# CS 147:
# Computer Systems Performance Analysis
## Summarizing Variability and Determining Distributions





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# Overview







Overview

Introduction
Indices of Dispersion
Range
Variance, Standard Deviation, C.V.
Quantiles
Miscellaneous Measures
Choosing a Measure
Identifying Distributions
Histograms
Kernel Density Estimation
Quantile-Quantile Plots
Statistics of Samples
Meaning of a Sample
Guessing the True Value

# Summarizing Variability

- A single number rarely tells entire story of a data set
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# Why Is Variability Important?

- Consider two Web servers:
  - Server A services all requests in 1 second
  - Server B services 90% of all requests in .5 seconds
    - But 10% in 55 seconds
  - Both have mean service times of 1 second
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# Indices of Dispersion

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  - Range
  - Variance and standard deviation
  - Percentiles
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- Minimum & maximum values in data set
- Can be tracked as data values arrive
- Variability characterized by difference between minimum and maximum
- Often not useful, due to outliers
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# Example of Range

- For data set 2, 5.4, -17, 2056, 445, -4.8, 84.3, 92, 27, -10
  - Maximum is 2056
  - Minimum is -17
  - Range is 2073
  - While arithmetic mean is 268





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# Variance (and Its Cousins)

- Sample variance is

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

- Expressed in units of the measured quantity, squared
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# Relatives of Percentiles

- Quantiles - fraction between 0 and 1
  - Instead of percentage
  - Also called fractiles
- Deciles—percentiles at 10% boundaries
  - First is 10-percentile, second is 20-percentile, etc.
- Quartiles—divide data set into four parts
  - 25% of sample below first quartile, etc.
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# Calculating Quantiles

To estimate $\alpha$-quantile:

- First sort the set
- Then take $[(n-1)\alpha + 1]^{\text{th}}$ element
  - 1-indexed
  - Round to nearest integer index
  - Exception: for small sets, may be better to choose “intermediate” value as is done for median





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- For data set 2, 5.4, -17, 2056, 445, -4.8, 84.3, 92, 27, -10 (10 observations)
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- First quartile, Q1, is -4.8
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# Interquartile Range

- Yet another measure of dispersion
- The difference between Q3 and Q1
- Semi-interquartile range is half that:

$$\text{SIQR} = \frac{Q_3 - Q_1}{2}$$

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# Semi-Interquartile Range Example

For data set -17, -10, -4.8, 2, 5.4, 27, 84.3, 92, 445, 2056

- Q3 is 92
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$$\text{SIQR} = \frac{Q_3 - Q_1}{2} = \frac{92 - (-4.8)}{2} = 48$$

- Compare to standard deviation of 643
  - Suggests that much of variability is caused by outliers





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# Sensitivity To Outliers

- From most to least,
- Range
- Variance
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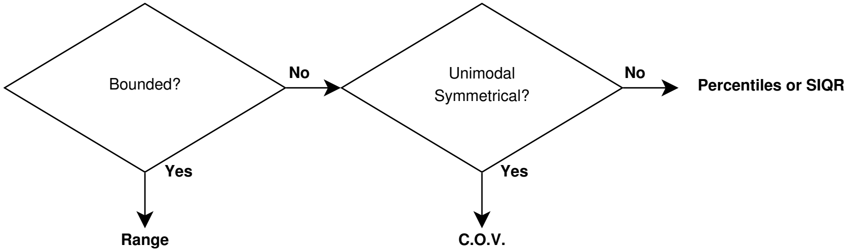
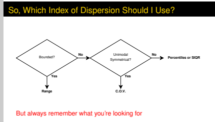
# So, Which Index of Dispersion Should I Use?




But always remember what you're looking for







# Finding a Distribution for Datasets

- If a data set has a common distribution, that's the best way to summarize it
- Saying a data set is uniformly distributed is more informative than just giving mean and standard deviation
- So how do you determine if your data set fits a distribution?





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# Methods of Determining a Distribution

- Plot a histogram
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# Plotting a Histogram

Suitable if you have relatively large number of data points

Procedure:

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# Problems With Histogram Approach

- Determining cell size
  - If too small, too few observations per cell
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# Kernel Density Estimation

- Basic idea: any observation represents probability of high *near* near that observation
- Example:
  - Seeing 7 means pdf is high all around 7
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# KDE Equations

- Want to estimate continuous $p(x)$:

$$\hat{p}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

- Where $K(x)$ is *kernel function*
- Must integrate to unity: $\int_{-\infty}^{\infty} K(x)\,dx = 1$
- Purpose is to select nearby samples
- $h$ is *bandwidth* parameter
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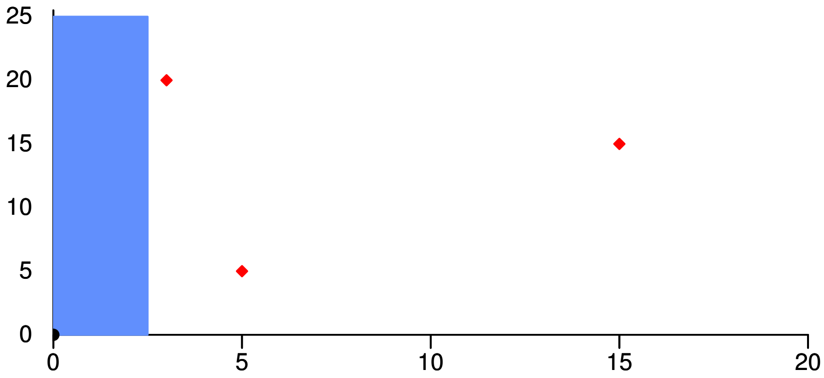




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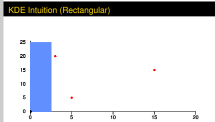

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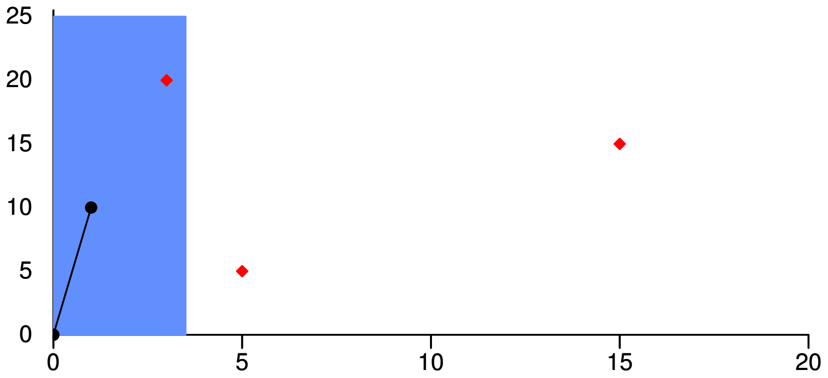






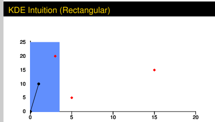

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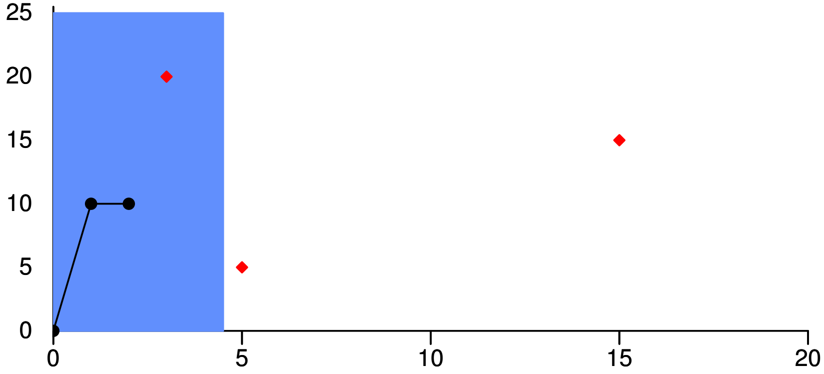




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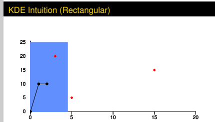

