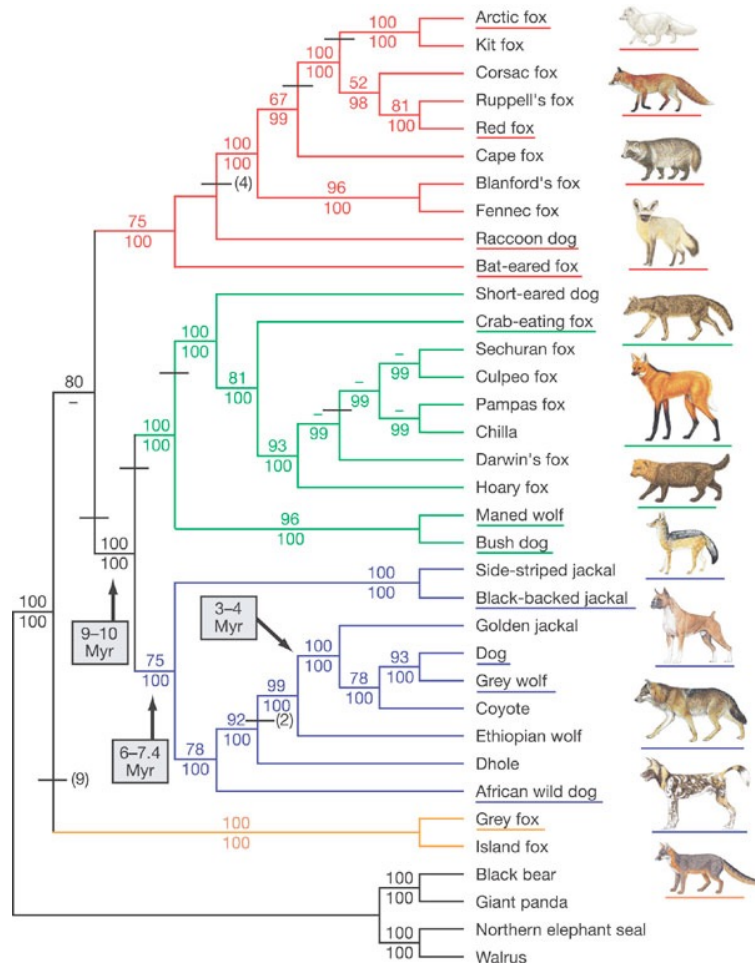
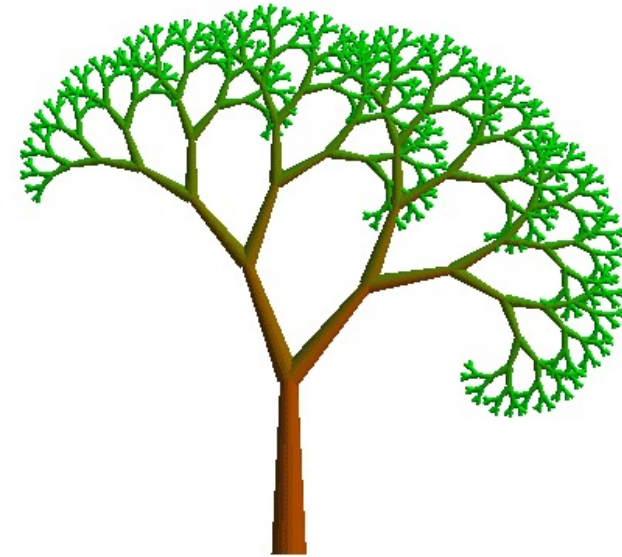


# “I wonder about Trees” –Robert Frost



“We wonder about Robert Frost” - Trees



Hey! How come no turtles in this tree?





**CS 5 Green**

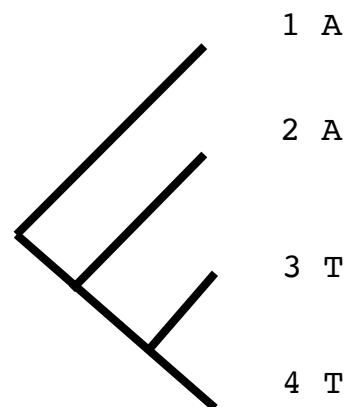
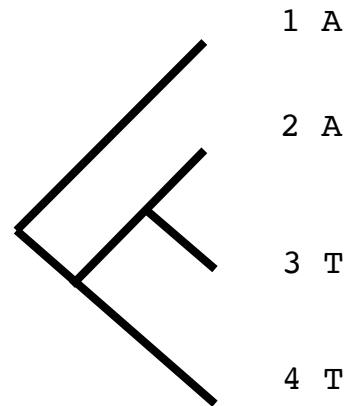
## **Learning Goals**

