CS140: Algorithms

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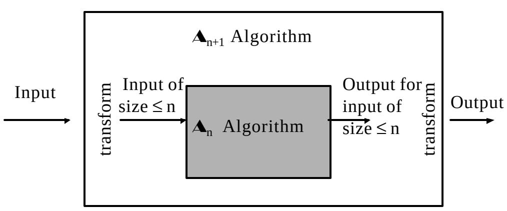
Lecture 2
1/23/01

Today

- **Inductive Design of Algorithms**

$\mathcal{A}_{n+1}$ Algorithm

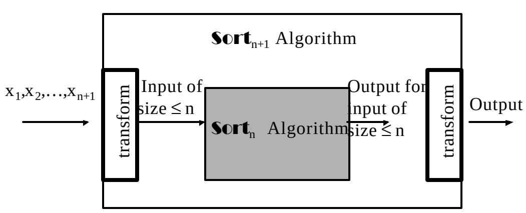
Solves instance(s) of size n or smaller



Inductive Design

- Objective: Define the solution for input of size n+1 in terms of solutions for inputs of size ≤ n

$\textbf{Sort}_{n+1}$ Algorithm



Inductive Sort

Transform Input	Transform output
• Remove element x	• ______________
• Remove max	• ______________
• Remove min	• ______________
• Partition input into 2 sets	• ______________
• Partition input into 2 sets about pivot	• ______________

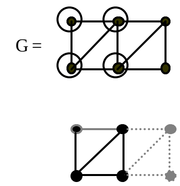
Induced Subgraphs

- Let G=(V,E) be a graph and let W be a subset of V.
- The subgraph of G induced by W is the graph with

  Vertex set: W

  Edge set: {(x,y) ε E such that x and y are in W}

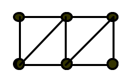
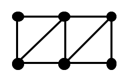
Example



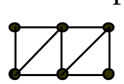
Maximal Induced Subgraph

- Input: A graph G=(V,E) and an integer k.
- Output: A largest subgraph G′=(V′,E′) of G such that every vertex of G′ has degree at least k.

Maximal Induced Subgraph Example

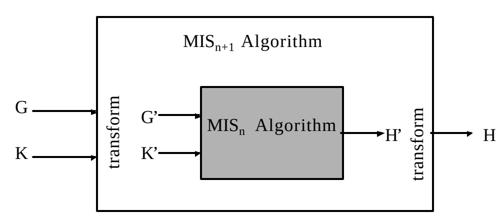
- Input: G =  K=2
- Output: 

Maximal Induced Subgraph Example

- Input: G =  K=3
- Output: ϕ

MIS_{n+1} Algorithm

Finds MIS on graphs with n+1 or fewer node



MIS(G,k)

- If every vertex has degree at least k
  - Return G
- Else
  - Let x be a vertex with degree less than k
  - Let G′ be the subgraph of G induced by V-{x}
  - Return $MIS_n(G',k)$

Correctness

Let H be a maximal induced subgraph of G with degree at least k.

- Case 1: Every vertex of G has degree at least k:
- Case 2: A vertex x of G has degree less than k:

Running Time

(Assume G has n vertices and m edges)

- Adjacency Matrix: $T(n) \le T(n-1)+cn^2$
- Adjacency List: $T(n,m) \le T(n-1,m-1) + m$
- Other:

Polynomial Evaluation

- Input: Integers $a_n, a_{n-1}, \ldots, a_0$ and an integer x.
- Output: $P_n(x) = \Sigma_{i=o\ldots n} a_i x^i$

Polynomial Evaluation Example

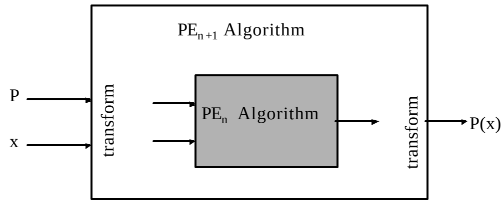
- Input: 3,-1,0,2 and 2
- Output: 22
- Explanation: $P(x)=3x^3-x^2+2$; $P(2)=22$

Naïve Approach

- Number of multiplications:
- Number of additions:

Pe_{n+1} Algorithm

Evaluation of polynomials of degree at most n+1



$PE(a_n,\ldots,a_0,x)$

- If n=0 then return a_0
- Else return $a_0 + x \cdot PE(a_n, \ldots, a_1, x)$

Inductive Approach

- Number of multiplications:

  $M(n) = 1 + M(n-1)$, $M(0) = 0$

  Closed form: $M(n)=n$
- Number of additions:

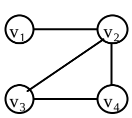
  $A(n) = 1 + A(n-1)$, $A(0) = 0$

  Closed form: $A(n)=n$

Vertex Cover

- Let G=(V,E) be a graph
- A vertex cover of G is a subset $W \subseteq V$ such that for every edge e=(x,y) of G either x is in W or y is in W

Vertex Cover Example



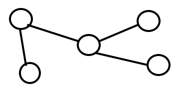
Some vertex covers:
$\{v_1, v_2, v_3, v_4\}$,
$\{v_2, v_3\}$

Vertex Covers in Forests

- Input: Forest F=(V,E)
- Output: A smallest vertex cover of F

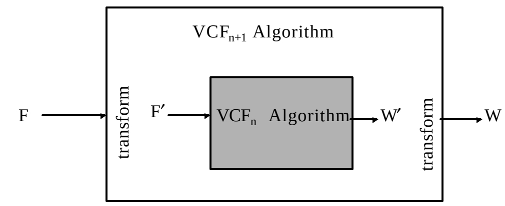
A few observations and definitions

- A collection of trees is a forest
- A tree on n nodes has n-1 edges
- A tree never has a cycle
- A tree is always connected
- A tree need not be rooted
- A node with 0 or 1 edges in a tree is a leaf
- A (non-empty) tree always has a leaf



VCF_{n+1} Algorithm

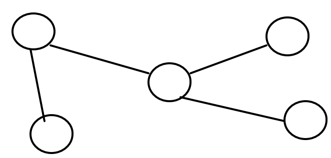
Vertex Cover in forests with at most n+1 nodes



Simple Case

- If F has no edges then return __________

Suppose F has an edge: Find something to keep or throw away …



Claim

- If u is a leaf of F and v is adjacent to u then some smallest vertex cover of F includes v.
- Proof:
  - Let W be a smallest vertex cover of F.
  - If v is not in W then u must be.
  - But then W-{u}+{v} is also a smallest vertex cover of F.

Vertex Cover in Forests

If F has no edges return φ

Else {

Let u be a leaf with an edge in F

Let v be the node adjacent to u in F

Let F′ be the subgraph of F induced by V-{u,v}

Return {v}∪VCT(F′)

}