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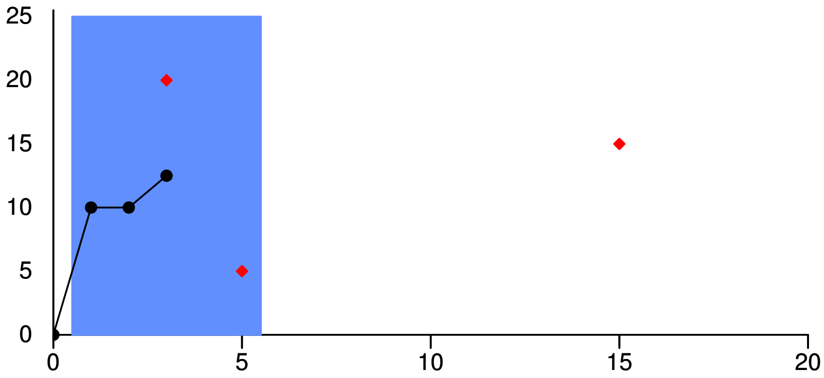






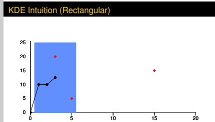

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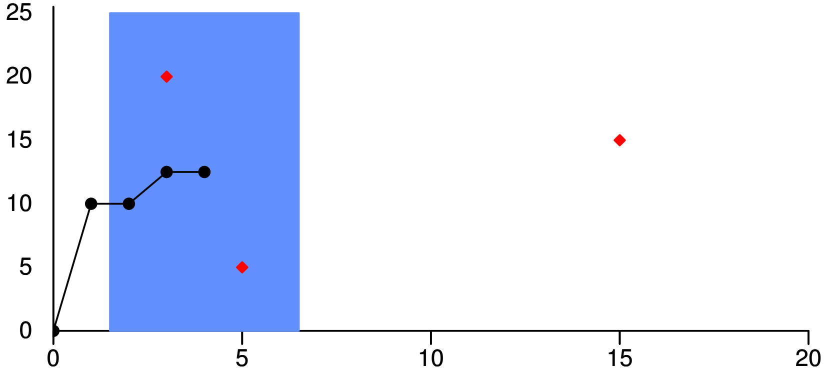






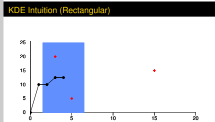

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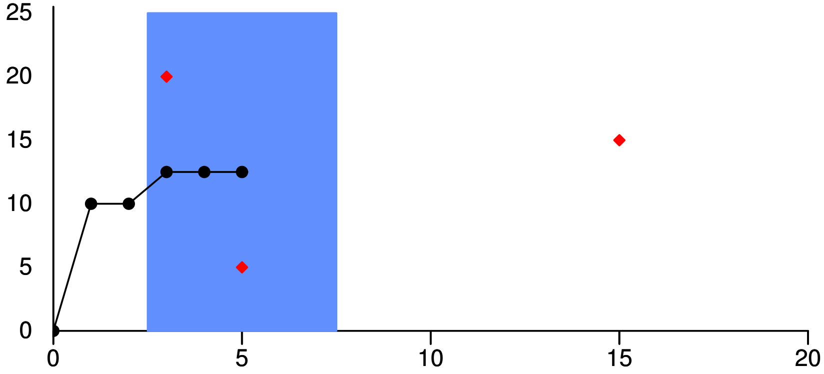






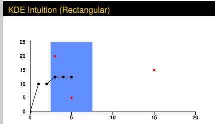

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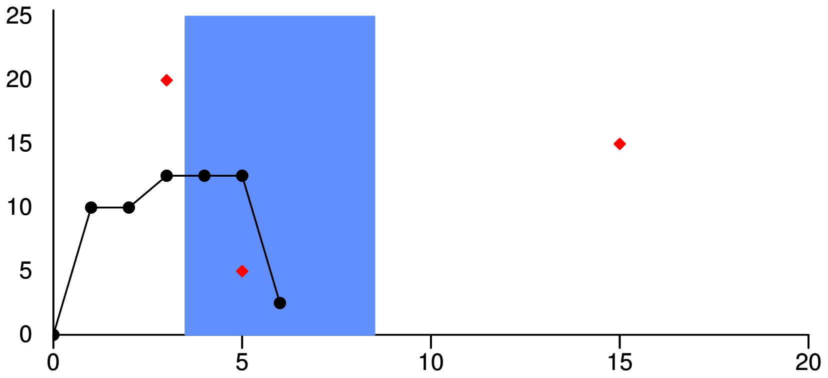






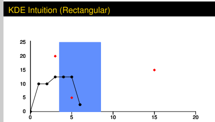

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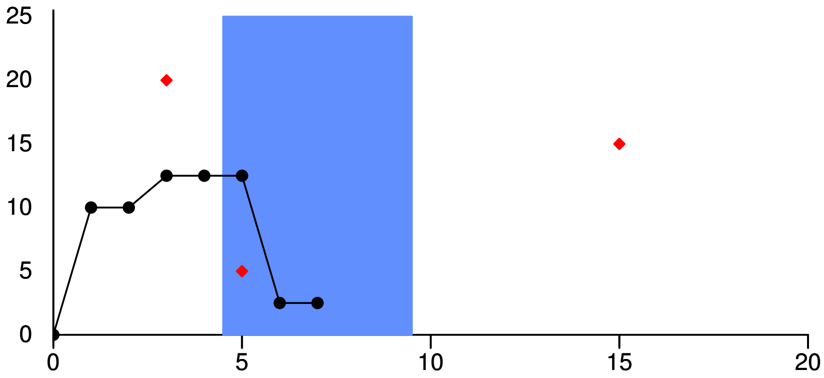






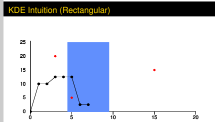

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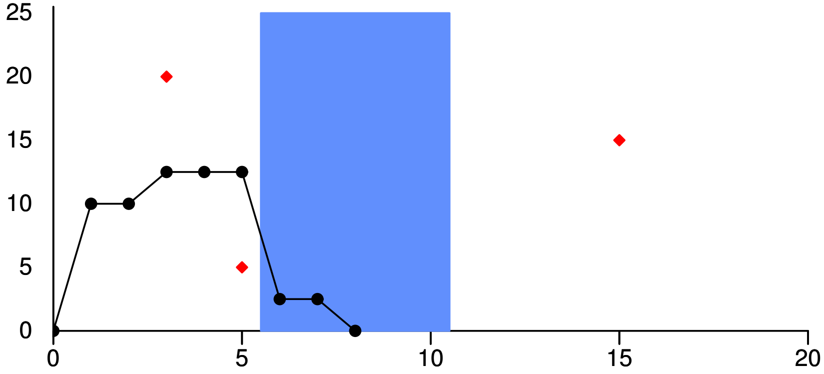






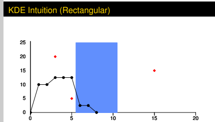

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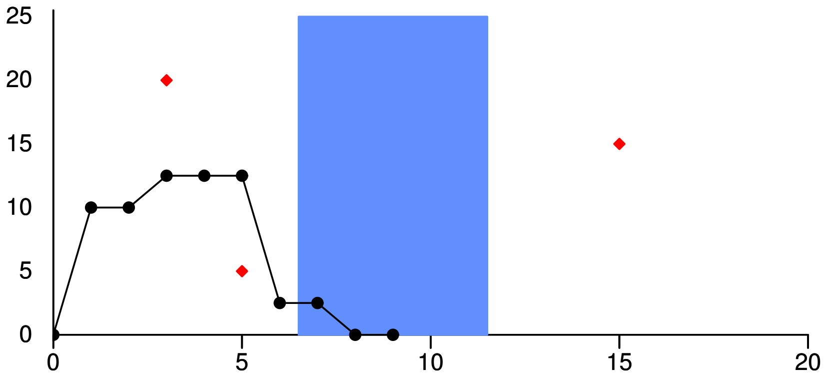






