

## The Principle of Complete Induction

**Definition:** Given a set  $A$ , a *property of  $A$*  is a function  $P : A \rightarrow \{\perp, \top\}$ . We say that *the property  $P$  holds for an element  $x \in A$*  if  $P(x) = \top$ .

### **The Principle of Complete Induction:**

Given a well-founded poset  $(A, \leq)$ , to prove that a property  $P$  holds for every element of  $A$ , it suffices to show that, for every element  $x \in A$ ,

if  $P(y)$  holds for all  $y < x$  then  $P(x)$  holds

This whole statement can be written notationally as:

$$(\forall x \in A)[(\forall y \in A)(y < x \Rightarrow P(y)) \Rightarrow P(x)] \Rightarrow (\forall z \in A)P(z)$$

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**Lemma:** The Principle of Complete Induction holds for every well founded set.

**Proof:**

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