Pre-Optimizations of Resolution

The Resolution algorithm is exponential in the number and size of the clauses. Thus, minimizing the number and size of the input clauses is potentially extremely beneficial.

A number of theorems allow us to reduce the scale of the input set. We first need to define the following notations:

Definition: Given a literal l we will denote its complement by l^c.

Definition:(2.10.4) Let S and S' be sets of clauses. The notation $S \approx S'$ denotes that S is satisfiable if and only if S' is satisfiable.

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Lemma:(2.10.5) Given a set of clauses S, if there is a literal l that occurs among the clauses of S, but its complement, l^c, does not occur in S, then the set of clauses S' obtained from S by deleting every clause that includes l is such that $S \approx S'$.

Proof.

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Lemma:(2.10.6) Given a set of clauses S, if S includes a unit clause (a clause with only one literal), $\{l\}$, then the set of clauses S' obtained from S by deleting every clause containing l, as well as deleteing l^c from every remaining clause, is such that $S \approx S'$.

Proof.

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Lemma:(2.10.7) Given a set of clauses S, if S includes a clause that contains both a literal, l, and its complement l^c, then the set of clauses S' obtained from S by deleting that clause is such that $S \approx S'$.

Proof.

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Definition:(2.10.8) Given two clauses C_1 and C_2, if all the literals in C_1 are in C_2 —That is, $C_1 \subseteq C_2$—, we say that C_1 *subsumes* C_2, and C_2 *is subsumed by* C_1.

Lemma:(2.10.9) Given a set of clauses S, if S includes clauses C_1 and C_2 such that C_1 subsumes C_2, then the set of clauses S' obtained from S by deleting C_2 (the larger clause) is such that $S \approx S'$.

Proof.

An Alternate Mechanical Proof Method

We can show that given a set S of clauses, the corresponding CNF formula is valid, if and only iff each clause of S contains a clashing pair of literals.

This seems to be a MUCH cheaper method. Why go to the bother and expense of resolution refutation?