# Complete Heuristics

**Definition:** (4.5.1 – **Set-of-Support Resolution**) Let $\Gamma$ be a set of Clauses, and let $\Delta$ be a subset of $\Gamma$ such that the set $\Gamma - \Delta$ is satisfiable. Then, at each resolution step, pick the clauses $C_1$ and $C_2$ to be resolved such that at least one of them is not a member of $\Gamma - \Delta$.

**Definition:** (4.5.3 – **Linear Resolution**) Let the original goal $A$ of the attempt to prove $\Gamma \Rightarrow A$ be called the *center clause*. Then, at each resolution step, pick the clauses $C_1$ and $C_2$ to be resolved such that at least one of them is the center clause. The resolvent then becomes the center clause in the next step.

Both of these restricted methods are complete: if there is a resolution refutation, then one can be found using either of these methods. Linear Resolution would also have enabled us to terminate the attempt to show that 3 was not in the list $[1, 2, 4]$ and report it unprovable.

# Incomplete Heuristics

**Definition:** (4.5.4 – **Input Resolution**) At each resolution step, pick the clauses $C_1$ and $C_2$ to be resolved such that at least one of them is one of the original clauses (given as input to the refutation procedure).

In general, input resolution is not complete. However, it is often extremely efficient.

# Horn Clauses

**Definition:** A *Horn Clause* is a formula of the form:

$$\forall \bar{x}((A_1 \wedge \cdots \wedge A_n) \Rightarrow A_0)$$

where each of the formulas $A_i$ is atomic, and $\bar{x}$ contains exactly the free variables of the underlying implication.

Note that in clausal form, a Horn clause appears as:

$$\neg A_1 \vee \cdots \vee \neg A_n \vee A_0$$

**Lemma:** If we restrict the clauses of $\Gamma$ to be Horn clauses, and the consequence $A$ to be of the form:

$$\exists \bar{x}(A_1 \wedge \cdots \wedge A_n)$$

then linear, input resolution is complete. Further, if there is refutation, then there is a refutation which produces an answer substitution.