CS 147:
Computer Systems Performance Analysis

Advanced Regression Techniques

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Overview

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Curvilinear Regression
Common Transformations
General Transformations
Handling Outliers
Common Mistakes

Curvilinear Regression

- Linear regression assumes a linear relationship between predictor and response
- What if it isn't linear?
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When To Use Curvilinear Regression

- Easiest to tell by sight
- Make a scatter plot
  - If plot looks non-linear, try curvilinear regression
- Or if non-linear relationship is suspected for other reasons
- Relationship should be convertible to a linear form

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Types of Curvilinear Regression

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  - $y = ax^b$
  - $y = a + b/x$
  - $y = ab^x$
  - Etc., ad infinitum

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  - Or a similar form
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Sample Transformations

- For $y = ae^{bx}$ take logarithm of y, do regression on $\log y = b_0 + b_1 x$, let $b = b_1$, $a = e^{b_0}$
- For $y = a + b \log x$, take log of x before fitting parameters, let $b = b_1$, $a = b_0$
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Corrections to Jain p. 257

(Early Editions)

Nonlinear	Linear
$y = a + b/x$	y = a+b(1/x)
$y = 1/(a + bx)$	$(1/y) = a + bx$
$y = x(a + bx)$	$(x/y) = a + bx$
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What Transformation To Use?

- Compute standard deviation of residuals
  - Plot as function of mean of observations
    - Assuming multiple experiments for single set of predictor values
  - Check for linearity: if linear, use a log transform
- If *variance* against mean of observations is linear, use square-root transform
- If standard deviation against mean *squared* is linear, use inverse ($1/y$) transform
- If standard deviation against mean *to a power* is linear, use power transform
- More covered in the book

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General Transformation Principle

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let $h(y) = \int \frac{1}{g(y)}\, dy$

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Example: Log Transformation

If standard deviation against mean is linear, then $g(y) = a\overline{y}$

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Outliers

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  - By chance, several standard deviations out
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Deciding How To Handle Outliers

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2. Check carefully for experimental error
3. Repeat experiments at predictor values for each outlier
4. Decide whether to include or omit outliers
   - Or do analysis both ways

Question: Is last point in last lecture's example an outlier on rating vs. year plot?

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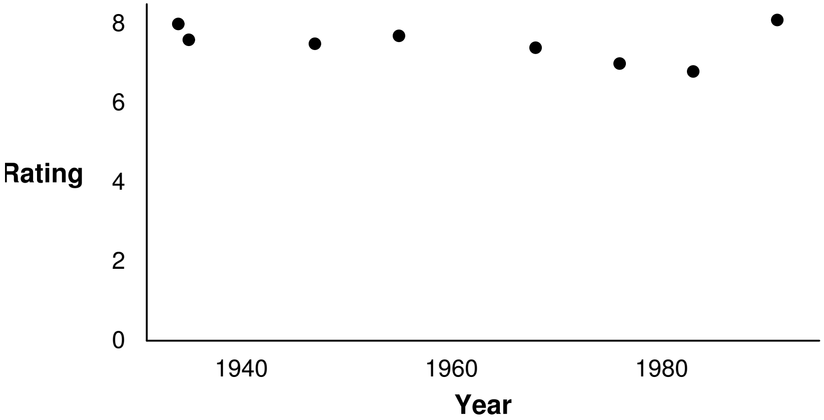
Question: Is last point in last lecture's example an outlier on rating vs. year plot?

Rating vs. Year



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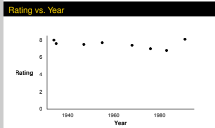
Handling Outliers

Rating vs. Year



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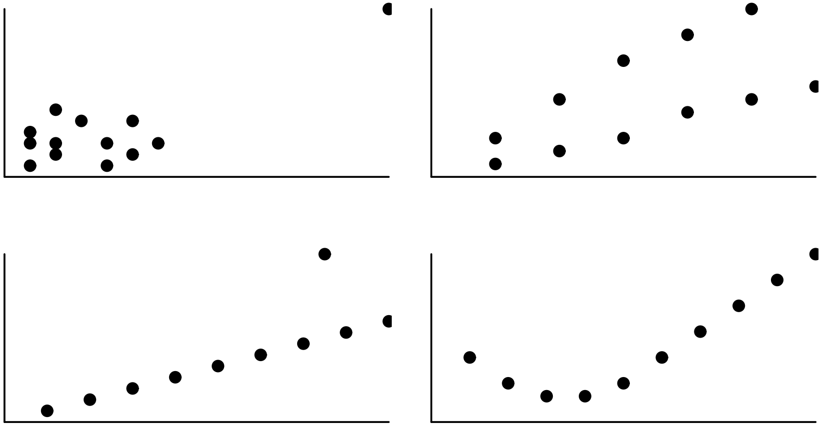

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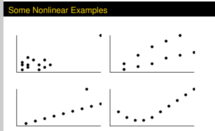

Attaching Importance to Parameter Values

- Numerical values of regression parameters depend on scale of predictor variables
- So just because a particular parameter's value seems "small" or "large," not necessarily an indication of importance
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