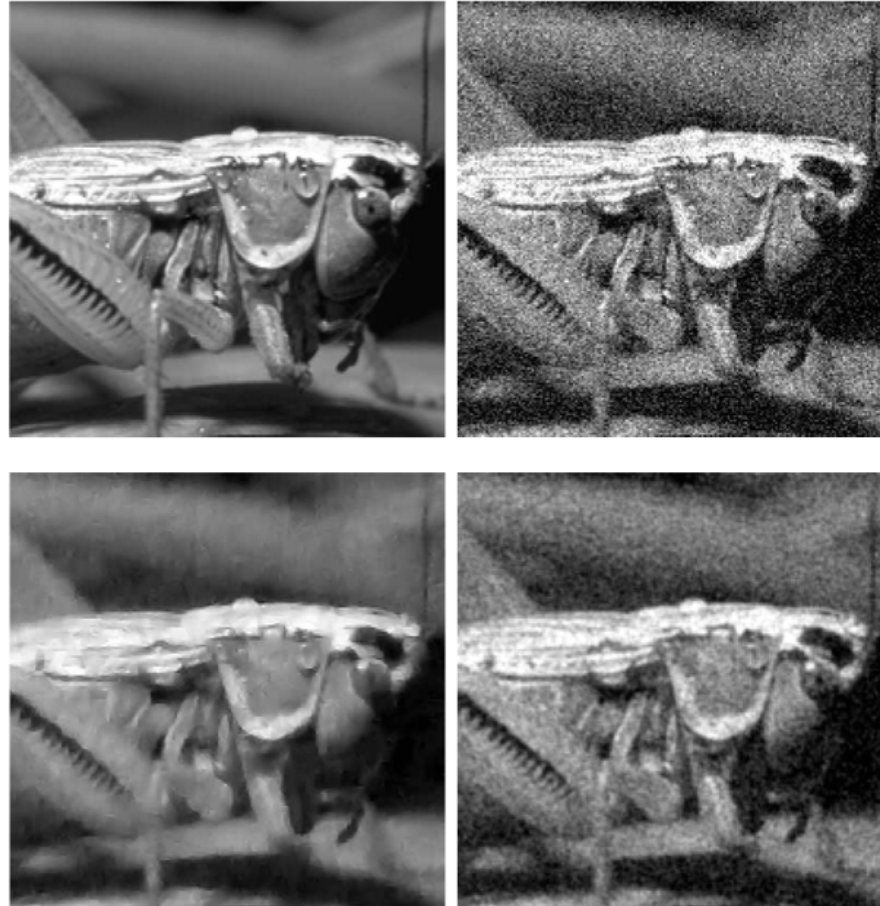


Figure 15: (from Hyvärinen, 1999d). *An experiment in denoising. Upper left: original image. Upper right: original image corrupted with noise; the noise level is 50 %. Lower left: the recovered image after applying sparse code shrinkage. Lower right: for comparison, a wiener filtered image.*

From Aapo Hyvärinen and Erkki Oja  
Neural Networks, 13(4-5):411-430, 200

# Amari's Recurrent Neural IC

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Fully recurrent neural network with self-inhibitory connections.

