Peano Arithmetic

Peano laid out the following set of axioms, intending them to describe the natural numbers under successor, addition, and multiplication (up to isomorphism). It turns out, however, that they are modeled by a large class of non-isomorphic structures (called arithmetic structures or Peano structures).

Language:

$\mathcal{A} = \{ \qquad \qquad \}$ $\quad \mathcal{F} = \{ \qquad \qquad \}$ $\quad \mathcal{P} = \{ \qquad \qquad \}$

Γ**:**

- Zero is the successor of no number.

- If the successors of two numbers are equal, then the two numbers are equal.

- The sum of any number and zero is that number.

Peano Arithmetic

- The sum of a number and the successor of another number is the successor of the sum of the two numbers.

- The product of any number and zero is zero.

- The product of any number and the successor of another number is the sum of the first number and the product of the two numbers.

Peano Arithmetic

- (The Principle of Mathematical Induction)
  If a property holds for zero, and holds for the successor of a number whenever it holds for the number, then it holds for all numbers.

Peano Arithmetic

We can write down a variety of properties of numbers as the definitions of predicates. These definitions can be at the meta-level, as a sort of macro, or at the object level as new axioms.

- A number is even if:

- A number is divisible by another number if:

- A number is prime if:

Plane Projective Geometry

The axiomatization of plane projective geometry is built around an asymmetric *incedence* relation between points and lines.

Language:

$\mathcal{A} = \{ \quad \}$ $\mathcal{F} = \{ \quad \}$ $\mathcal{P} = \{ \quad \}$

Γ:

- A point is something that is incident to a line.
- A line is something that has a point incident to it.
- Eveything is either a point or a line, but not both.

Plane Projective Geometry

- For every pair of points, there is a line to which both are incident.

- For every pair of lines, there is a point that is incident to both.

- If two points are incident to two lines, then either the points are the same or the lines are the same (i.e. the point and line in the last two axioms are unique).

Plane Projective Geometry

- There are enough distinct points and lines (3 of each) to make the structure non-trivial.