Assignment #2 – Lumberjacking
Due 11:00am, Thursday September 23, 1999

1. For each of the following trees give the tree domain and the graph of the labeling function:

(a) 
```
    S
   /\  
  NP VP
 / \ / \ 
PN V NP
 |  |  |
John loves NP
`'

(b) 
```
    NP
   /\  
  DET ADJ N
 /   /\  /\ 
the big dog
`'

2. Assuming the trees above are called a and b, let the tree \(a' = a[22 \leftarrow b]\). Draw the tree \(a'\) and give its domain and the graph of its labeling function.

3. Assuming a restriction to binary trees, give the specification of the total order \(\leq_{inorder}\) on the nodes of a tree \(t\) corresponding to the ordering generated by an inorder depth-first traversal of the tree. That is, given addresses \(a, b \in dom(t)\), give conditions on the form of those addressed such that \(a \leq_{inorder} b\) iff \(a = b\) or node \(a\) would come before node \(b\) in an inorder traversal of the tree \(t\).

4. Prove that though tree addresses are finite strings, trees may have both (or either) infinitely long branches and nodes with an infinite number of children.

5. We say that tree \(t\) is a subtree of tree \(s\) if there is a tree address \(u\) in the domain of \(s\) such that \(t\) is the subtree rooted at \(u\) (as defined in class). Prove that the relation “is a subtree of” is transitive. That is, suppose that if \(s\) is a subtree of \(r\), then \(t\) (which is a subtree of \(s\)) is also a subtree of \(r\). You must show (supposing \(s\) is rooted at \(v\) in \(r\)) that given the definitions of the domain and labeling of a subtree, that \(t\) is the subtree of \(r\) rooted at \(v.u\).