- Describe the parsimony principle
- Introduce method for enumerating all trees

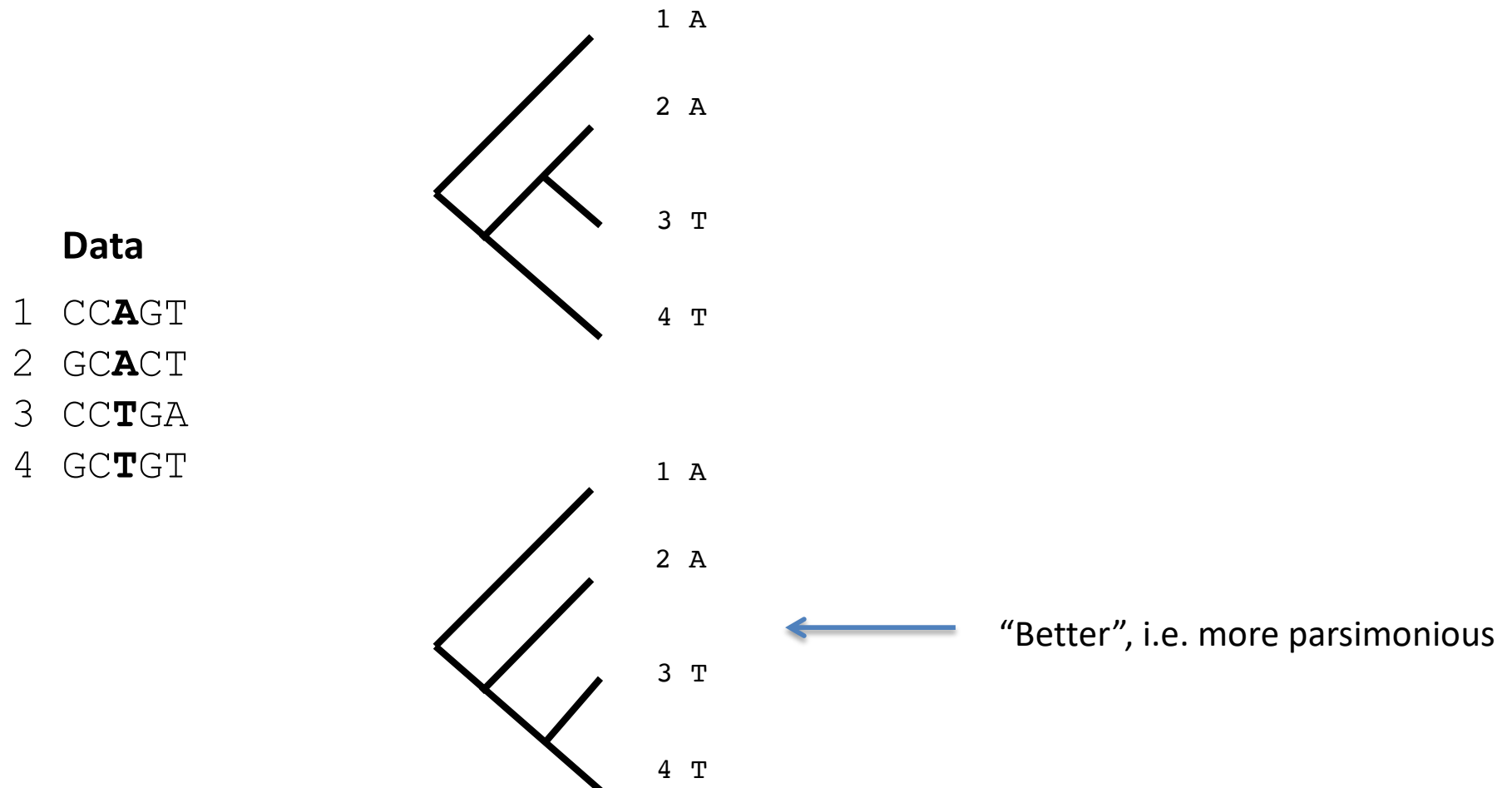
# Trees and the parsimony principle

## Data

1 CC**A**GT  
2 GC**A**CT  
3 CC**T**GA  
4 GC**T**GT



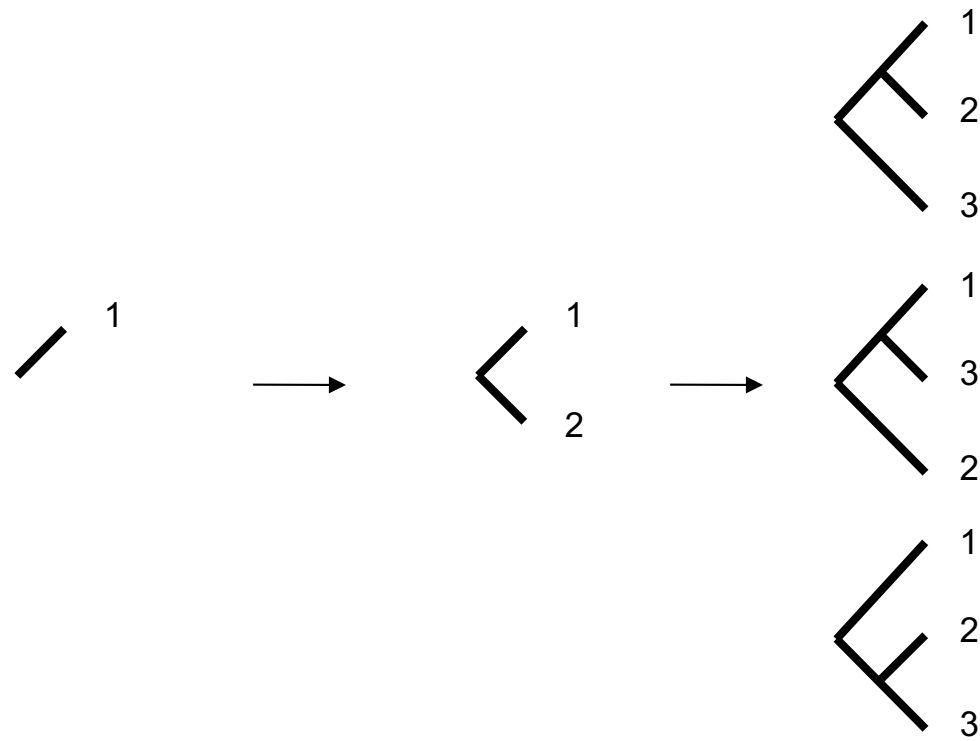
# Trees and the parsimony principle



# Another general strategy for inferring phylogenies

- Generate all possible trees
- Pick the most parsimonious given some data

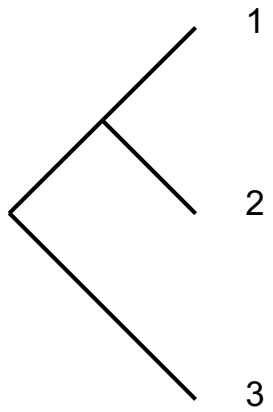
# Generating all possible trees from the ground up



# species (leaves)	1	2	3	4
# possible trees	1	1	3	<div data-bbox="1759 1349 1866 1451" style="border: 1px solid black; width: 50px; height: 60px; display: flex; align-items: center; justify-content: center;"> </div>

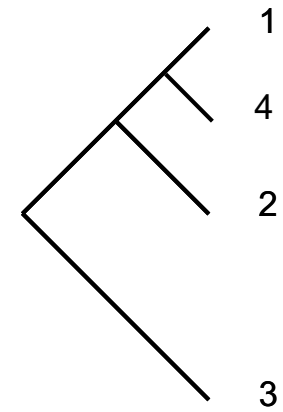
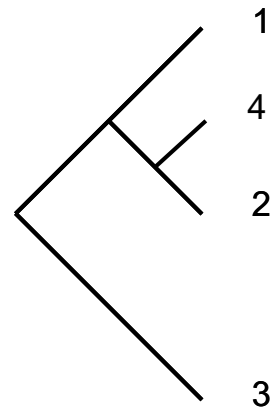
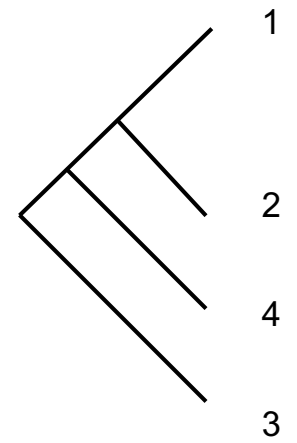
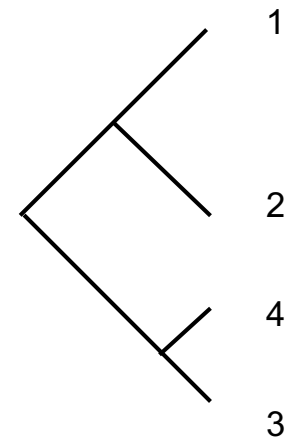
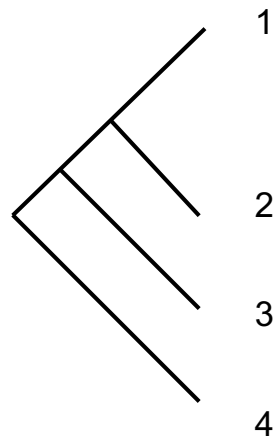
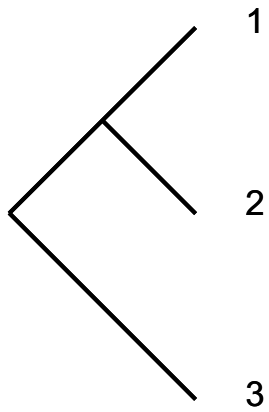


Draw all possible trees that result from adding a species 4 to this tree.

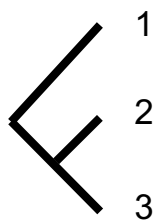
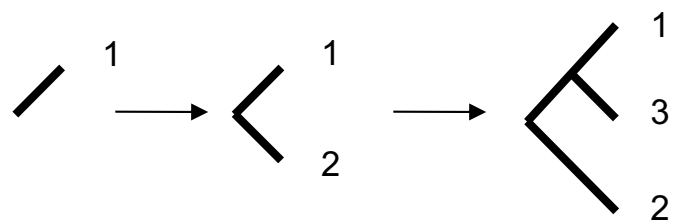
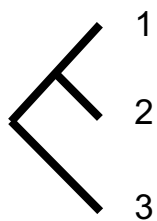




Draw all possible trees that result from adding a species 4 to this tree.





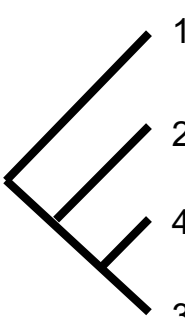
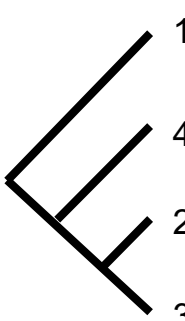
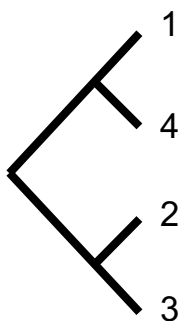
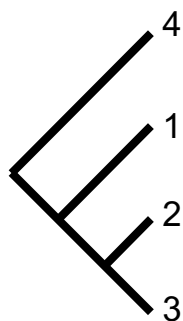
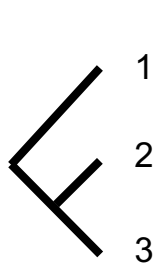
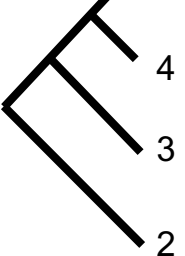
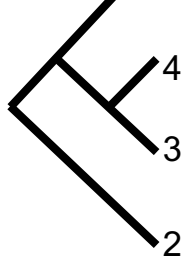
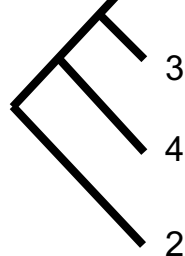
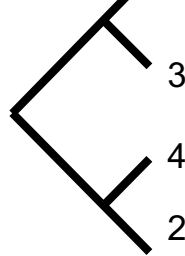
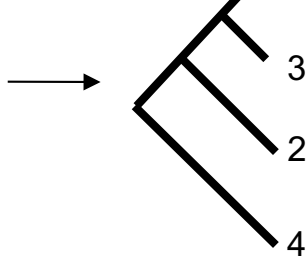
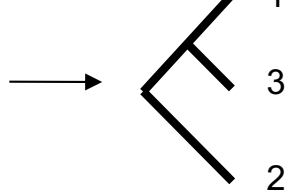
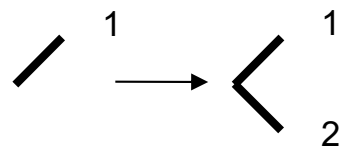
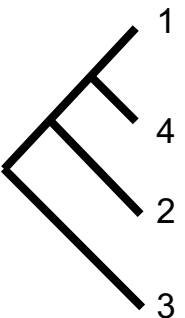
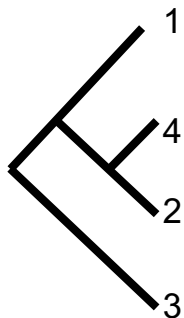
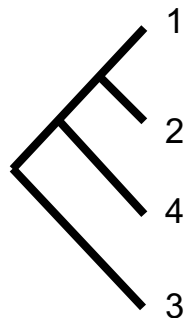
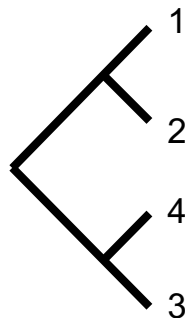
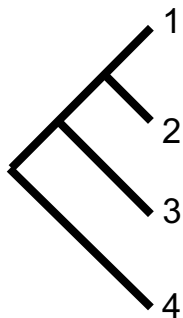
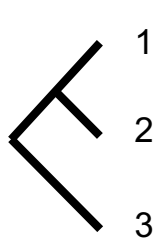