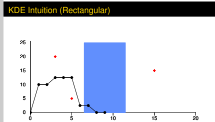

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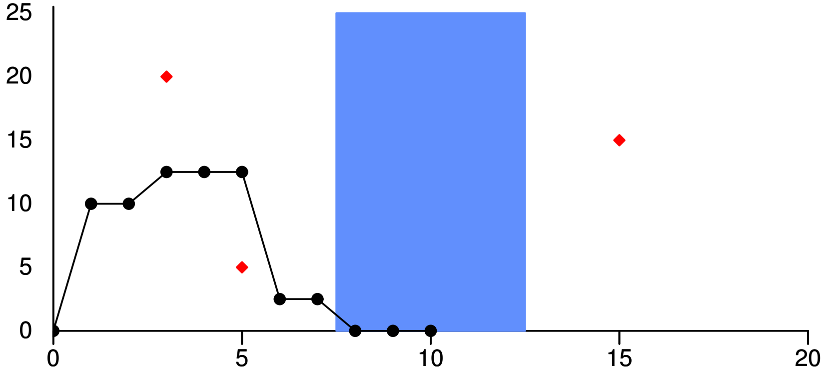






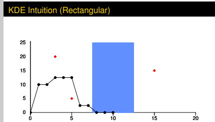

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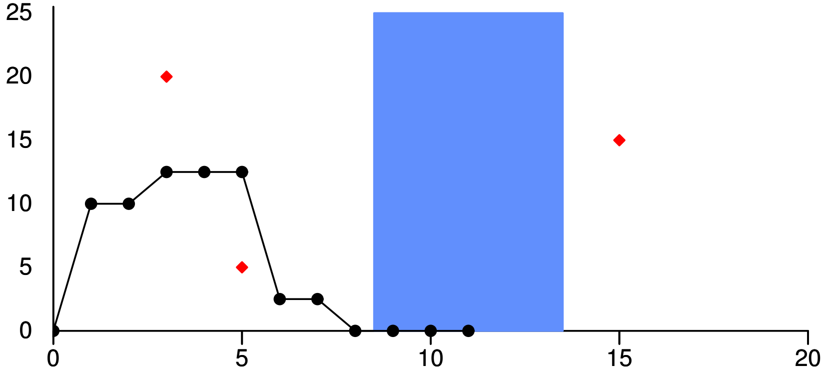






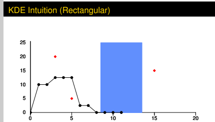

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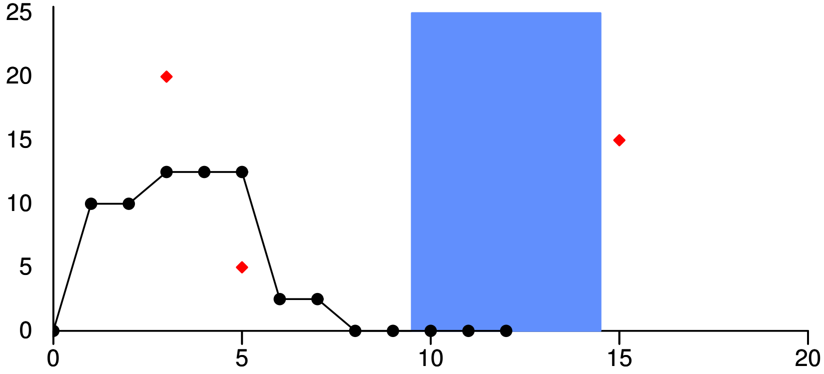




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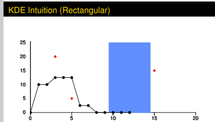

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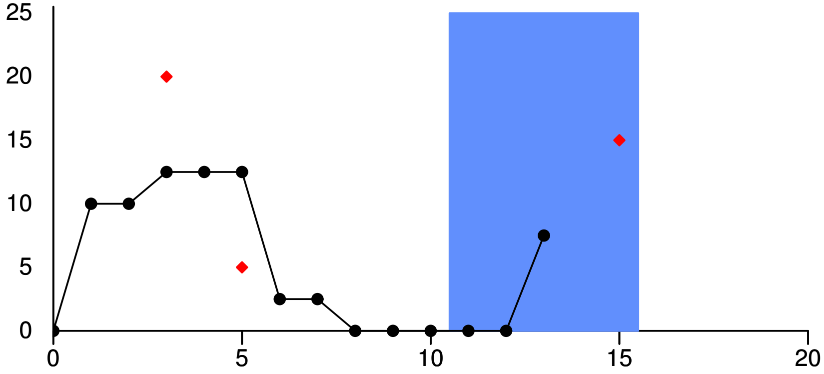






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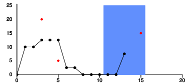

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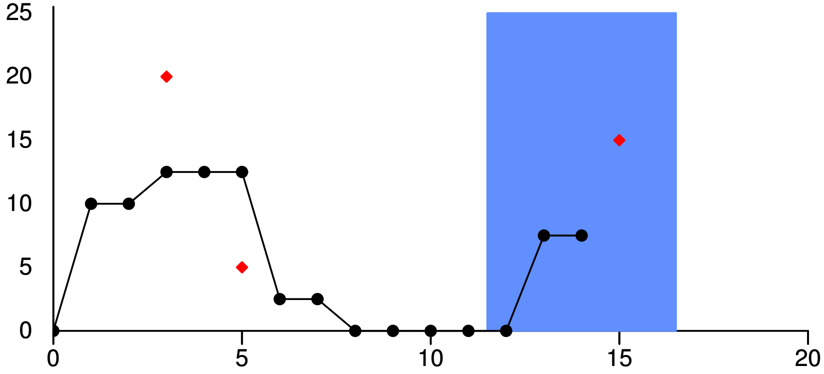




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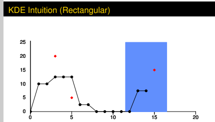

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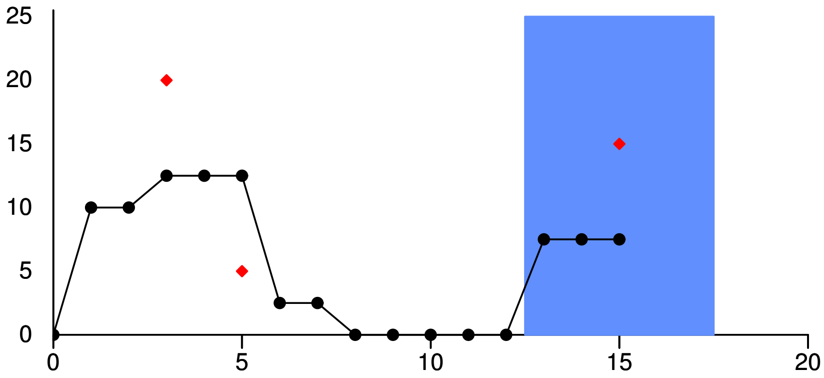






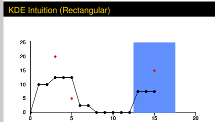

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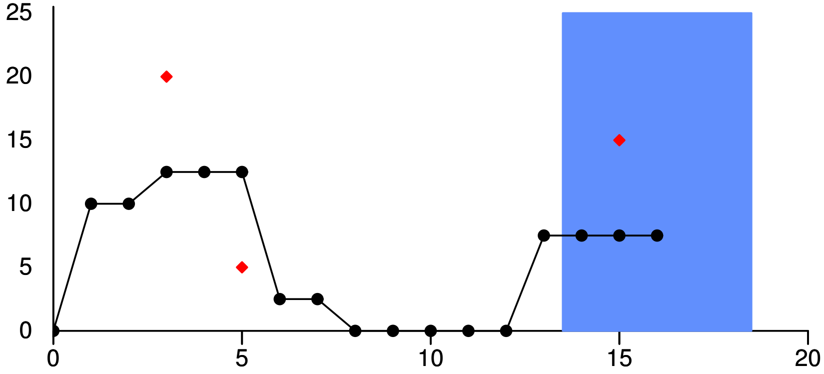




