Some Sequent Calculus Proofs

$$\Xi_1 = \dfrac{\vdots}{((\phi \wedge \psi) \vee \sigma) \longrightarrow ((\phi \vee \sigma) \wedge (\psi \vee \sigma))}$$

$$\Xi_2 = \dfrac{\vdots}{((\phi \vee \sigma) \wedge (\psi \vee \sigma)) \longrightarrow ((\phi \wedge \psi) \vee \sigma)}$$

$$\dfrac{\overset{\Xi_1}{((\phi \wedge \psi) \vee \sigma) \longrightarrow ((\phi \vee \sigma) \wedge (\psi \vee \sigma))} \quad \overset{\Xi_2}{((\phi \vee \sigma) \wedge (\psi \vee \sigma)) \longrightarrow ((\phi \wedge \psi) \vee \sigma)}}{\longrightarrow ((\phi \wedge \psi) \vee \sigma) \equiv ((\phi \vee \sigma) \wedge (\psi \vee \sigma))} \equiv_R$$

Some Sequent Calculus Proofs

$$\dfrac{\vdots}{\longrightarrow (a \Rightarrow (b \wedge c)) \Rightarrow (a \Rightarrow b)}$$

Some Intrisically Classical Sequent Proofs

$$\frac{\vdots}{\longrightarrow \phi \vee \neg\phi}$$

$$\frac{\vdots}{\longrightarrow (a \Rightarrow b) \equiv (\neg b \Rightarrow \neg a)}$$

Pierce's Formula

Now is a good time to revisit Pierce's Formula:

$$((p \Rightarrow q) \Rightarrow p) \Rightarrow p$$

The NK proof of the formula is not very edifying:

$$\frac{\vdots}{((p \Rightarrow q) \Rightarrow p) \Rightarrow p}$$

The Sequent Calculus (LK) proof is more informative:

$$\frac{\vdots}{\longrightarrow ((p \Rightarrow q) \Rightarrow p) \Rightarrow p}$$

Sequent Calculus Rules

In addition to the other readings, the rules can also be read as specifying how to search for a falsifying valuation.

$$\frac{}{\Gamma, A \longrightarrow \Delta, A}\, id \qquad \frac{}{\Gamma \longrightarrow \Delta, \top}\, \top \qquad \frac{}{\Gamma, \bot \longrightarrow \Delta}\, \bot$$

$$\frac{\Gamma, A, B \longrightarrow \Delta}{\Gamma, A \wedge B \longrightarrow \Delta}\, \wedge_L \qquad \frac{\Gamma \longrightarrow \Delta, A \quad \Gamma \longrightarrow \Delta, B}{\Gamma \longrightarrow \Delta, A \wedge B}\, \wedge_R$$

$$\frac{\Gamma, A \longrightarrow \Delta \quad \Gamma, B \longrightarrow \Delta}{\Gamma, A \vee B \longrightarrow \Delta}\, \vee_L \qquad \frac{\Gamma \longrightarrow \Delta, A, B}{\Gamma \longrightarrow \Delta, A \vee B}\, \vee_R$$

$$\frac{\Gamma \longrightarrow \Delta, A \quad \Gamma, B \longrightarrow \Delta}{\Gamma, A \Rightarrow B \longrightarrow \Delta}\, \Rightarrow_L \qquad \frac{\Gamma, A \longrightarrow \Delta, B}{\Gamma \longrightarrow \Delta, A \Rightarrow B}\, \Rightarrow_R$$

$$\frac{\Gamma \longrightarrow \Delta, A}{\Gamma, \neg A \longrightarrow \Delta}\, \neg_L \qquad \frac{\Gamma, A \longrightarrow \Delta}{\Gamma \longrightarrow \Delta, \neg A}\, \neg_R$$

$$\frac{\Gamma \longrightarrow \Delta, A, B \quad \Gamma, A, B \longrightarrow \Delta}{\Gamma, A \equiv B \longrightarrow \Delta}\, \equiv_L \qquad \frac{\Gamma, A \longrightarrow \Delta, B \quad \Gamma, B \longrightarrow \Delta, A}{\Gamma \longrightarrow \Delta, A \equiv B}\, \equiv_R$$

Some Sequent Calculus Proofs and Non-Proofs

$$\frac{\vdots}{\longrightarrow a \wedge b \Rightarrow a \vee b}$$

$$\frac{\vdots}{\longrightarrow a \vee b \Rightarrow a \wedge b}$$

$$\frac{\vdots}{\longrightarrow (a \Rightarrow b) \Rightarrow (b \Rightarrow a)}$$

$$\frac{\vdots}{\longrightarrow (a \Rightarrow b) \Rightarrow (\neg a \Rightarrow \neg b)}$$