CS 147:
Computer Systems Performance Analysis
Networks of Queues

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Overview

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Types of Networks

Queues in Computer Systems
Operational Quantities
Operational Laws
Bottleneck Analysis

Tricks for Solving Networks
Mean Value Analysis
Hierarchical Decomposition

Limitations

Networks of Queues

- Many systems consist of interconnected queueing systems
  - CPU→disk→network
  - Web client→Web server→Web client
  - Network of freeways
- Fortunate property: M/M/m queues have Poisson departures
  - ⇒ Next queue is M/*/m
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Open and Closed Networks

- Closed network recirculates jobs
- Open network has external arrivals and departures
  - May also allow recycling
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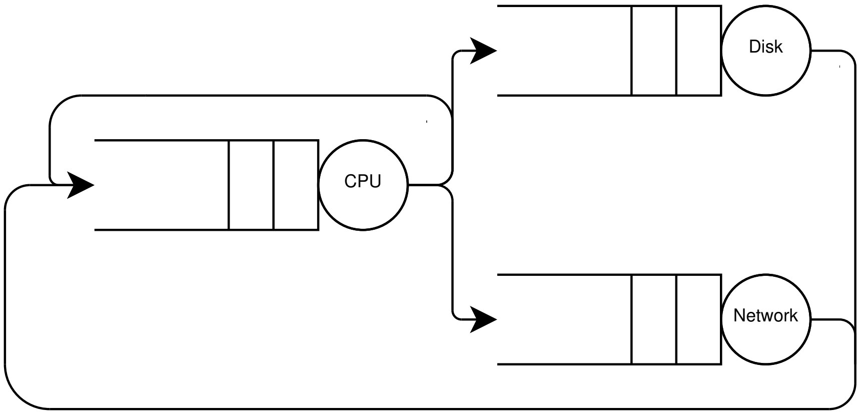
An Example Closed Network



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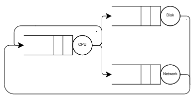
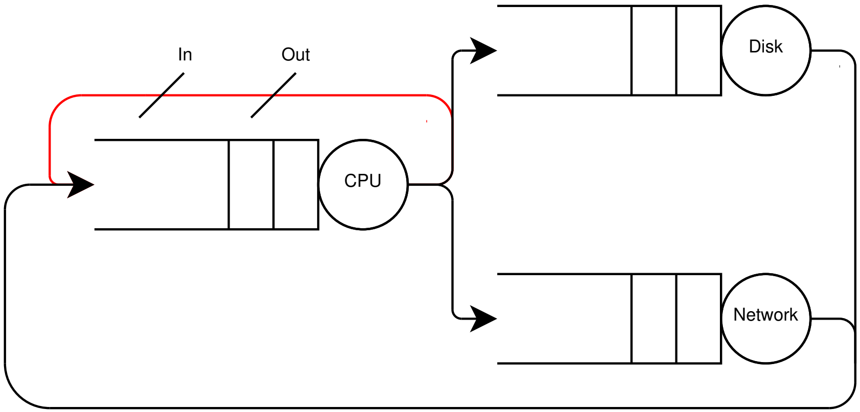
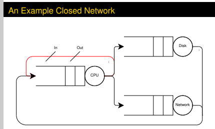


A closed network can be converted into an open one by cutting any arbitrary flow path; see next slide.

An Example Closed Network

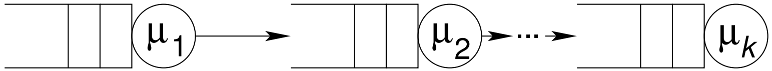




A closed network can be considered as an open network in which jobs leaving "Out" immediately reenter "In", i.e., an equilibrium network in which $\mu_{\text{Out}} = \lambda_{\text{In}}$.

Product-Form Networks

- We are interested in $P(n_1, n_2, \ldots, n_k)$, i.e., the probability that there are n_1 customers in the first queue, n_2 in the second, etc.
- Consider simple linear network:



- Arrival rate for each queue is λ (why?)
- Utilization $\rho_i = \lambda/\mu_i$
- P(n_i jobs in i^{th} queue $= p_i(n_i) = (1-\rho_i)\rho_i^{n_i}$
- $P(n_1, n_2, \ldots, n_k) = p_1(n_1)p_2(n_2)\cdots p_k(n_k)$

2015-06-15

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Generalizing Product-Form Networks

- General form of equilibrium probability:

$$P(n_1, n_2, \ldots, n_k) = \frac{1}{G(N)} \prod_{i=1} kf_i(n_i)$$

- $G(N)$ is normalizing constant, function of total jobs in system
- $f_i(n_i)$ is function of (only) system parameters and n_i
- Not always true that each queue behaves as M/M/1
- ... But analysis of each queue is separable
- Surprisingly large classes of networks are product-form

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Computer Systems as Queueing Networks

Three general types of queues appear in computer systems:

Fixed-capacity service center Service time doesn't depend on number of jobs; i.e., single server with queueing

Delay center Service time is random but no queueing; i.e. infinite number of servers (sometimes called *IS*)

Load-dependent service center Service rate depends on load; e.g., M/M/m with $m > 1$ (runs faster as more servers used)