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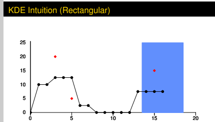

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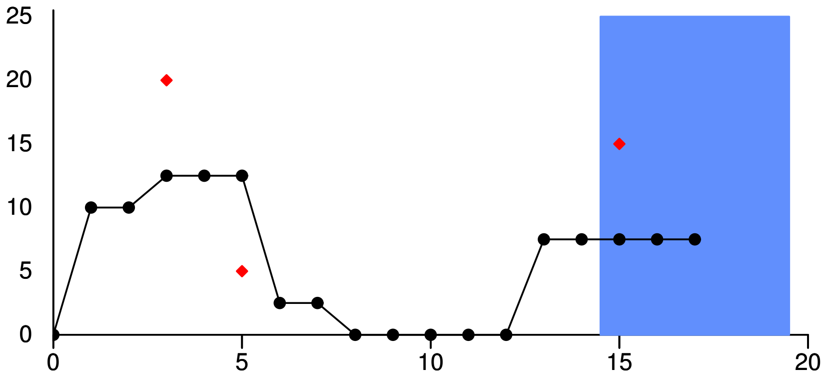






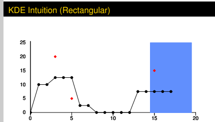

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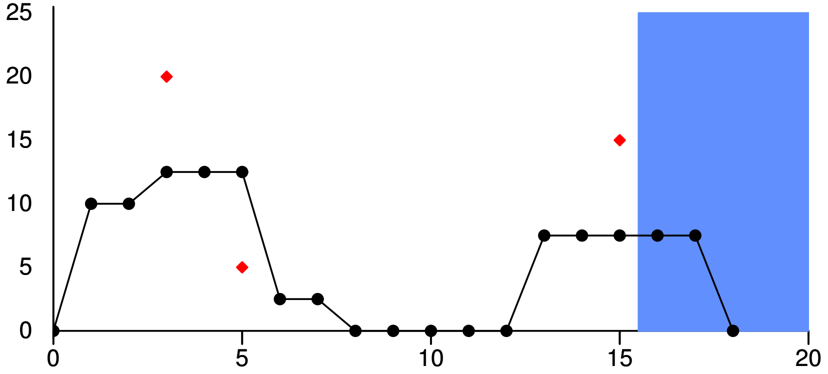
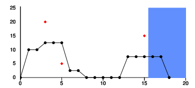







KDE Intuition (Rectangular)



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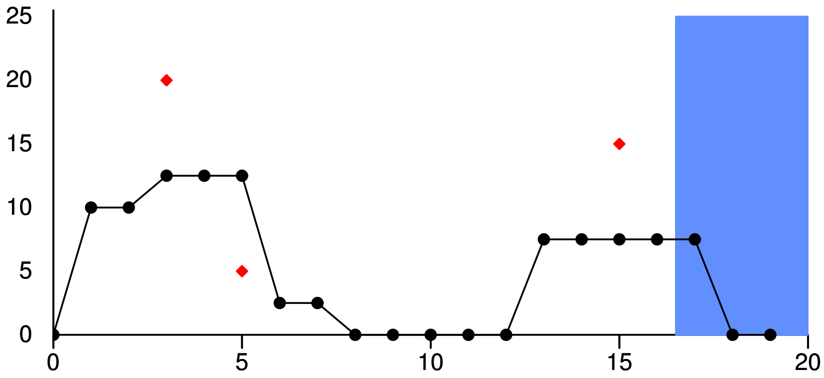






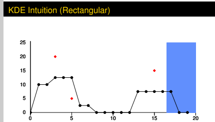

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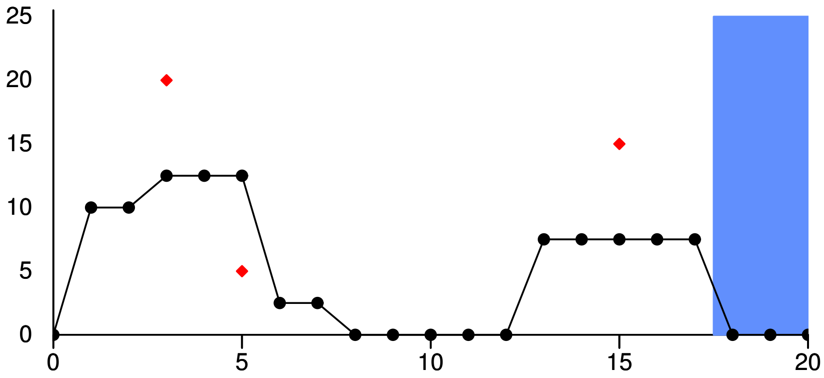




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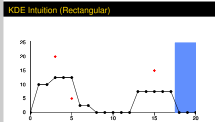
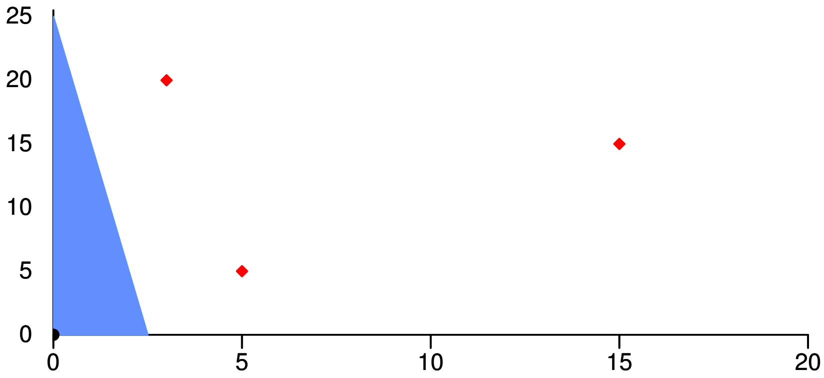
└Identifying Distributions
 └Kernel Density Estimation
  └KDE Intuition (Rectangular)

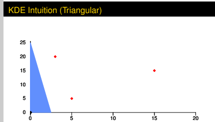


# KDE Intuition (Triangular)







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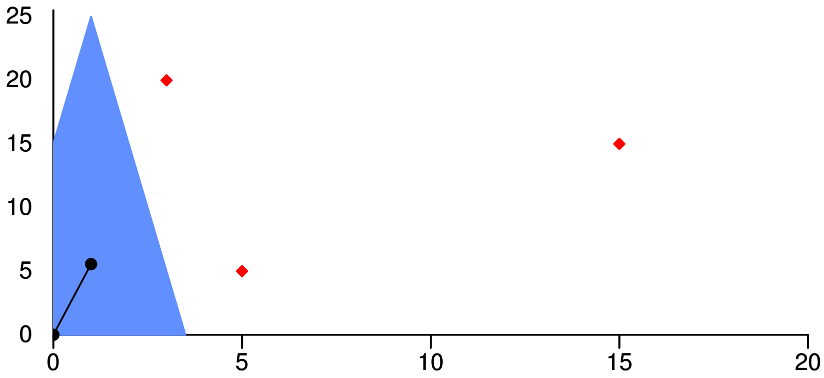




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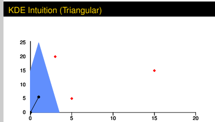

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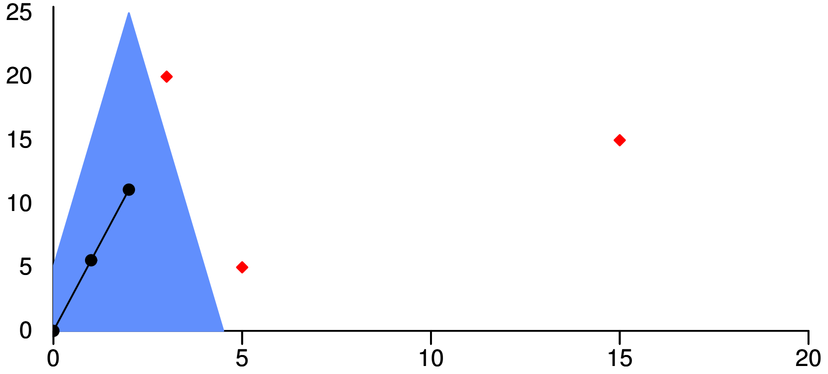