$$\tau_i \frac{dy_i}{dt} + y_i = x_i(t) - \sum_{j=1}^n \hat{w}_{ij}(t) y_j,$$

$$y(t) = (I + \hat{W}(t))^{-1} x(t),$$

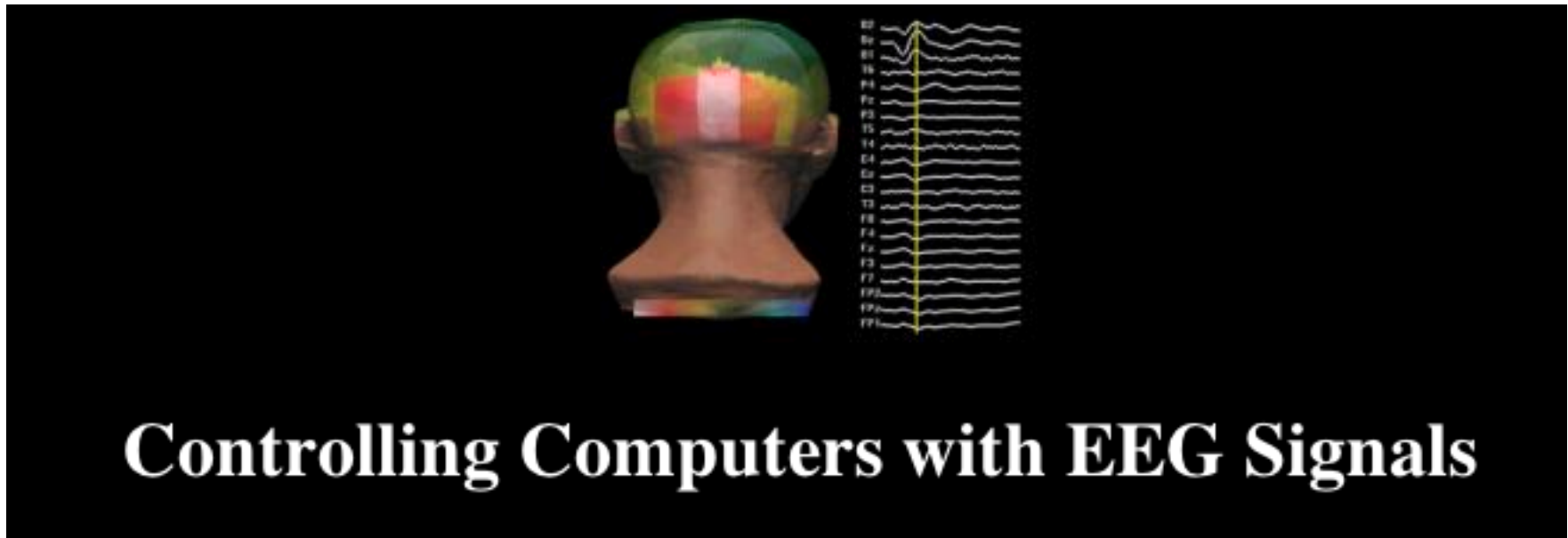
$$y(t) = x(t) - \hat{W}(t) y(t - \tau),$$

$$\frac{d\hat{W}}{dt} = -\mu(t) [I + \hat{W}] [\Lambda - f(y(t)) g^T(y(t))].$$

# Culpepper & Keller Project using ICA

<http://www.cs.hmc.edu/~bjc/eeg/>

<http://ieeexplore.ieee.org/iel5/7333/28199/01261745.pdf?arnumber=1261745>



ICA was used to achieve a separation among 12 EEG channels, used to discriminate between two or three modes of thought.

