CS140: Algorithms

Fall 2000

Assignment 8

This assignment is due at the start of class on Wed., Oct. 25. You may collaborate with your classmates to complete this assignment but you must reference your collaborators. In addition, you must do your own write-up. This assignment is worth 100 points (50 points if handwritten). Your answers will be judged for clarity, conciseness, and correctness. You algorithms and bounds should be, respectively, as fast and as tight as possible.

Each of the following problems can be solved using dynamic programming by strengthening the inductive hypothesis. For each, clearly state the alternative problem you propose to solve. Give a dynamic programming algorithm for the alternative problem. Show how to use the algorithm for the alternative problem to solve the original problem. Analyze the running time of the final algorithm.

1. Problem 16-4(a) on page 326 of the text but, rather than outputting the guest list, your algorithm need only compute the maximum achievable conviviality score of any guest list.

   Following the procedure described above, design a dynamic programming algorithm for this problem and analyze its running time.

   For each node v in the tree we will compute two functions: $PE(v)$ and $PI(v)$. $PE(v)$ is the conviviality rating of the best party on the subtree rooted at v in which v *is not* invited. $PI(v)$ is the conviviality rating of the best party on the subtree rooted at v in which v *is* invited. We can define these functions recursively as follows.

If v is a leaf:	$PE(v)$	$=$	0
	$PI(v)$	$=$	$c(v)$
Else	$PE(v)$	$=$	$\sum_{\text{children } w \text{ of } v} \max(PE(w), PI(w))$
	$PI(v)$	$=$	$\sum_{\text{children } w \text{ of } v} PE(w)$

   where $c(v)$ is the conviviality rating of v.

   We can compute these function from the leaves up. The value of these function for any node are used a constant number of times in computing the functions for the parent. Thus the time to compute these functions at all nodes of the tree is $O(n)$.

   The best possible conviviality rating of any party is $\max(PE(p), PI(p))$, where p is the root (or president node) of the tree.

2. In this problem you are given n pairs of integers: $(a_{1,1}, a_{1,2}), \ldots, (a_{n,1}, a_{n,2})$. Your objective is to find the maximum product that can be formed using exactly one elment of each pair. For example, consider the input

$(-2, 1), (2, 100), (-1, 1)$. The maximimum possible product is $(-2)*(100)*(-1) = 200$.

Following the procedure described above, design a dynamic programming algorithm for this problem and analyze its running time.

For each $i \in [1, n]$ we'll compute two functions, $M(i)$ and $m(i)$. $M(i)$ is the maximum product that can be obtained by multiplying one element of each of the first i pairs. Similarly, $m(i)$ is the minimum product that can be obtained. Then $M(1) = \max(a_{1,1}, a_{1,2})$ and $m(1) = \min(a_{1,1}, a_{1,2})$. For $i > 1$ we can compute $M(i)$ and $m(i)$ using the following recursive definition.

$$\begin{aligned} M(i) &= \max(M(i-1) * a_{i,1}, M(i-1) * a_{i,2}, m(i-1) * a_{i,1}, m(i-1) * a_{i,2}) \\ m(i) &= \min(M(i-1) * a_{i,1}, M(i-1) * a_{i,2}, m(i-1) * a_{i,1}, m(i-1) * a_{i,2}) \end{aligned}$$

We can compute $M(i)$ and $m(i)$ in $O(1)$ based on $M(i-1)$ and $m(i-1)$. If we compute these functions for increasing values of i we can compute all values in $O(n)$ time.

The solution to the original problem is $M(n)$.