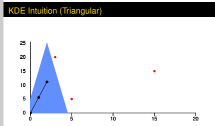

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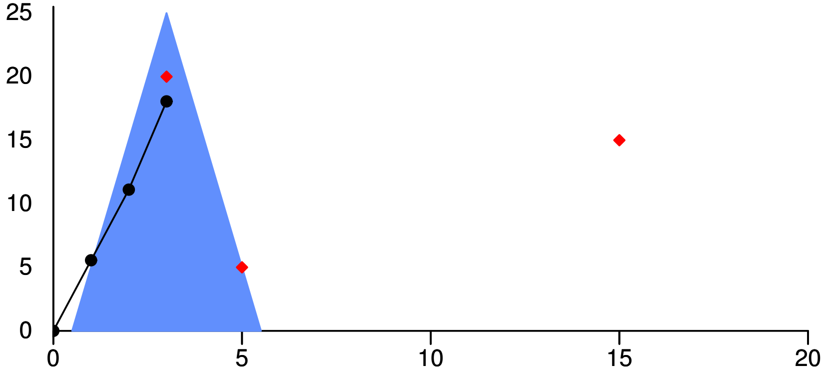




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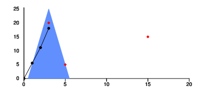

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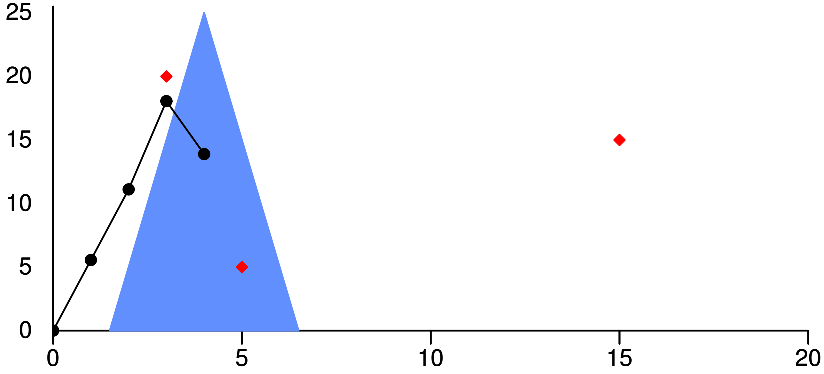






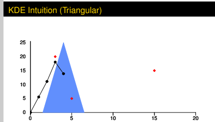

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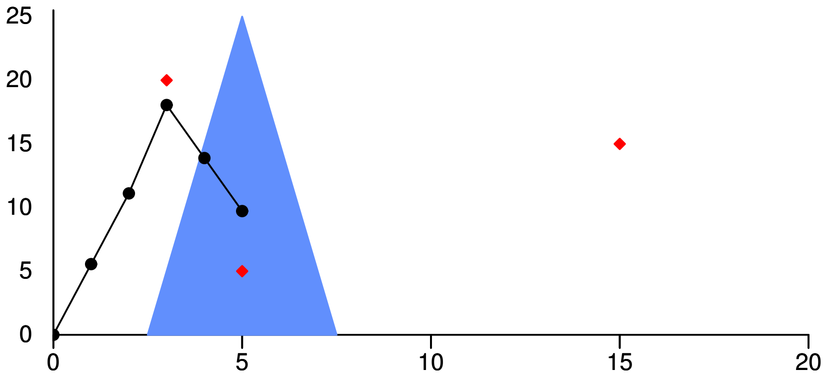




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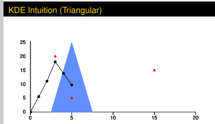

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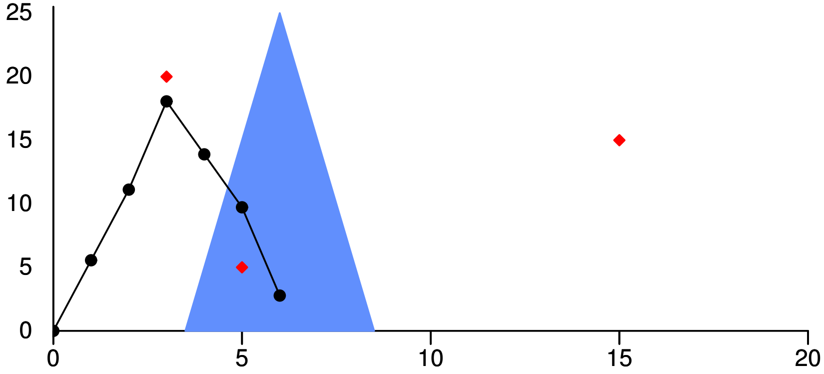




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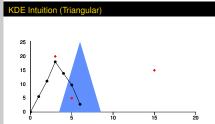

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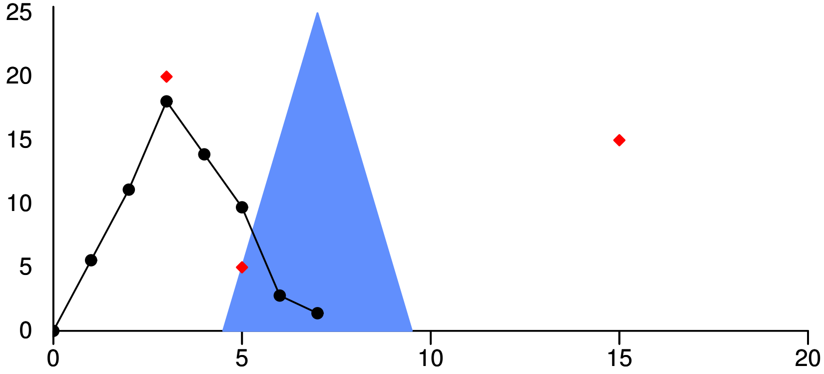






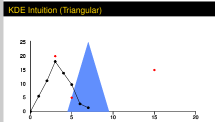

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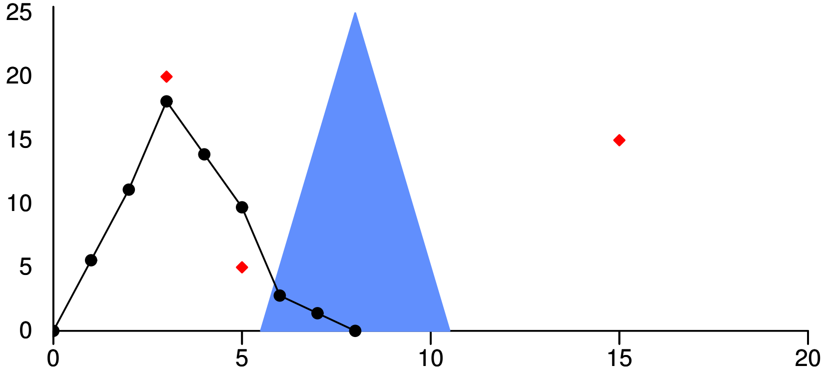






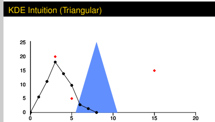

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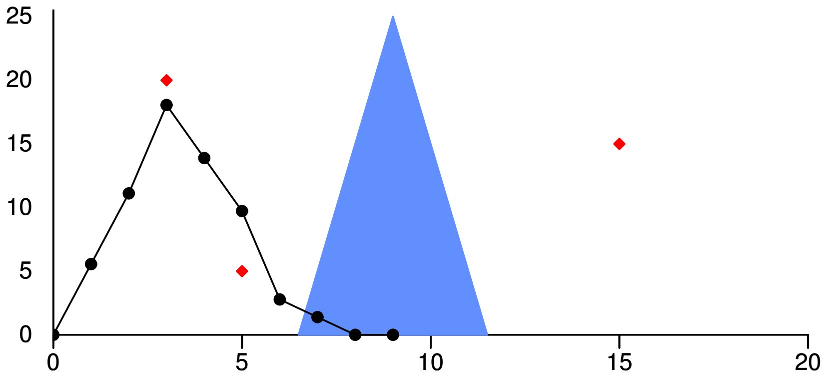






