Harvey Mudd College
Computer Science 80
Logic for Computer Science
Fall Semester 1999

Assignment #8 – Predicate Logic: Proof Theory

Due 11:00am, Monday, December 6, 1999

1. Give Natural Deduction proofs of each of the following formulas (brackets are used in place of parentheses in some formulas to make the structure clearer):

   (a) $\exists x[\forall y(p(x,y))] \Rightarrow \forall y[\exists x(p(x,y))]$

   (b) $\exists x[p(x) \Rightarrow q(x)] \Rightarrow [\forall x(p(x)) \Rightarrow \exists x(q(x))]$

   (c) Prove that any boletus is poisonous. That is, from open assumptions:

   - $\forall x(f(x) \Rightarrow [m(x) \vee t(x)])$
   - $\forall x(b(x) \Rightarrow f(x))$
   - $\forall x([t(x) \vee pp(x)] \Rightarrow p(x))$
   - $\forall x(b(x) \Rightarrow \neg m(x))$

   prove the conclusion $\forall x(b(x) \Rightarrow p(x))$

   (d) (extra credit) $[\forall x(p(x)) \Rightarrow \exists x(q(x))] \Rightarrow \exists x[p(x) \Rightarrow q(x)]$

2. Give Gentzen Sequent Calculus proofs for each of the conclusions in the last problem, including the extra credit (which is not extra credit in this problem).

3. Give Resolution Refutaion proofs for each of the conclusions in the first problem, including the extra credit (which is not extra credit in this problem).