1

2

3

4



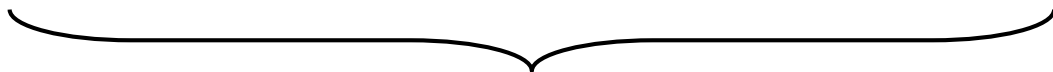
1



2



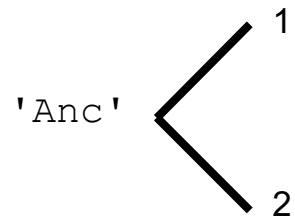
3



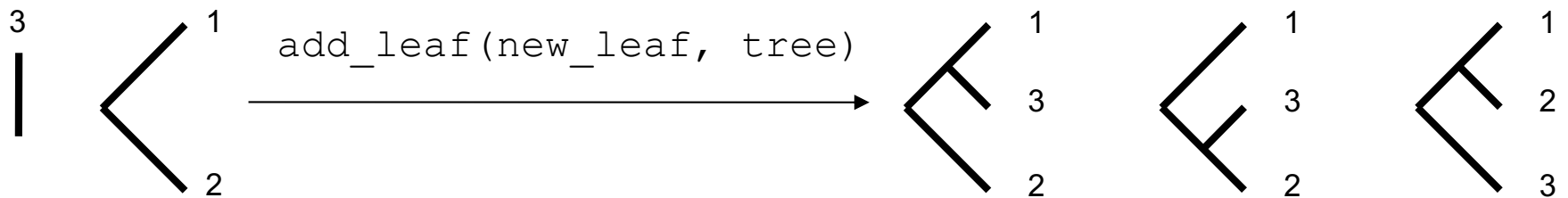
4

# A convention for naming internal/ancestral nodes

```
tree = ( 'Anc', (1, (), ()) , (2, (), ()) )
```



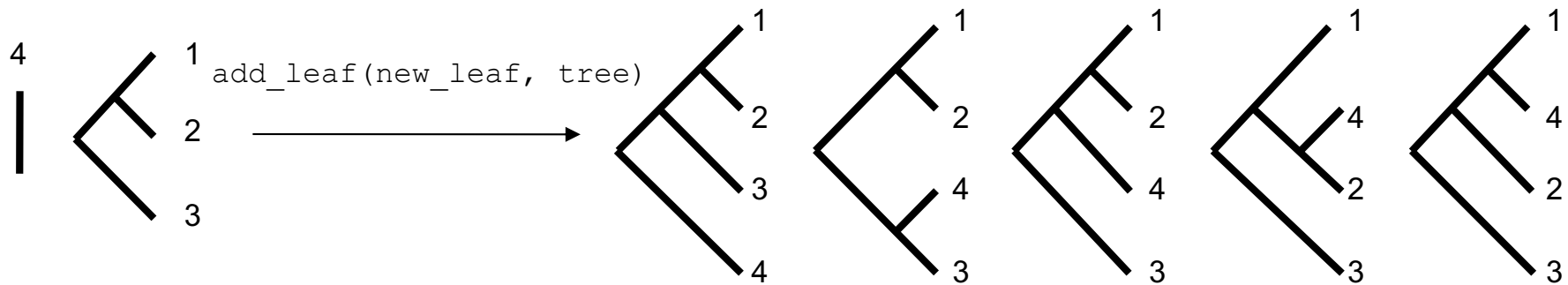
# The add\_leaf function



```
>>> leaf = ( 3, (), () )
>>> tree = ( 'Anc', (1,(),()), (2,(),()) )
>>> add_leaf(leaf, tree)
[
  ('Anc', (3, (), ()), ('Anc', (1, (), ()), (2, (), ()))) ,
  ('Anc', ('Anc', (3, (), ()), (1, (), ())), (2, (), ())) ,
  ('Anc', (1, (), ()), ('Anc', (3, (), ()), (2, (), ()))) ,
]
```

Which illustration does the last tuple tree correspond to?

# The add\_leaf function



```
>>> leaf = ( 4, (), () )
>>> tree = ('Anc', (3, (), ()) , ('Anc', (1, (), ()), (2, (), ()))) )
>>> add_leaf(leaf, tree)
[
  ('Anc', (4, (), ()), ('Anc', (3, (), ()), ('Anc', (1, (), ()), (2, (), ())))),
  ('Anc', ('Anc', (4, (), ()), (3, (), ())), ('Anc', (1, (), ()), (2, (), ())),
  ('Anc', (3, (), ()), ('Anc', (4, (), ()), ('Anc', (1, (), ()), (2, (), ())))),
  ('Anc', (3, (), ()), ('Anc', ('Anc', (4, (), ()), (1, (), ())), (2, (), ())),
  ('Anc', (3, (), ()), ('Anc', (1, (), ()), ('Anc', (4, (), ()), (2, (), ())))
]
```

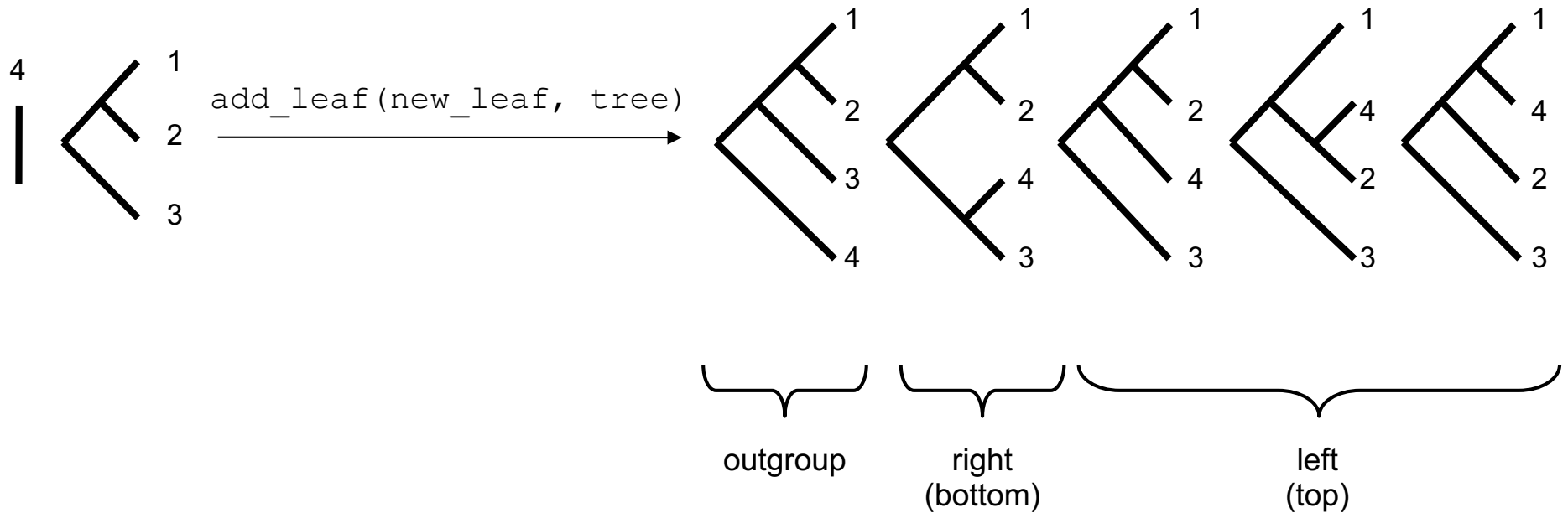
```
def add_leaf(new_leaf, tree):  
    """Returns a list of all possible trees that result from  
    adding new_leaf to tree."""  
    root, left, right = tree  
    anc = "Anc"
```



```
def add_leaf(new_leaf, tree):  
    """Returns a list of all possible trees that result from  
    adding new_leaf to tree."""  
    root, left, right = tree  
    anc = "Anc"  
    if left == (): # a leaf.  
        new_tree = (anc, new_leaf, tree)  
        return [new_tree] # wrap it in a list!
```



# General case: three steps at each node



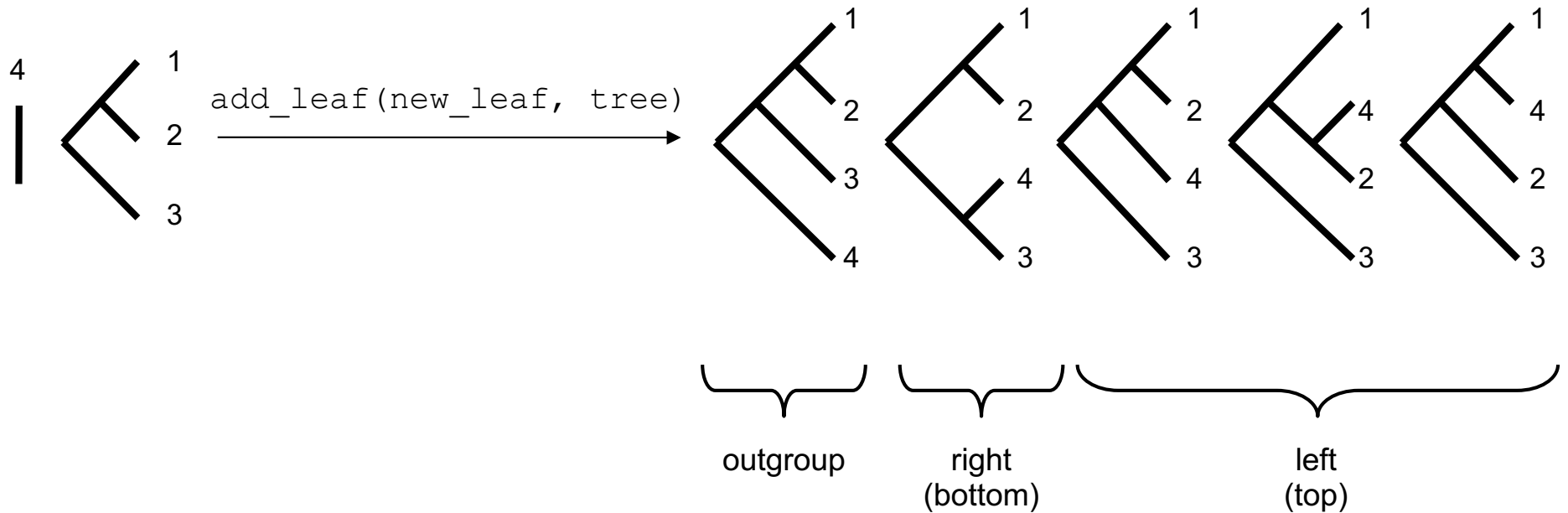


```
def add_leaf(new_leaf, tree):
    """Returns a list of all possible trees that result from
    adding new_leaf to tree."""
    root, left, right = tree
    anc = "Anc"
    if left == (): # a leaf.
        new_tree = (anc, new_leaf, tree)
        return [new_tree] # wrap it in a list!
    else:
        output_trees = []

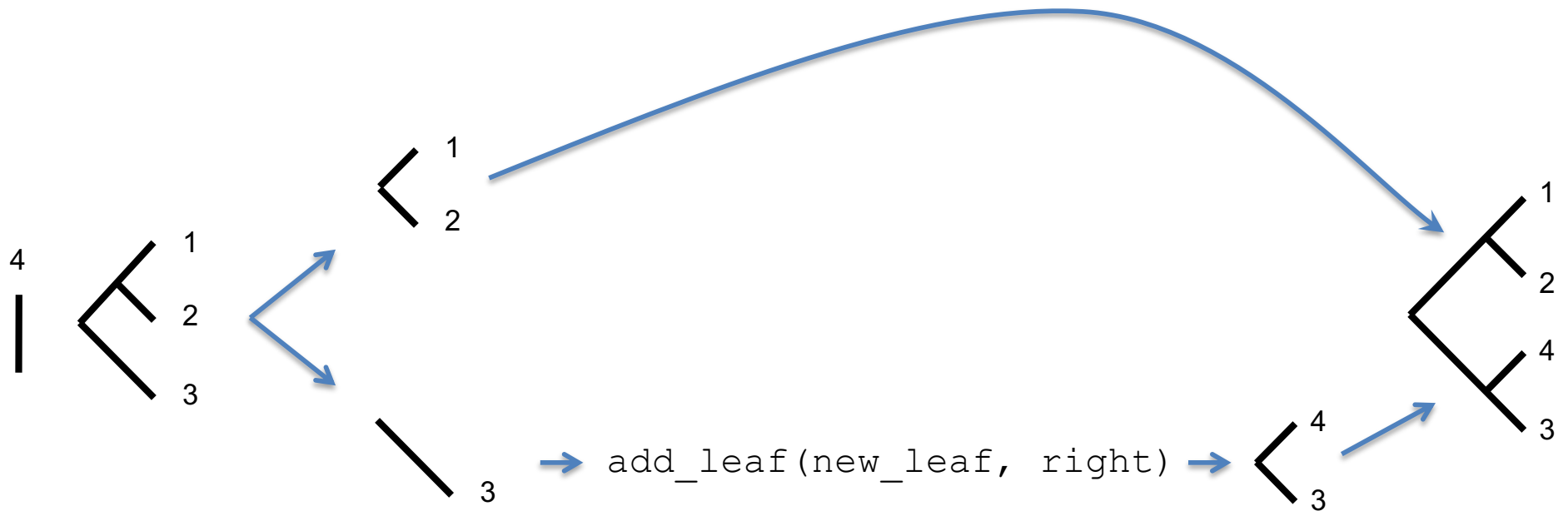
        # put new_leaf as outgroup
        output_trees.append((anc, new_leaf, tree))
```