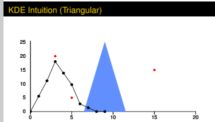

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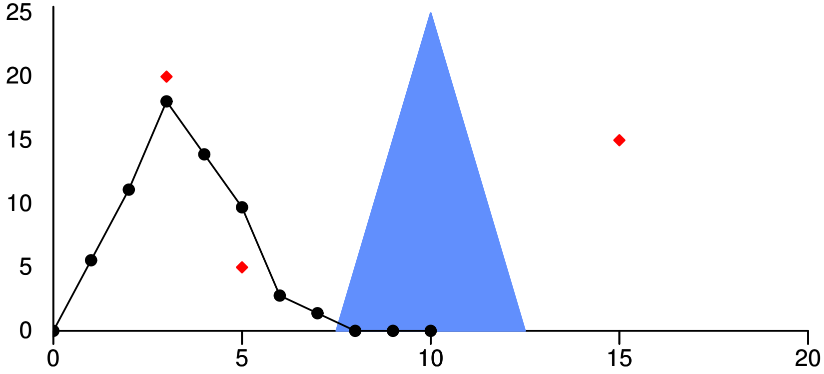






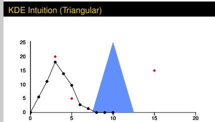

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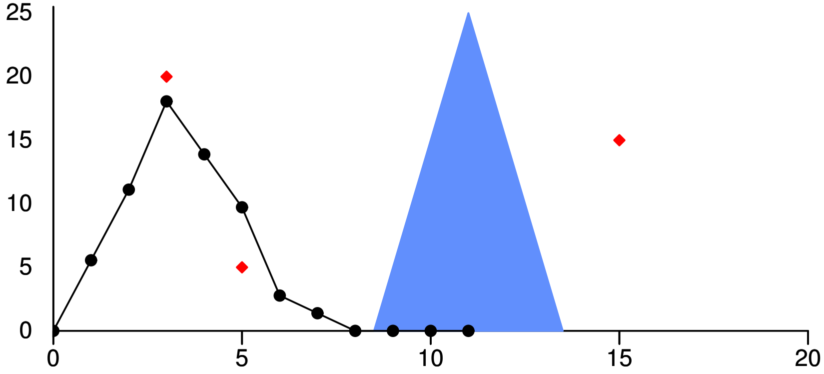






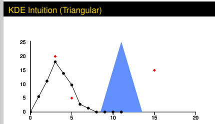

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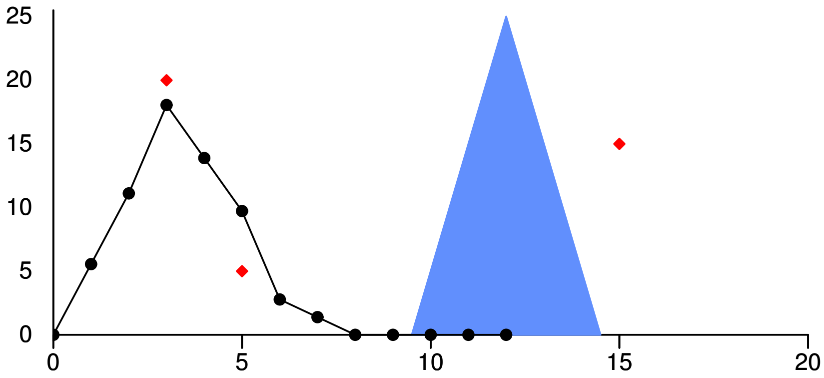






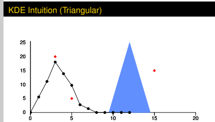

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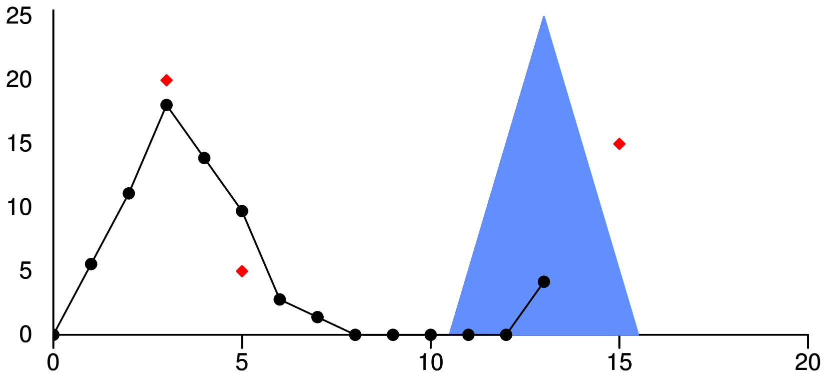






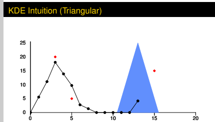

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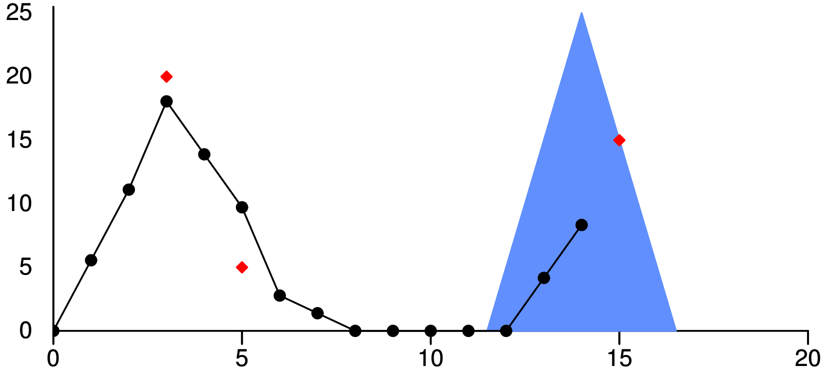






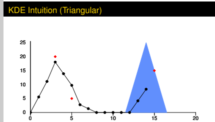

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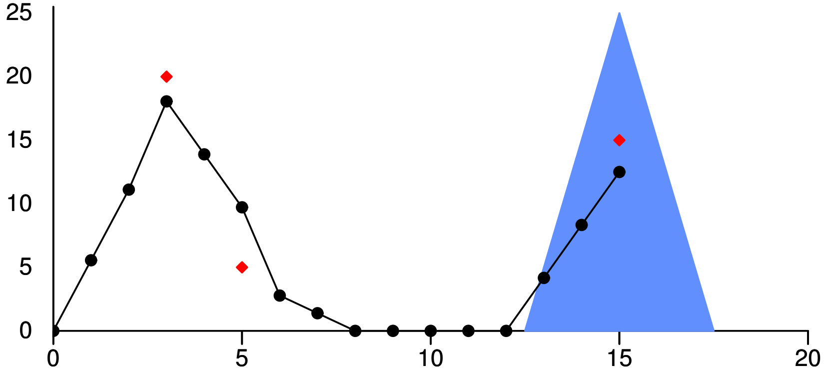




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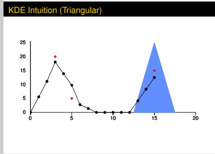

