Sets

Definition (Ben-Ari): A *set* is composed of *elements*. Notation: $a \in S$, a is an element of set S, and $a \notin S$, a is *not* an element of S. The set with no elements is called the *empty set*, denoted $\emptyset$.

- The set of traffic light colors:

  $$\{\textit{red},\textit{green},\textit{yellow}\}$$

- The set of strings of names of CS faculty:

  {`"Wing Tam"`, `"Bob Keller"`, `"Margaret Fleck"`, `"Mike Erlinger"`, `"Josh Hodas"`, `"Ran Libeskind-Hadas"`}

- The set of CS faculty:

  {*Wing Tam*, *Bob Keller*, *Margaret Fleck*, *Mike Erlinger*, *Josh Hodas*, *Ran Libeskind-Hadas*}

Repetition and order are meaningless in sets:

$$\{1, 2, 3, 3, 2, 1\} = \{1, 2, 3\}$$

Sets

Sets may also be infinite:

- The set of *integers*:

$$\mathcal{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

- The set of *natural numbers*:

$$\{0, 1, 2, 3, \ldots\}$$

Sets are often *described* rather than *enumerated*:

- The set of even integers:

$$\mathcal{EV} = \{n | n \in \mathcal{Z} \text{ and } n \; mod \; 2 = 0\}$$

- The set of prime numbers:

$$\mathcal{PN} = \{n| \; n \in \mathcal{N} \text{ and } \neg\exists f.(f \in \mathcal{N} \text{ and } f > 1 \text{ and } n \; mod \; f = 0)\}$$

This is called *set comprehension* notation.

Sets

Set operations:

$$\cup, \cap, \overline{A}$$

Set relations:

$$\subset, \subseteq$$

In most cases there is an implicit assumption that the members of a set belong to the same "type".

Ordered Sets

"If we impose an order on elements of a set, we obtain an *ordered set* which will be denoted:

$$(a_1, \ldots, a_n)$$

Using parentheses rather than braces." (Ben-Ari)

Definition (Ben-Ari): A finite ordered set of n elements is called an *n-tuple* or a *finite sequence*. A 2-tuple is called an [(*ordered*)] *pair*, a 3-tuple is called a *triple*.... An infinite ordered set is called an *infinite sequence*.

- (*red*, *green*, *yellow*)
- (*red*, *yellow*, *green*)
- (*yellow*, *red*, *yellow*, *red*)
- $(1, 2, 2, 3, 3, 3, \ldots)$

Cartesian Products

Definition (Ben-Ari): Let S and T be sets. $S \times T$, the *Cartesian product* of S and T is the set of all (ordered) pairs whose first element is from S and whose second element is from T.

In general, given sets $S_1, \ldots, S_n$, the Cartesian product $S_1 \times \ldots \times S_n$ is the set of ordered n-tuples whose i-th element is in S_i.

- $\mathcal{N} \times \mathcal{N}$
- $\{\mathit{red}, \mathit{yellow}, \mathit{green}\} \times \{1, 2\}$

Relations

Definition: Given sets $S_1, \ldots, S_n$, an *n-ary relation* $\mathcal{R}$ is a subset of $S_1 \times \ldots \times S_n$.

The subset may be totally arbitrary:

- $\{(1, 2), (9, 12), (5, 3)\}$

Or it may be formed according to some pattern:

- $\{(1, 2), (2, 3), (3, 4), (4, 5), \ldots\}$
- $\{(a, b) | a \in \mathcal{N} \text{ and } b \in \mathcal{N} \text{ and } b = a + 1\}$
- $\{(a, b) | a \in \mathcal{N} \text{ and } b \in \mathcal{N} \text{ and } a < b\}$

Properties of relations:

- Symmetry
- Reflexivity
- Transitivity