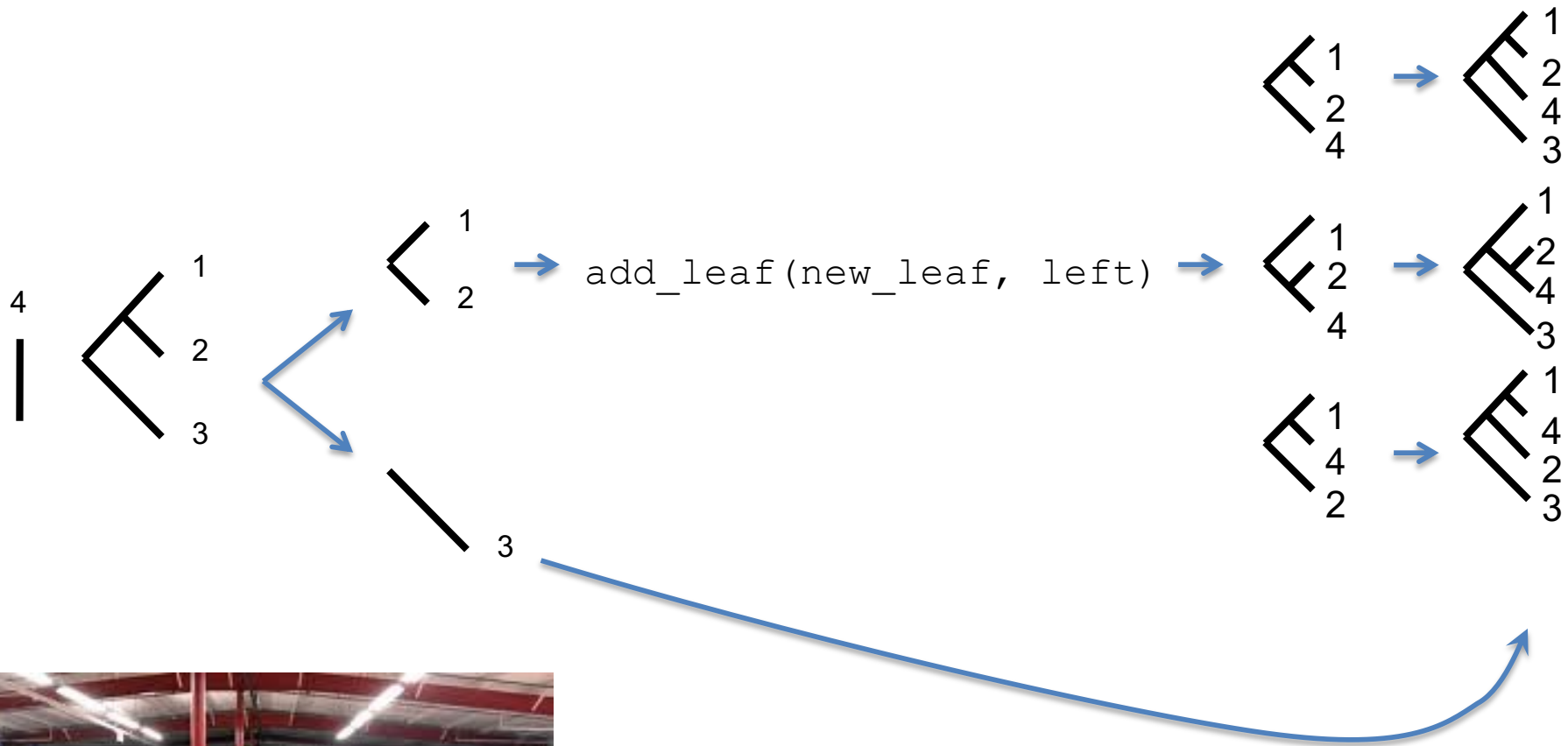
# General case: three steps at each node



# The add\_leaf chop shop: right tree



# The add\_leaf chop shop: left tree





```
def add_leaf(new_leaf, tree):  
    """Returns a list of all possible trees that result from  
    adding new_leaf to tree."""  
    root, left, right = tree  
    anc = "Anc"  
    if left == (): # a leaf.  
        new_tree = (anc, new_leaf, tree)  
        return [new_tree] # wrap it in a list!  
    else:  
        output_trees = []  
  
        # put new_leaf as outgroup  
        output_trees.append((anc, new_leaf, tree))
```

```
def add_leaf(new_leaf, tree):  
    """Returns a list of all possible trees that result from  
    adding new_leaf to tree."""  
    root, left, right = tree  
    anc = "Anc"  
    if left == (): # a leaf.  
        new_tree = (anc, new_leaf, tree)  
        return [new_tree] # wrap it in a list!  
    else:  
        output_trees = []  
  
        # put new_leaf as outgroup  
        output_trees.append((anc, new_leaf, tree))  
  
        # recur to add new_leaf on branches of right subtree  
        temp_right_trees = add_leaf(new_leaf, right)  
        for temp_right_tree in temp_right_trees:  
            new_tree = (anc, left, temp_right_tree)  
            output_trees.append(new_tree)
```

```
def add_leaf(new_leaf, tree):
    """Returns a list of all possible trees that result from
    adding new_leaf to tree."""
    root, left, right = tree
    anc = "Anc"
    if left == (): # a leaf.
        new_tree = (anc, new_leaf, tree)
        return [new_tree] # wrap it in a list!
    else:
        output_trees = []

        # put new_leaf as outgroup
        output_trees.append((anc, new_leaf, tree))

        # recur to add new_leaf on branches of right subtree
        temp_right_trees = add_leaf(new_leaf, right)
        for temp_right_tree in temp_right_trees:
            new_tree = (anc, left, temp_right_tree)
            output_trees.append(new_tree)

        # recur to add new_leaf on branches of left subtree
        temp_left_trees = add_leaf(new_leaf, left)
        for temp_left_tree in temp_left_trees:
            new_tree = (anc, temp_left_tree, right)
            output_trees.append(new_tree)

    return output_trees
```

Demo!