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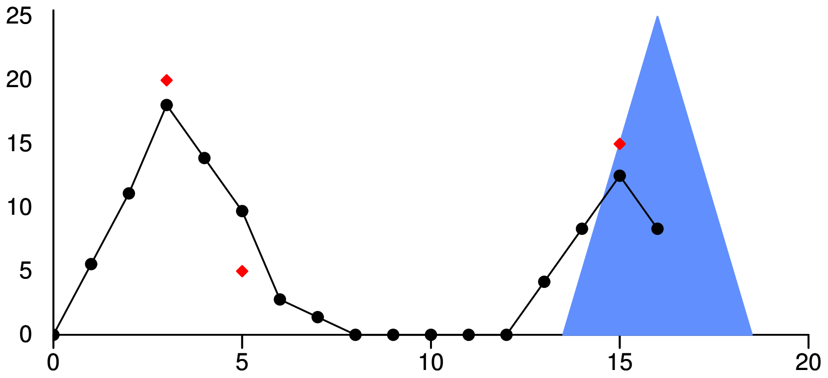






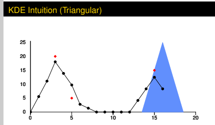

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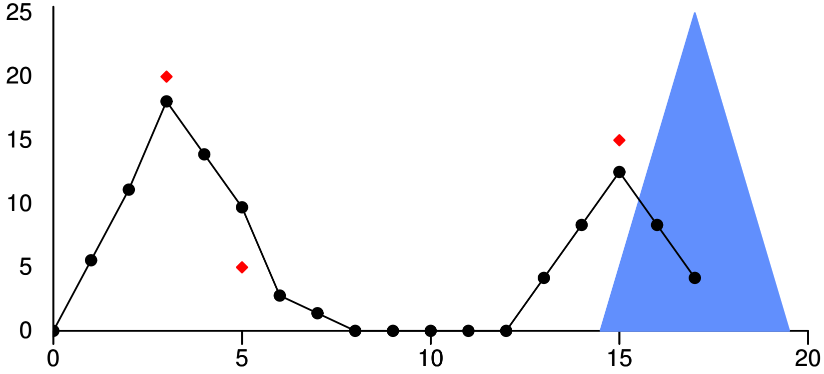




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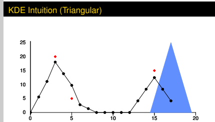

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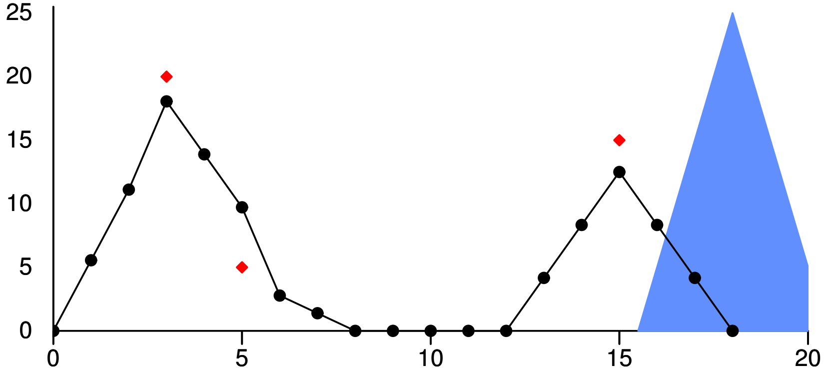






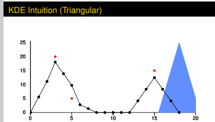

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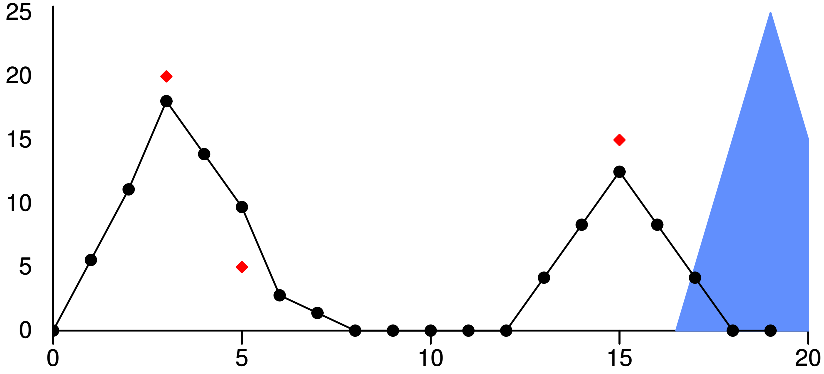






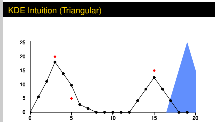

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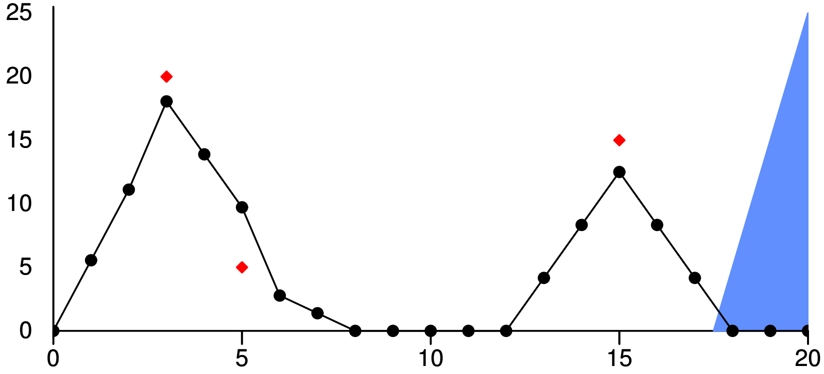




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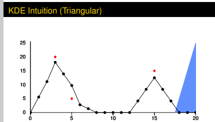
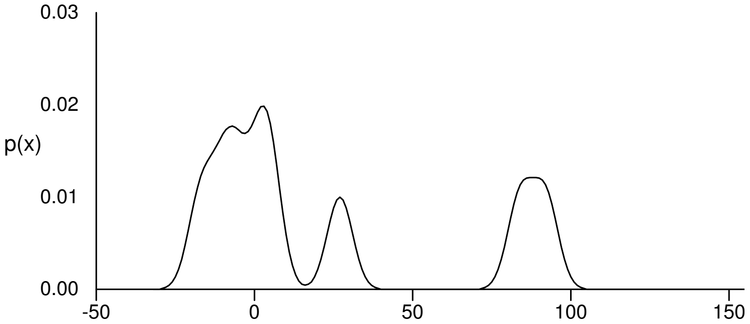


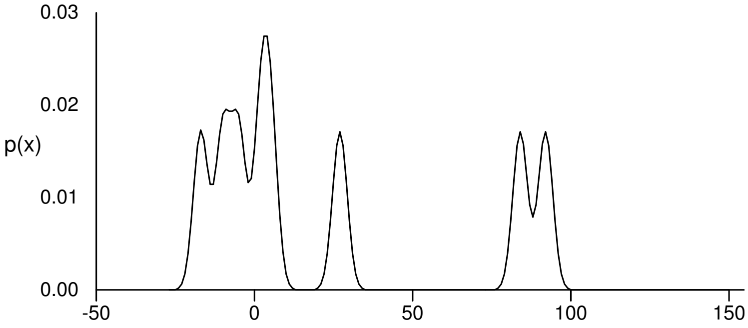
# KDE Example

- Sample data set: -17, -10, -4.8, 2, 5.4, 27, 84.3, 92, 445, 2056
- One observation per sample
- KDE with Gaussian window (RHS dropped):



# KDE Example #2

- Same data set
- Narrower Gaussian window
- (Again, RHS dropped):



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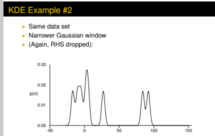

# Quantile-Quantile Plots

- More suitable than KDE for small data sets
- Basically, guess a distribution
- Plot where quantiles of data should fall in that distribution
  - Against where they actually fall
- If plot is close to linear, data closely matches guessed distribution





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# Obtaining Theoretical Quantiles

- Need to determine where quantiles should fall for a particular distribution
- Requires inverting CDF for that distribution
  - Then determining quantiles for observed points
  - Then plugging quantiles into inverted CDF





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- Many common distributions have already been inverted (how convenient...)
- For others that are hard to invert, tables and approximations often available (nearly as convenient)





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# Is Our Sample Data Set Normally Distributed?