# Eye-blink EEG removal (Peterson and Anderson, 1999)

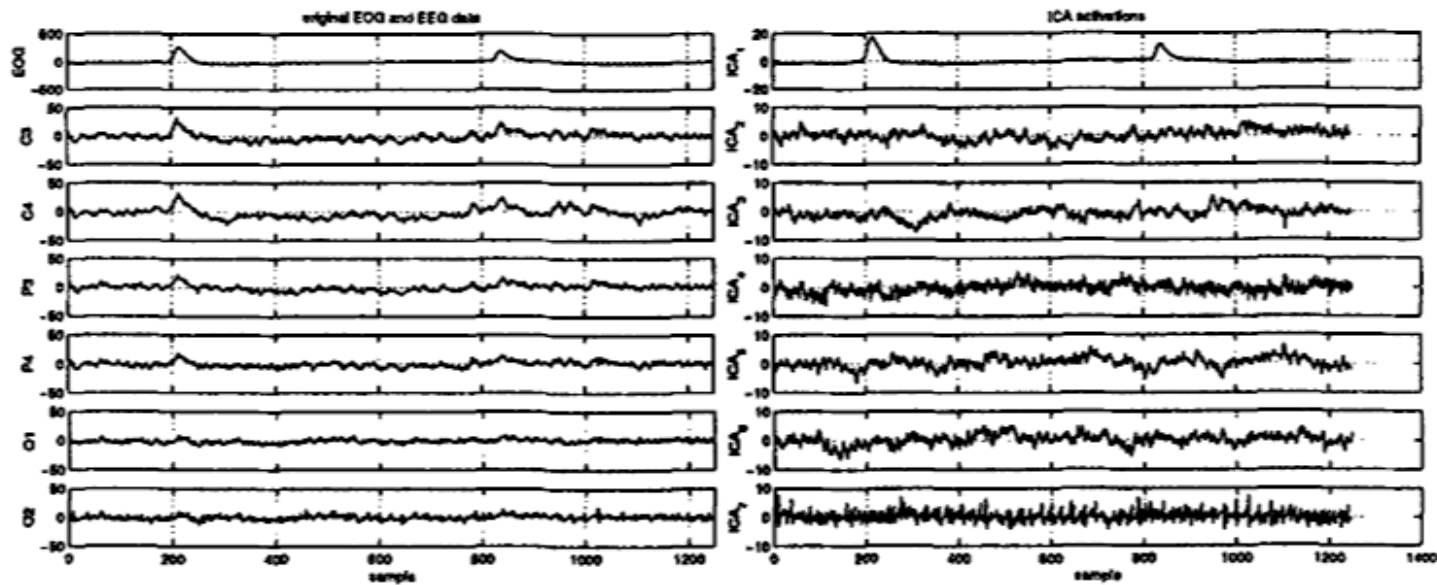


Fig. 1. Eye blink subtraction with ICA

# Financial Application of ICA to Time Series Analysis

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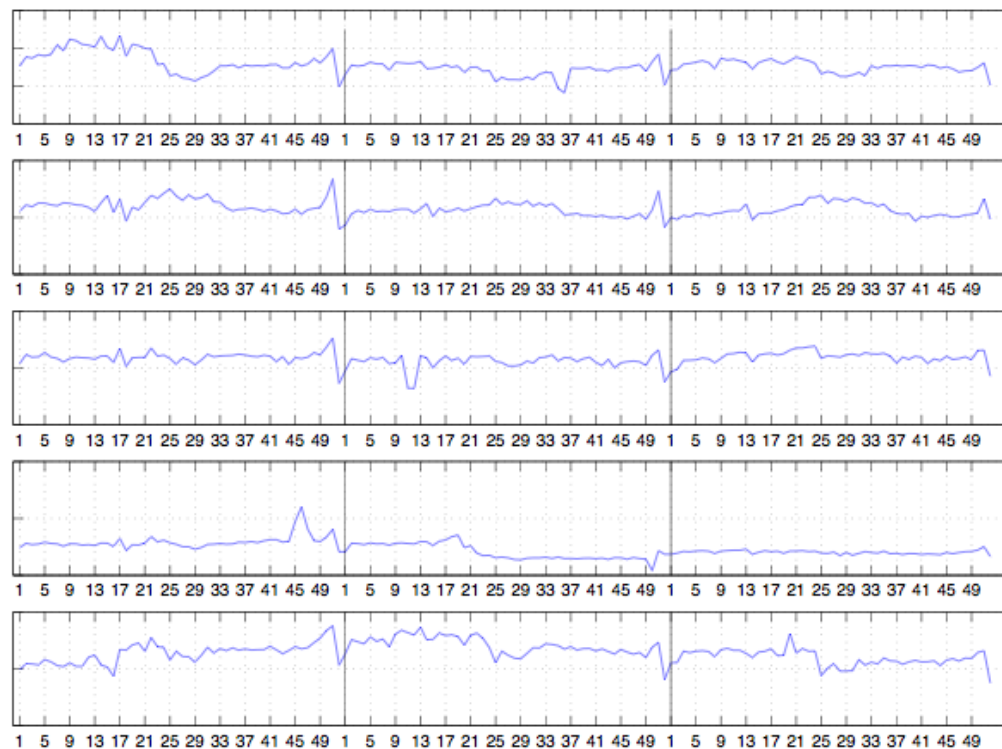


Figure 13: (from Kiviluoto and Oja, 1998). *Five samples of the original cashflow time series (mean removed, normalized to unit standard deviation). Horizontal axis: time in weeks.*