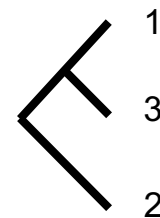
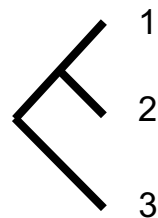
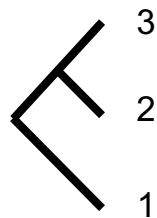




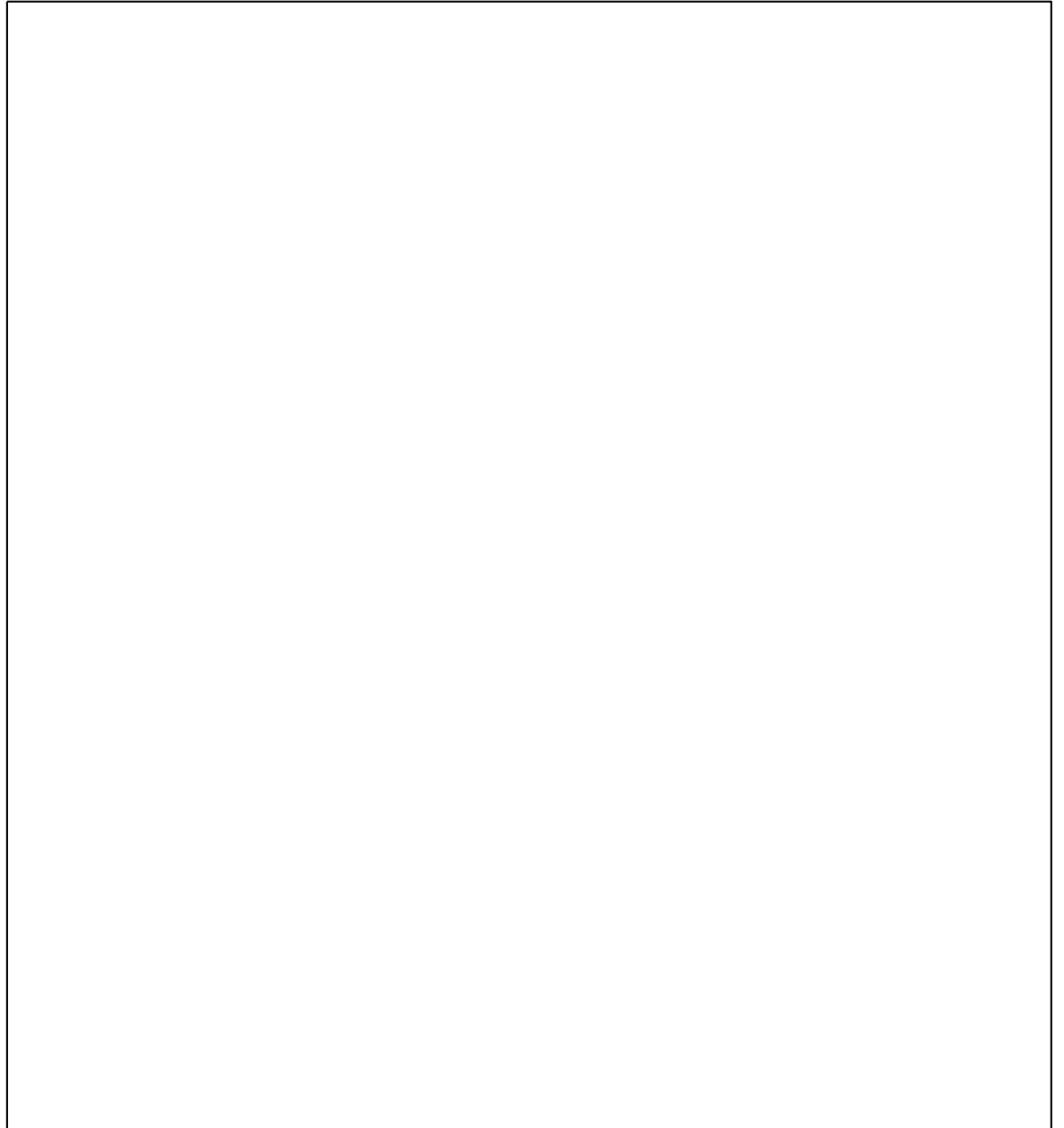
# (Bonus) homework problem:

## `all_trees`

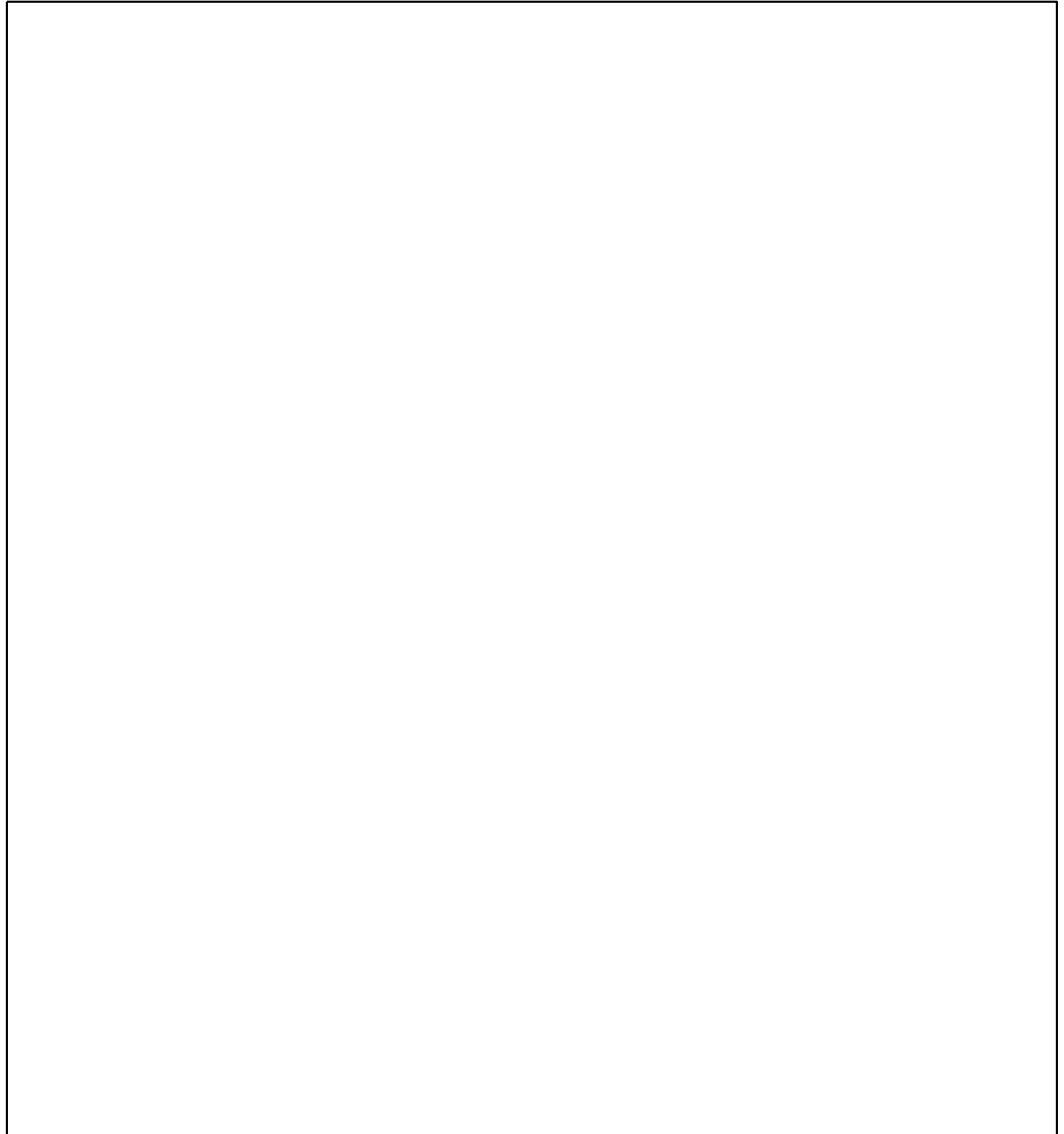
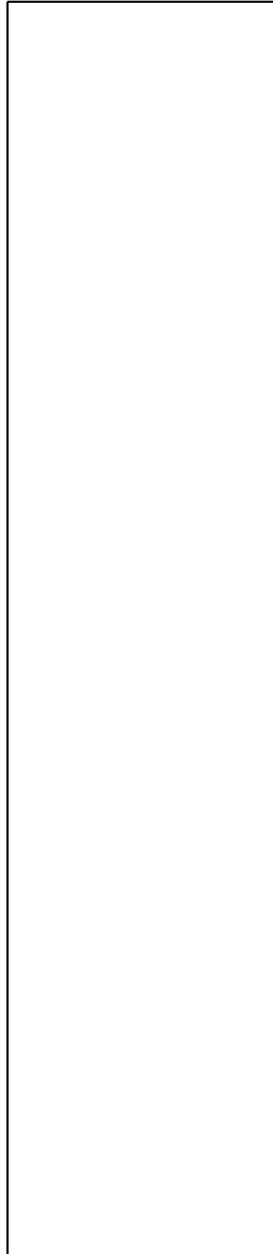
```
def all_trees(leaf_names):  
    """Given a list of species, returns a list of all  
    possible tree topologies."""  
  
>>> all_trees( [1,2,3] )  
[  
    ('Anc', (1, (), ()) , ('Anc', (2, (), ()), (3, (), ()))),  
    ('Anc', ('Anc', (1, (), ()), (2, (), ())), (3, (), ())),  
    ('Anc', (2, (), ()), ('Anc', (1, (), ()), (3, (), ())))  
]
```

