CS-155

Notes on clipping calculations

These notes summarize the calculation needed to clip for assignment 4. I believe these notes are error-free. If you find an error or have a problem please contact me. To clip a line segment against a plane you need to be able to do the following.

1. Compute the equation of a plane:

   Let $p_0 = (x_0, y_0, z_0), p_1 = (x_1, y_1, z_1)$, and $p_2 = (x_2, y_2, z_2)$ be three points on the plane.

   Define two vectors $u =< u_0, u_1, u_2 >$ and $v =< v_0, v_1, v_2 >$ where $u_0 = x_1 - x_0, u_1 = y_1 - y_0, u_2 = z_1 - z_0$ and $v_0 = x_2 - x_0, v_1 = y_2 - y_0, v_2 = z_2 - z_0$.

   Compute the cross-product $u \times v =< A, B, C >$ where $A = u_1 v_2 - u_2 v_1$, $B = -(u_0 v_2 - u_2 v_0)$, and $C = u_0 v_1 - u_1 v_0$. Then let $D = Ax_0 + By_0 + Cz_0$. The equation of the plane is $Ax + By + Cz = D$.

   Here is an example. The plane contains the points $(1, 0, 1), (1, 0, 0)$ and $(1, 1, 0)$. The vectors are $u =< 0, 0, -1 >$ and $v =< 0, 1, -1 >$. Their cross-product is $n =< 1, 0, 0 >$. $D = 1$. The the equation of the plane is $x = 1$. (This should be obvious from the initial points.)

   A side note: The vector $n =< A, B, C >$ is normal to the plane. The vector $< -A, -B, -C >$ is also normal to the plane but points in the opposite direction of $< A, B, C >$. The vector $< -A, -B, -C >$ corresponds to the equation $-Ax - By - Cz = -D$ of the plane. This will be useful in the next section.

2. Determine which side of the plane an endpoints of L lies on:

   You must define what you mean by inside and outside for each boundary plane. Here is how you should do it. Imagine you are standing at a point "inside" and you are looking at the bounding plane. Consider the three defining points p_0, p_1, p_2. Are they oriented counter-clockwise or clockwise from where you are standing? If the answer is counter-clockwise then $< A, B, C >$ as computed in the previous section is an inward-pointing normal. If the answer is clockwise then $< A, B, C >$ is an outward-pointing normal; in this case $< -A, -B, -C >$ is an inward-pointing normal.

   For each bounding plane you should compute an inward-pointing normal. (You can let the program do this from three points on the plane that you choose and order so that they are oriented counter-clockwise with respect to "inside.")

   Now, suppose you want to test if a point (p, q, r) is "inside", "outside", or on the plane. Choose a point (u, v, w) that lies on the plane (use one of your initial points). Let $< A, B, C >$ be the inward-pointing normal.

If $A(p-u) + B*(q-v) + C*(r-w) > 0$ the point lies inside. If $A(p-u) + B*(q-v) + C*(r-w) < 0$ the point lies outside. If $A(p-u) + B*(q-v) + C*(r-w) = 0$ the point lies on the plane.

3. Compute the intersection of a line and plane:

   Let $p_0 = (x_0, y_0, z_0)$ and $p_1 = (x_1, y_1, z_1)$ be points on the line and let $Ax + By + Cz = D$ be the equation of the plane.

   Let $E = D - A*x_0 - B*y_0 - C*z_0$ and $F = A*(x_1 - x_0) + B*(y_1 - y_0) + C*(z_1 - z_0)$. If $F = 0$ the line does not intersect the plane or it is contained in the plane. In either case you are in trouble. If $F! = 0$ then $t = E/F$ and the point of intersection of the line and the plane is $(x_0 + t(x_1 - x_0), y_0 + t(y_1 - y_0), z + t(z_1 - z_0))$.