# Four Independent Factors Found using ICA

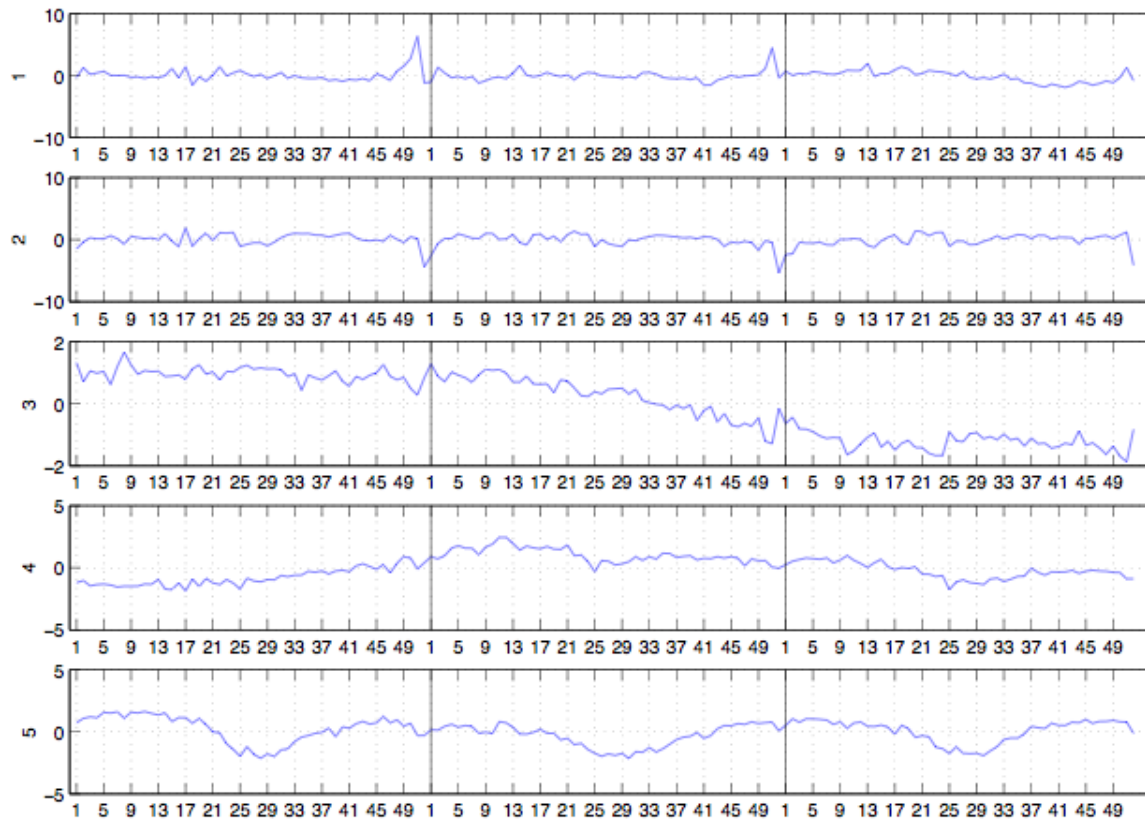


Figure 14: (from Kiviluoto and Oja, 1998). *Four independent components or fundamental factors found from the cashflow data.*

# ICA Sources

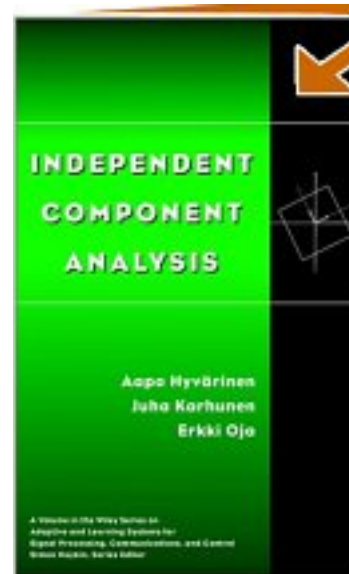
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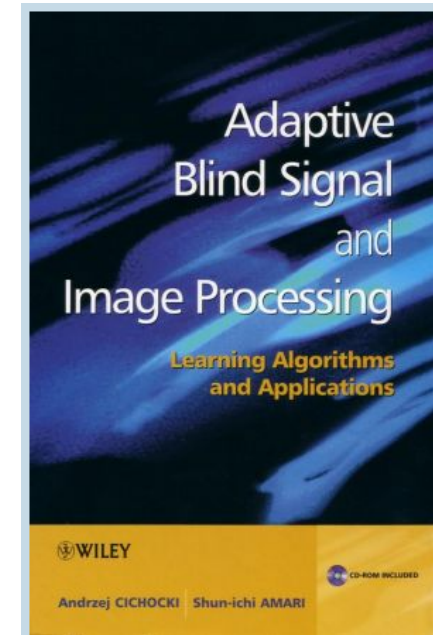
Links: <http://cni.salk.edu/~tony/ica.html>



James V. Stone



Aapo Hyvärinen,  
Juha Karhunen,  
Erkki Oja



Andrzej Cichocki,  
Shun-ichi Amari



# IB2010

June 12-13, Jyväskylä, Finland

UNIVERSITY ALLIANCE FINLAND

**The 1st InterBrain Symposium  
ICA conference and 10th EEGLAB workshop  
June 12-17, 2010  
Jyväskylä, Finland**

The 2-day International ICA conference will focus on theory of independent components analysis and its applications in brain research, mainly in EEG/ ERP analysis. The conference consists of talks by invited experts of the field as well as of oral sessions dealing with ICA and other related analysis techniques.

The conference is followed by hands-on EEGLAB workshop (for a limited number of participants).

## **Welcome**

Independent Component Analysis and Signal Separation is one of the most exciting current research areas in neural signal processing. We would like to welcome you to the **ICA Conference, The 1st InterBrain Symposium, and the 10th EEGLAB workshop**. These consecutive events will take place in Jyväskylä, Finland, a university town surrounded by lakes.

**The 2-day International ICA conference will focus on theory of independent components analysis and its applications in brain research, mainly in EEG/ ERP analysis.** The conference

# ICA Extensions

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- ICA does not take sequence or structure into account; it is purely statistical.
- It might be possible to do better by considering non-statistical information.