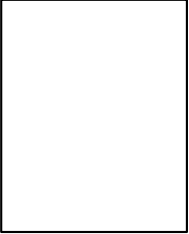


```
all_trees([4, 3, 2, 1])
```



`all_trees([3, 2, 1])` ← `all_trees([4, 3, 2, 1])`

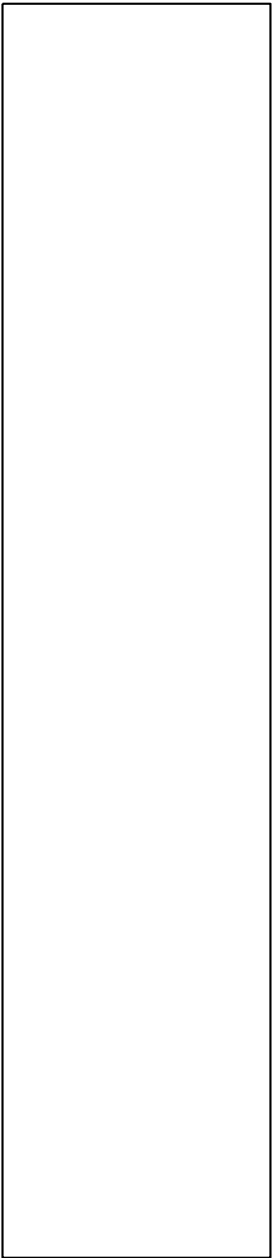




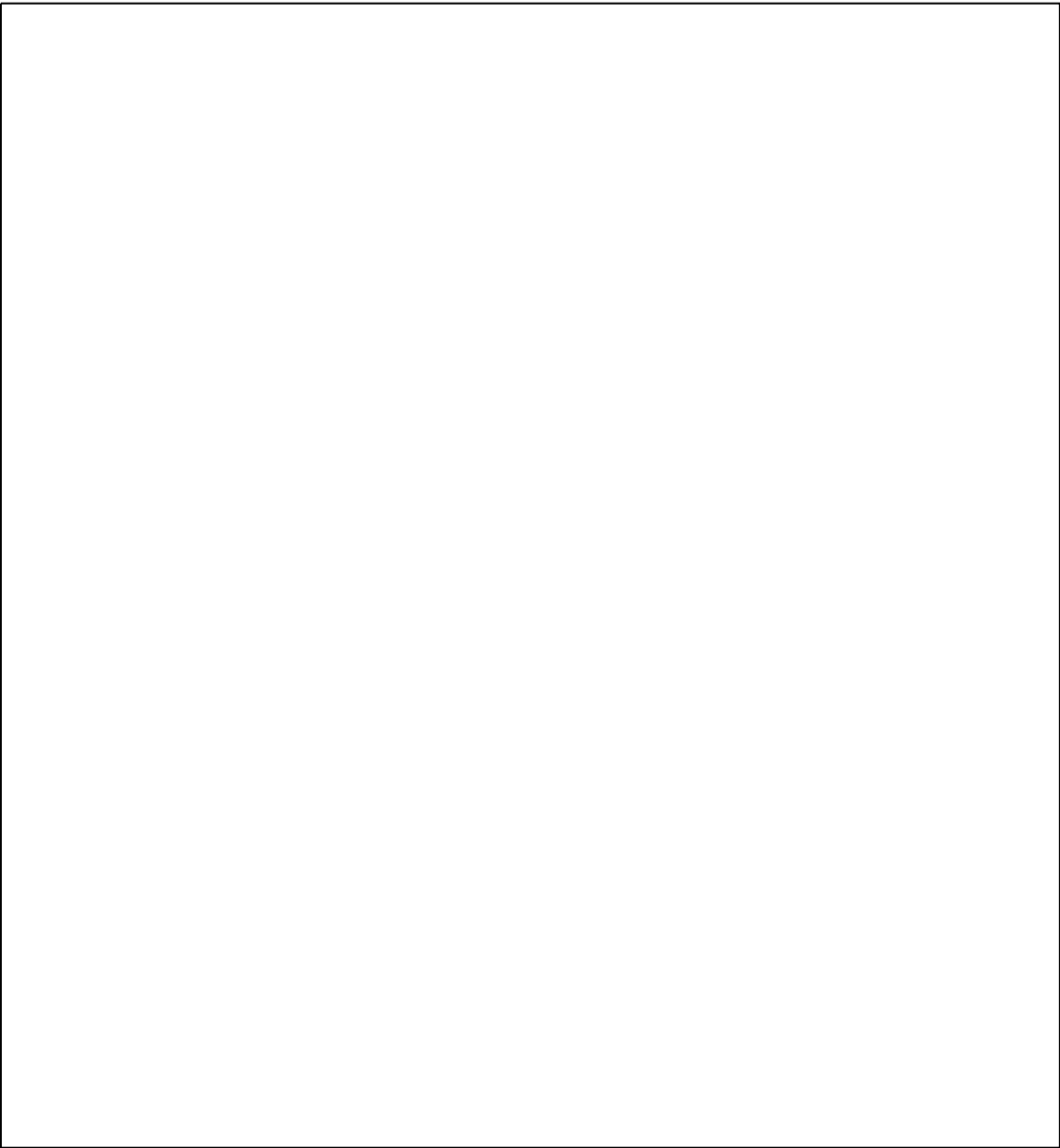
`all_trees([2, 1])`

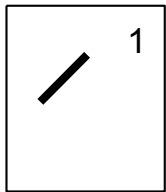


`all_trees([3, 2, 1])`

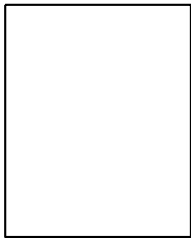


`all_trees([4, 3, 2, 1])`





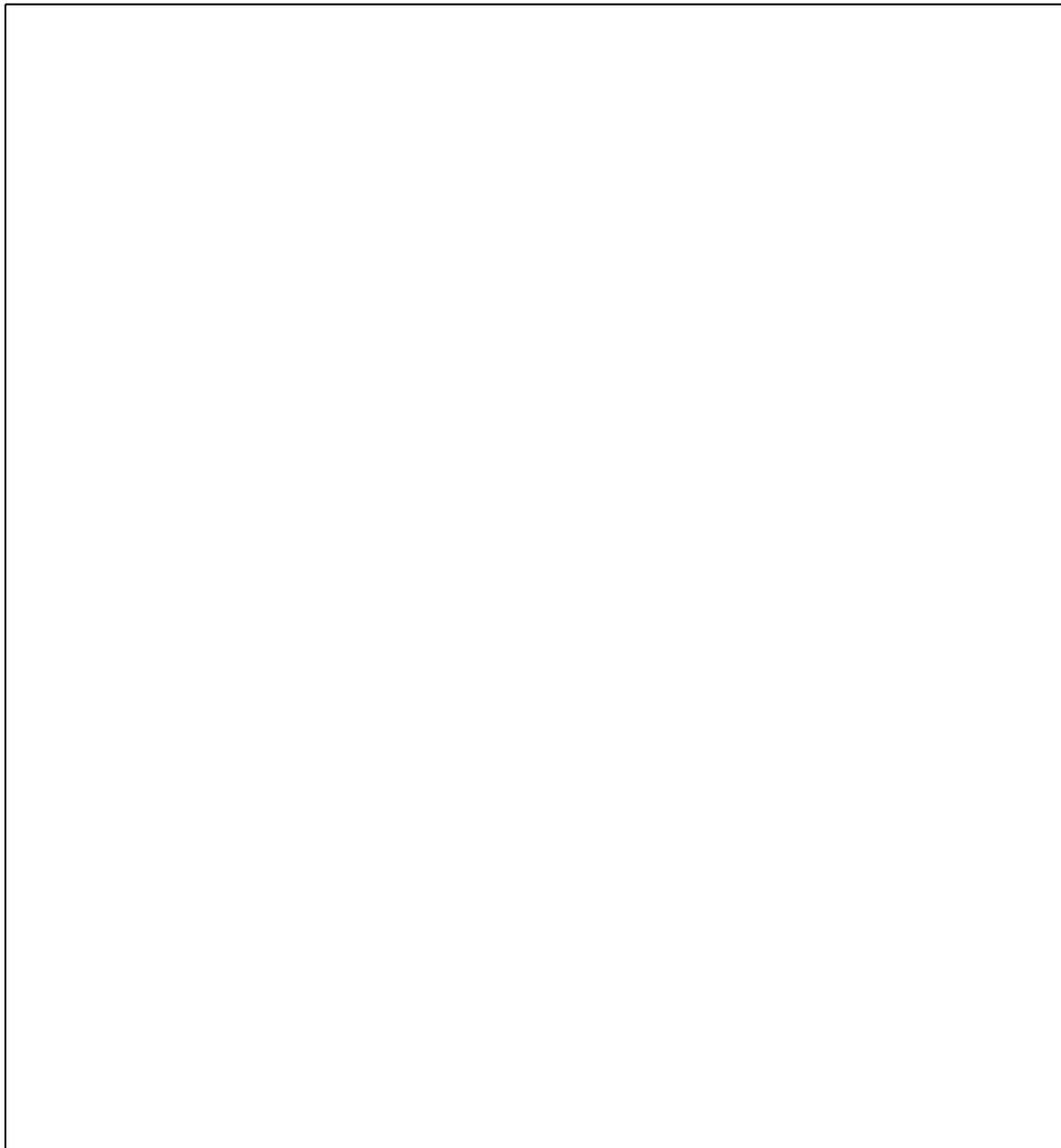
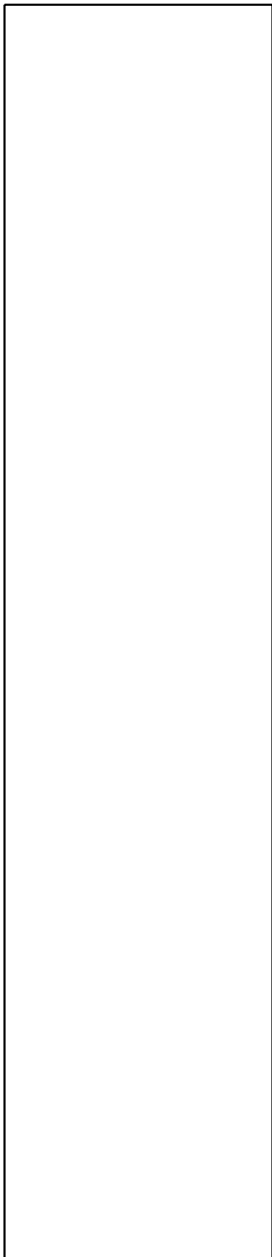
`all_trees([1])`

