

Harvey Mudd College
 Computer Science 80
 Logic for Computer Science
 Spring Semester 1998

Assignment #6 – Propositional Logic: Gentzen Sequent Calculus
 Due 11:00am, Wednesday October 27, 1999

1. Give Classical Gentzen Sequent Calculus proofs of each of the following formulas (brackets are used in place of parentheses in some formulas to make the structure clearer):

- (a) $[(p \wedge q) \Rightarrow r] \equiv [p \Rightarrow (q \Rightarrow r)]$
- (b) $(p \Rightarrow q) \Rightarrow [(r \Rightarrow p) \Rightarrow (r \Rightarrow q)]$
- (c) $[(p \Rightarrow q) \Rightarrow p] \Rightarrow p$
- (d) $(p \Rightarrow r) \equiv \neg(p \wedge \neg r)$
- (e) $(p \Rightarrow r) \equiv (\neg p \vee r)$

2. Consider the intuitionistic sequent calculus in which the right hand side of the sequent is restricted to a single formula (Greek letters denote sets of formulas, roman letters denote individual formulas):

$$\frac{}{\Gamma, A \longrightarrow A} \textit{id} \quad \frac{}{\Gamma, \perp \longrightarrow A} \perp$$

$$\frac{\Gamma, A, B \longrightarrow C}{\Gamma, A \wedge B \longrightarrow C} \wedge_L \quad \frac{\Gamma \longrightarrow A \quad \Gamma \longrightarrow B}{\Gamma \longrightarrow A \wedge B} \wedge_R$$

$$\frac{\Gamma, A \longrightarrow C \quad \Gamma, B \longrightarrow C}{\Gamma, A \vee B \longrightarrow C} \vee_L \quad \frac{\Gamma \longrightarrow A}{\Gamma \longrightarrow A \vee B} \vee_{R1} \quad \frac{\Gamma \longrightarrow B}{\Gamma \longrightarrow A \vee B} \vee_{R2}$$

$$\frac{\Gamma \longrightarrow A \quad \Gamma, B \longrightarrow C}{\Gamma, A \Rightarrow B \longrightarrow C} \Rightarrow_L \quad \frac{\Gamma, A \longrightarrow B}{\Gamma \longrightarrow A \Rightarrow B} \Rightarrow_R$$

$$\frac{\Gamma \longrightarrow A}{\Gamma, \neg A \longrightarrow C} \neg_L \quad \frac{\Gamma, A \longrightarrow \perp}{\Gamma \longrightarrow \neg A} \neg_R$$

Prove that this version of the sequent calculus is sound (for the fragment of propositional calculus it covers) by showing that whenever there is a derivation of a sequent $\Gamma \longrightarrow A$ in this system, then there is a Natural Deduction proof of the formula A in which the open assumptions (non-discharged leaves) form a set $\Delta \subseteq \Gamma$.

(Hint: Use complete induction on the height of the sequent calculus proof.)