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Operational Quantities

- An *operational quantity* is something that can be observed
  - Necessarily over some period of time
  - If period is long enough, approximates a system parameter
- Examples:
  - Arrival rate $\lambda_i = \dfrac{\text{number of arrivals}}{\text{time}} = \dfrac{A_i}{T} \approx \lambda$
  - Throughput $X_i = \dfrac{\text{number of completions}}{\text{time}} = \dfrac{C_i}{T} \approx \lambda$
  - Utilization $U_i = \dfrac{\text{busy time}}{\text{total time}} = \dfrac{B_i}{T} \approx \rho$
  - Mean service time $S_i = \dfrac{\text{total time served}}{\text{number served}} = \dfrac{B_i}{C_i} \approx \mu$

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Other Useful Quantities

- Number of devices M
- Visits per job V_i = Number of requests each job makes for device i (can be fractional)
- Demand D_i = Seconds of service needed from device i by each job = $V_i S_i$
- Overall system throughput $X = \frac{\text{jobs completed}}{\text{total time}} = \frac{C_0}{T}$
- Queue length at i: Q_i
- Response time at i: R_i
- Think time in interactive systems: Z

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Forced Flow Law $X_i = X V_i$

- In other words, device i's throughput had better be V_i times the system throughput or it won't be able to handle the load

Little's Law $Q_i = X_i R_i$

General Response Time Law $R = \sum_{i=1}^{M} R_i V_i$

Interactive Response Time Law For N users, $R = (N/X) - Z$

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Bottleneck Analysis

- Note that device demands D_i are total seconds of service needed from device i
- Some device (or devices) will be the max: $D_{\max}$
- This device is the *bottleneck device*
  - Improving other device performances can still improve response time, but most benefit will happen at bottleneck
- Asymptotic bounds on performance, as functions of N:

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X(N) &\leq \min\left\{\frac{1}{D_{\max}}, \frac{N}{D+Z}\right\} \\
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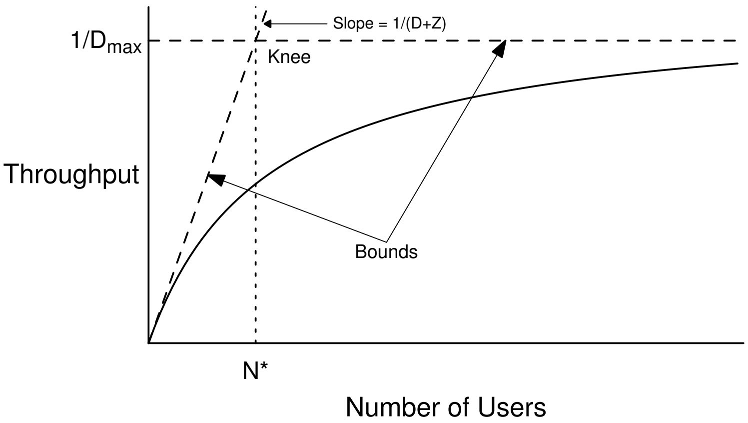
Asymptotic Bounds on Throughput



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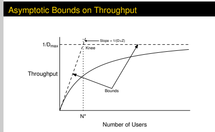
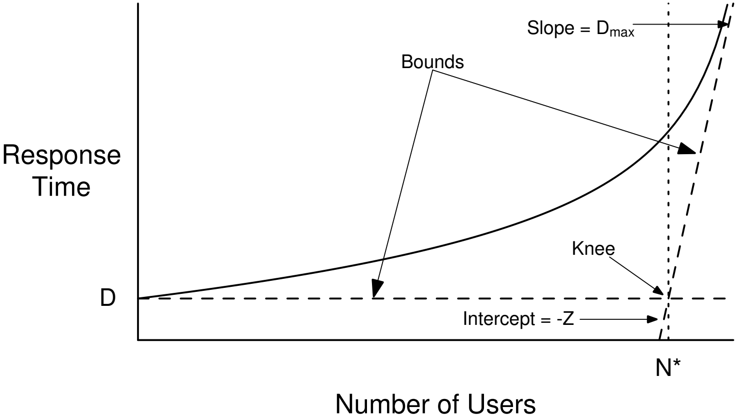


Asymptotic Bounds on Response Time



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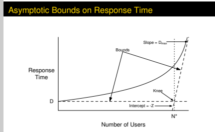

Mean Value Analysis

- Iterative procedure for calculating per-device parameters (response time, queue length, etc.)
- Basic approach:
  - Assume queue length = 0 for all devices
  - For increasing user counts, calculate response times, then new queue lengths
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Hierarchical Decomposition

- Large networks are hard to deal with
- Stems comes to the rescue!
  - In a queueing network, a complex subsystem with one input and one output can be replaced by a single queue tuned to the same behavior
  - In particular, if you're interested in device i, the *entire rest of the network* has just one input and output
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1. Pick a device to study (also works for subnetwork)
2. Set device's service times to zero, solve remaining network
3. Replace remaining network with single load-dependent queue, using solved parameters
4. Reset device's service time and solve result

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Limitations of Queueing Theory

Queueing theory is useful but has limitations:

- Nonexponential service times
- Self-similar (“train”) arrivals
- Load-dependent arrivals
- Response-dependent arrivals (e.g., retransmissions)
- Defections after joining queue
- Transient analysis generally not possible
- Fork and join make jobs interdependent
- Contention for resources
- Holding multiple resources
- Mutual exclusion among jobs
- Blocking of other devices

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