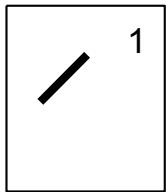


`all_trees([2, 1])`

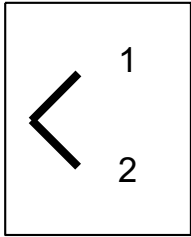
`all_trees([3, 2, 1])`

`all_trees([4, 3, 2, 1])`





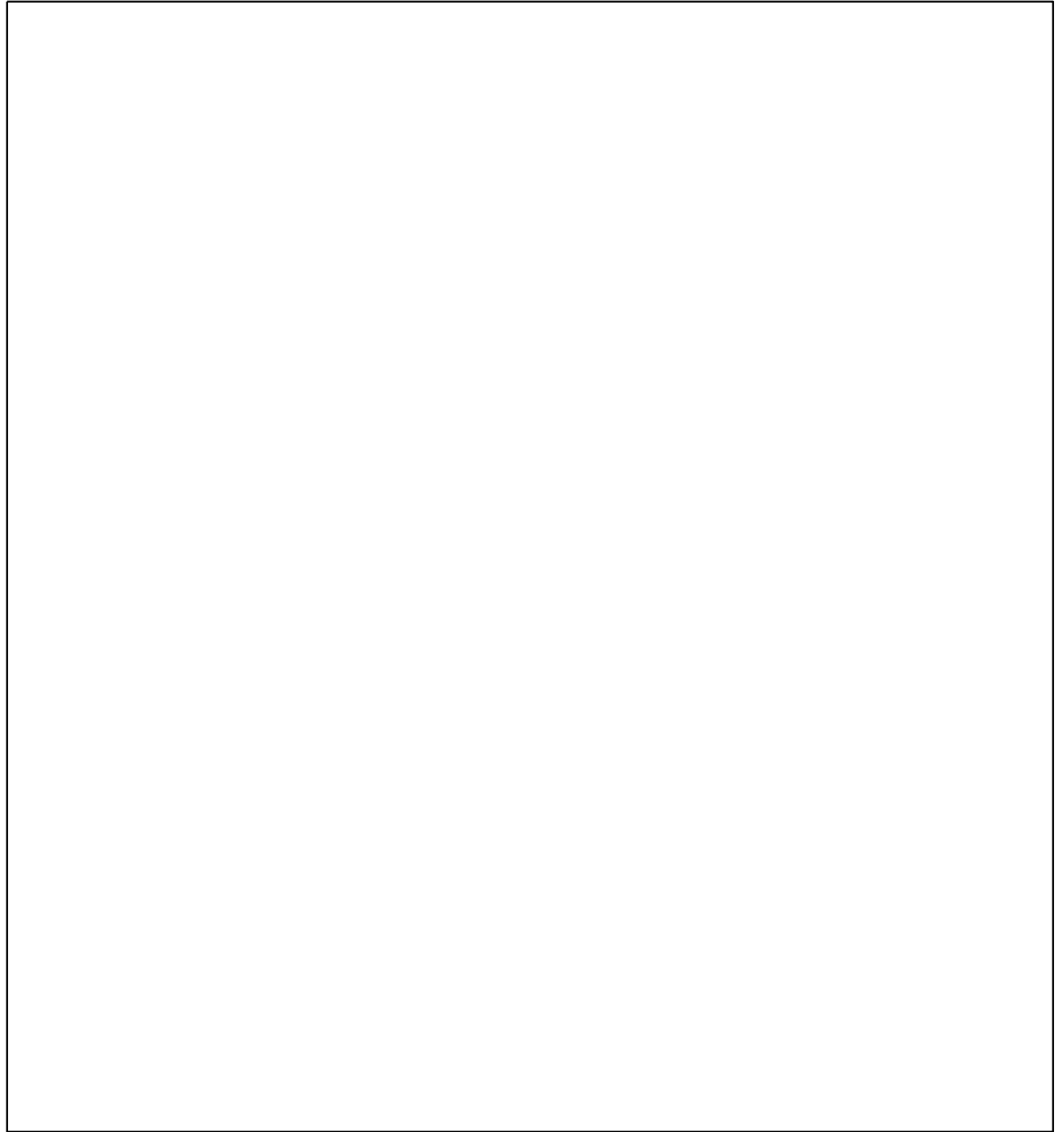
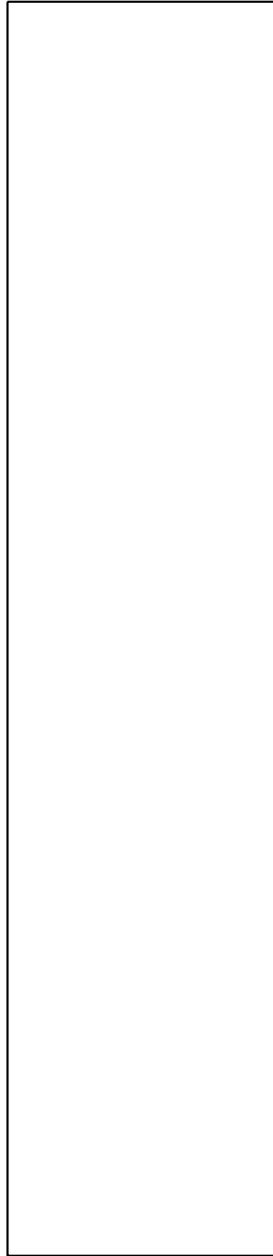
`all_trees([1])`

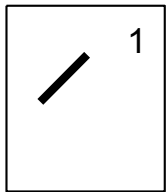


`all_trees([2, 1])`

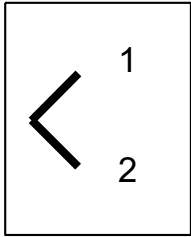
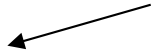
`all_trees([3, 2, 1])`