- Our data set was -17, -10, -4.8, 2, 5.4, 27, 84.3, 92, 445, 2056
- Does this match normal distribution?
- Normal distribution doesn't invert nicely
  - But there is an approximation:

$$x_i = 4.91 \left( q_i^{0.14} - (1 - q_i)^{0.14} \right)$$

  - Or invert numerically





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# Data For Example Normal Quantile-Quantile Plot

| $i$ | $q_i$ | $y_i$ | $x_i$ |
|---|---|---|---|
| 1 | 0.05 | -17.0 | -1.64684 |
| 2 | 0.15 | -10.0 | -1.03481 |
| 3 | 0.25 | -4.8 | -0.67234 |
| 4 | 0.35 | 2.0 | -0.38375 |
| 5 | 0.45 | 5.4 | -0.12510 |
| 6 | 0.55 | 27.0 | 0.12510 |
| 7 | 0.65 | 84.3 | 0.38375 |
| 8 | 0.75 | 92.0 | 0.67234 |
| 9 | 0.85 | 445.0 | 1.03481 |
| 10 | 0.95 | 2056.0 | 1.64684 |



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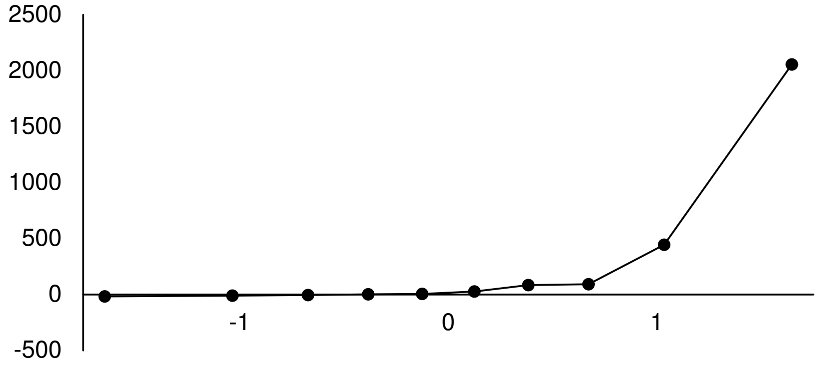
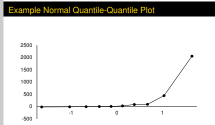
| 10 | 0.95 | 2056.0 | 1.64684 |

# Example Normal Quantile-Quantile Plot







# Analysis

- Definitely not normal
  - Because it isn't linear
  - Tail at high end is too long for normal
- But perhaps the lower part of graph is normal?





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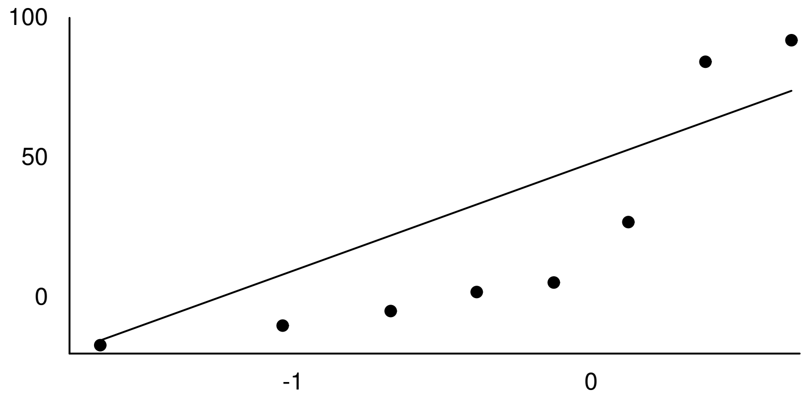
# Quantile-Quantile Plot of Partial Data



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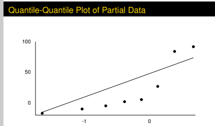
    └Quantile-Quantile Plot of Partial Data



# Analysis of Partial Data Plot

- Again, at highest points it doesn't fit normal distribution
- But at lower points it fits somewhat well
- So, again, this distribution looks like normal with longer tail to right





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# Analysis of Partial Data Plot

- Again, at highest points it doesn't fit normal distribution
- But at lower points it fits somewhat well
- So, again, this distribution looks like normal with longer tail to right
- (Really need more data points)
- You can keep this up for a good, long time



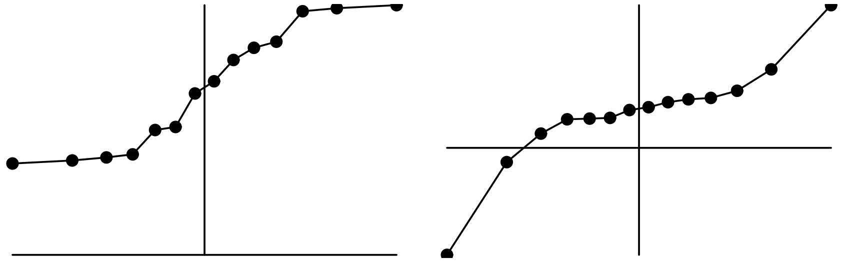


Analysis of Partial Data Plot

- Again, at highest points it doesn't fit normal distribution
- But at lower points it fits somewhat well
- So, again, this distribution looks like normal with longer tail to right
- (Really need more data points)
- You can keep this up for a good, long time

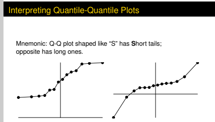
# Interpreting Quantile-Quantile Plots

Mnemonic: Q-Q plot shaped like “S” has **S**hort tails; opposite has long ones.









# What is a Sample?

- How tall is a human?
  - Could measure every person in the world
  - Or could measure everyone in this room
- Population has *parameters*
  - Real and meaningful
- Sample has *statistics*
  - Drawn from population
  - Inherently erroneous





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# Sample Statistics

- How tall is a human?
  - People in B126 have a mean height
  - People in Edwards have a different mean
- Sample mean is *itself* a random variable
  - Has own distribution





Sample Statistics

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# Estimating Population from Samples

- How tall is a human?
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  - Calculate sample mean $\overline{x}$
  - Assume population mean $\mu$ equals $\overline{x}$
- What is the error in our estimate?





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- Sample mean is a random variable
  - $\Rightarrow$ Mean has some distribution
  - $\therefore$ Multiple sample means have “mean of means”
- Knowing distribution of means, we can estimate error





Estimating Error

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# Estimating the Value of a Random Variable

- How tall is Fred?





Estimating the Value of a Random Variable

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# Estimating the Value of a Random Variable

- How tall is Fred?
- Suppose average human height is 170 cm





Estimating the Value of a Random Variable

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# Estimating the Value of a Random Variable

- How tall is Fred?
- Suppose average human height is 170 cm
  - $\therefore$ Fred is 170 cm tall





Estimating the Value of a Random Variable

- How tall is Fred?
- Suppose average human height is 170 cm
  - ∴ Fred is 170 cm tall

# Estimating the Value of a Random Variable

- How tall is Fred?
- Suppose average human height is 170 cm
  - $\therefore$ Fred is 170 cm tall
  - Yeah, right





Estimating the Value of a Random Variable

- How tall is Fred?
- Suppose average human height is 170 cm
  - ∴ Fred is 170 cm tall
  - Yeah, right

# Estimating the Value of a Random Variable

- How tall is Fred?
- Suppose average human height is 170 cm
  - ∴ Fred is 170 cm tall
  - Yeah, right
- Safer to assume a range





Estimating the Value of a Random Variable
- How tall is Fred?
- Suppose average human height is 170 cm
  - ∴ Fred is 170 cm tall
  - Yeah, right
- Safer to assume a range

# Confidence Intervals

- How tall is Fred?





Confidence Intervals

- How tall is Fred?

# Confidence Intervals

- How tall is Fred?
  - Suppose 90% of humans are between 155 and 190 cm





Confidence Intervals

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Confidence Intervals
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# Confidence Intervals

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- We are 90% *confident* that Fred is between 155 and 190 cm





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