Harvey Mudd College
Computer Science 80
Logic for Computer Science
Fall Semester 1999

Assignment #4 – Propositional Logic: Hilbert Systems

Due 11:00am, Wednesday October 6, 1999

Prove each of the following derived Hilbert rules sound by showing that when you have a proof of each of the premises (the formulas on the top) you can construct a proof of the conclusion (the formula on the bottom) without using the rule.

1. In class, we gave a derivation justifying the following rule which used the deduction rule. Please give a derivation that uses only the pure Hilbert system (consisting of the three axioms and *modus ponens*).

   The Transitivity Rule:

   $$\frac{A \Rightarrow B \quad B \Rightarrow C}{A \Rightarrow C}$$

For the rest, you may use any of the derived rules covered in class, and any of the rules you have already proved. You will find the framework of most of these proofs in section 2.9 of the text, but in a different style than used in class.

2. The Inverse Contrapositive Rule:

   $$\frac{A \Rightarrow B}{(\neg B) \Rightarrow (\neg A)}$$

3. The Inverse Double Negation Rule:

   $$\frac{A}{\neg\neg A}$$

4. The And Rule:

   $$\frac{A \quad B}{A \wedge B} \quad \text{[really} \quad \frac{A \quad B}{\neg(A \Rightarrow (\neg B))} \text{]}$$

5. The Commutativity of Or Rule:

   $$\frac{B \vee A}{A \vee B} \quad \text{[really} \quad \frac{((\neg B) \Rightarrow A)}{((\neg A) \Rightarrow B)} \text{]}$$