`all_trees([4, 3, 2, 1])`





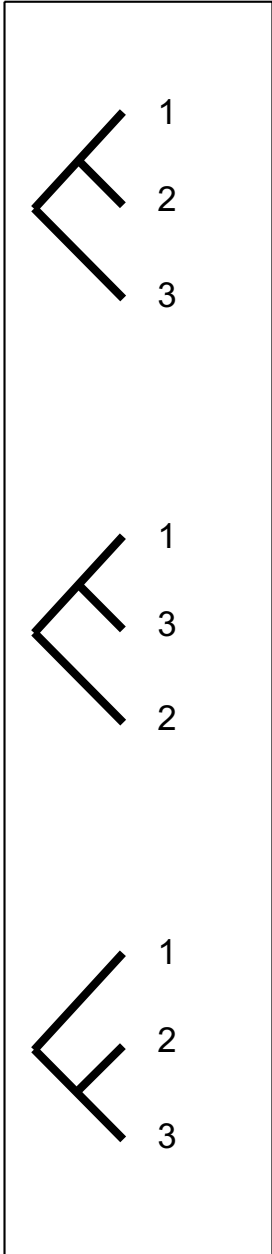
`all_trees([1])`



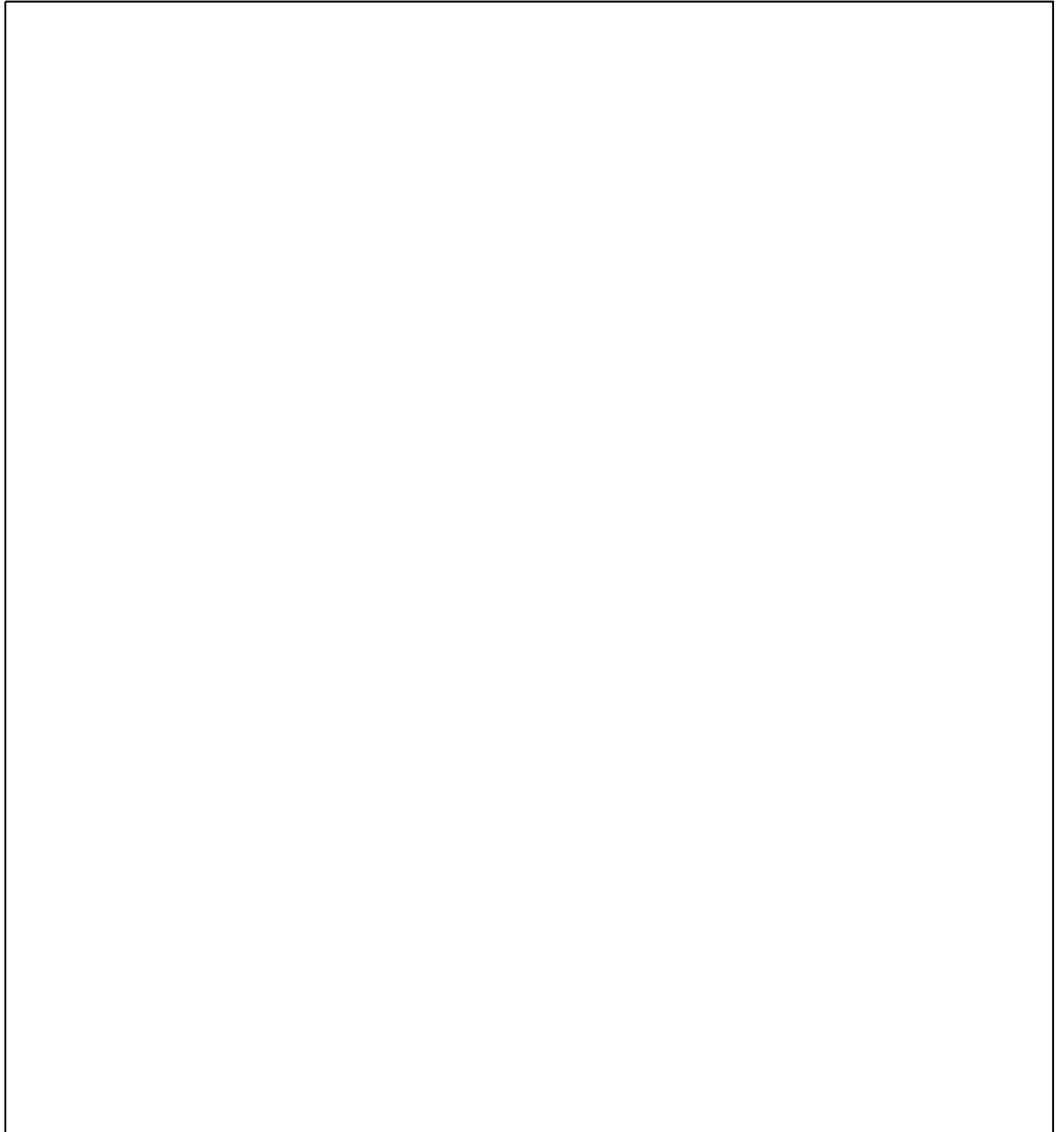
`all_trees([2, 1])`

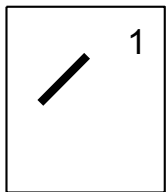


`all_trees([3, 2, 1])`

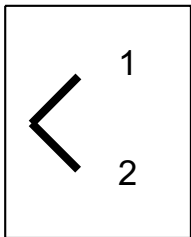


`all_trees([4, 3, 2, 1])`



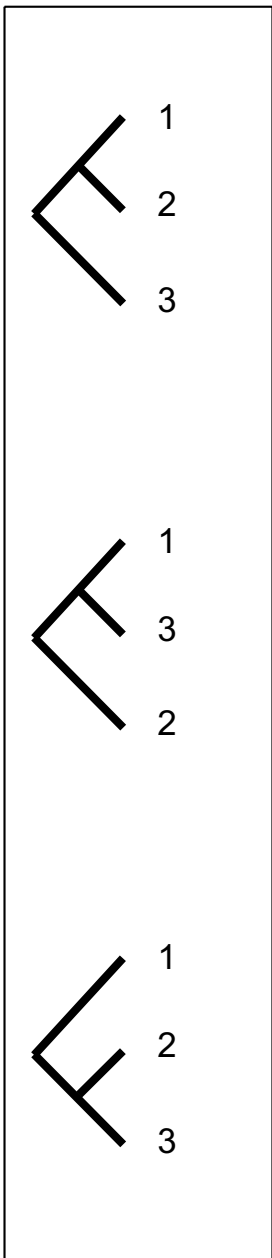


`all_trees([1])`

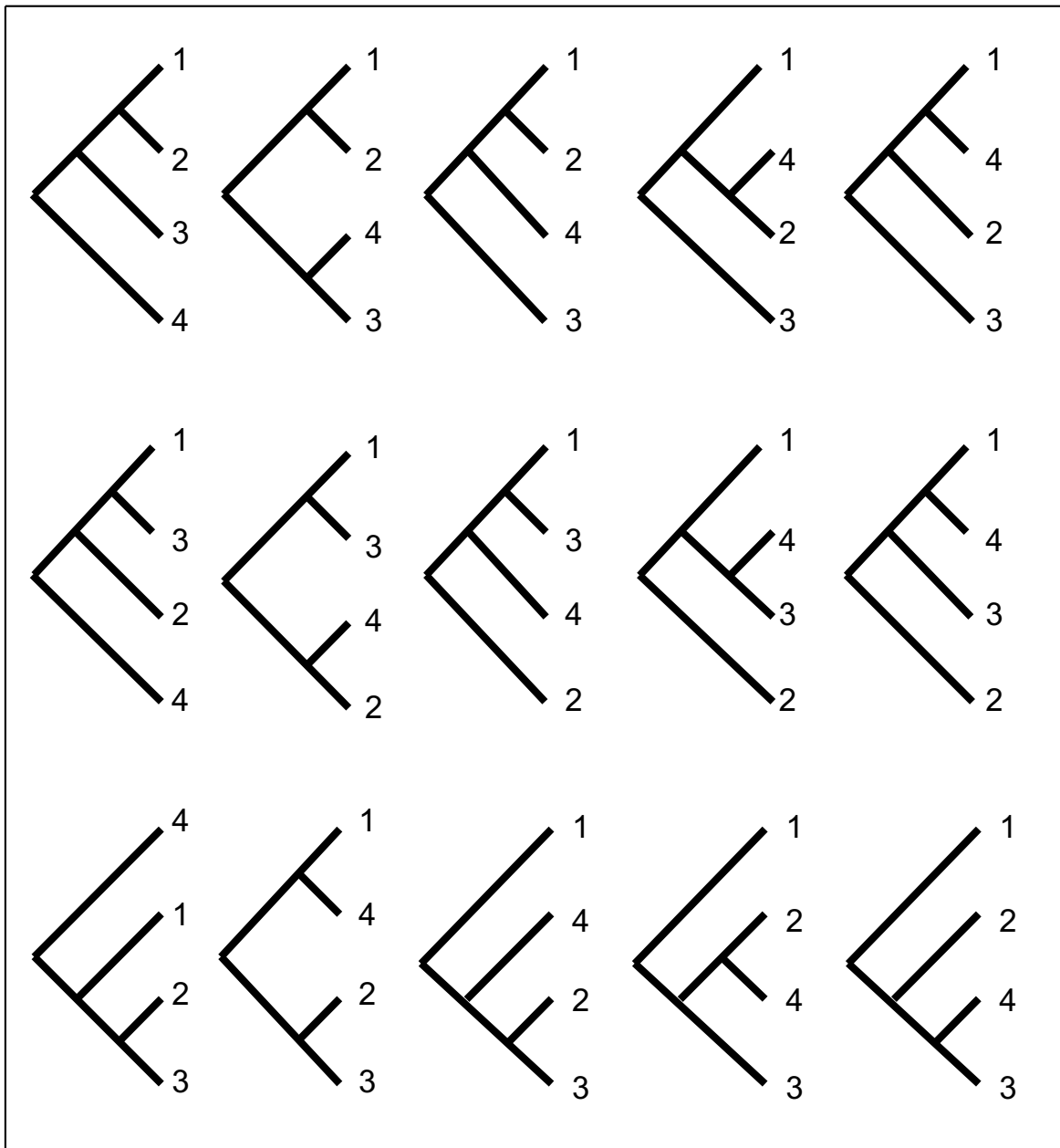


`all_trees([2, 1])`

`all_trees([3, 2, 1])`



`all_trees([4, 3, 2, 1])`





# One general strategy for inferring phylogenies

- Generate all possible trees
- Pick the most parsimonious given some data

# The number of trees grows quickly...

# species (leaves)	1	5	10	15	20	25
# possible trees	1	105	34,459,425	$2.13458 \times 10^{14}$	$8.200795 \times 10^{21}$	$1.192568 \times 10^{30}$





# Programming motifs: all vs. all

```
protsA = ['PLLYK', 'QSTE', 'NITQIVG', 'INE', 'QVAEA', 'YMSA']  
protsB = ['LAGADLEQ', 'LAL', 'EAMERY', 'ENLEL']
```

	B1	B2	B3	B4
A1				
A2				
A3				
A4				
A5				
A6				

```
d = {}  
for pA in protsA:  
    for pB in protsB:  
        d[(pA,pB)] = memoAlignScore(pA, pB, -9, blosum62, {})
```

# Programming motifs: running the gauntlet

```
rnas = ['AUGACGCAGUAGUCA', 'UAGACAGUA', 'AGGUACAUC'...]
```

- If no RNA has a fold score above 7, return False
- Otherwise return True

```
for rna in rnas:  
    if fold(rna) > 7:  
        return True  
return False
```

# Programming motifs: finding extremz

```
dictionary = [  
    "abdomen",  
    "abdominal",  
    "abduct",  
    "abduction",  
    "aberration",  
    "abet",  
    "abhor",  
    "abhorrence",  
    "abhorrent",  
    "abide",  
    "abiding",  
    "ability",  
    "abject",  
    "ablaze",  
    ...  
    ...  
    etc.  
    ...  
    ...]  
  
def z(input):  
    '''Count z's in a string'''  
    counter = 0  
    for symbol in input:  
        if symbol == 'z':  
            counter = counter + 1  
    return counter  
  
def extremz(words):  
    '''Find and return the word with the most z's'''  
    best_count = 0  
    best_word = ""  
    for word in words:  
        count = z(word)  
        if count > best_count:  
            best_count = count  
            best_word = word  
    return best_word
```


# Recursion on trees: graft

```
groodies = ("Q",
            ("R", (), ()),
            ("S",
             ("T", (), ()),
             ("U", (), ()))
           )

utree = ("U",
         ("V", (), ()),
         ("W", (), ()))
```

primary tree

graft tree



```
>>> graft(groodies, utree)
('Q', ('R', (), ()), ('S', ('T', (), ()), ('U', ('V', (), ()), ('W', (), ())))

>>> graft(groodies, ('X', (), ()))
('Q', ('R', (), ()), ('S', ('T', (), ()), ('U', (), ())))
```

- At most one *leaf* of `primary_tree` has the same name as the root of `graft_tree`.
- If there is one such match, the function returns a new tree that is identical to `primary_tree` but with that leaf in `primary_tree` replaced by the entire `graft_tree`.
- If there is no leaf in `primary_tree` that matches the name of the root of `graft_tree`, the function simply returns `primary_tree`.
- No internal node of the `primary_tree` will have a name that matches the root of the `graft_tree`.



```
def graft(primary_tree, graft_tree):  
    """If primary_tree has a leaf whose name is the same as the root of  
    graft_tree then we return a new tree identical to primary_tree  
    except with that leaf replaced by graft_tree. Otherwise, we  
    just return primary_tree."""
```

Try not to use any helper functions on this one!





```
def graft(primary_tree, graft_tree):  
    """If primary_tree has a leaf whose name is the same as the root of  
    graft_tree then we return a new tree identical to primary_tree  
    except with that leaf replaced by graft_tree. Otherwise, we  
    just return primary_tree."""  
  
    root, left, right = primary_tree  
    if root == graft_tree[0]:  
        return graft_tree  
  
    elif left == ():  
        return primary_tree  
  
    else:  
        left_graft = graft(left, graft_tree)  
        right_graft = graft(right, graft_tree)  
        return (root, left_graft, right_graft)
```

Try not to use any helper functions on this one!

## Reminder:

- Lecture feedback form  
(<https://forms.gle/aPmkpXDUTp4Xo4CV7>)