

CS140: Algorithms

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Lecture 8
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Outline

- Reductions to network flow
- Reductions between search and decision problems

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Reductions to Network Flow Problem

- **Bipartite Matching** μ **Network Flow**
- The Gee-ball Problem ∞ Network Flow

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Matching

- Let $G=(V,E)$ be a graph.
- $E' \subseteq E$ is a matching if every vertex of V is incident to at most one edge of E' .

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Matching Example



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Bipartite Graph

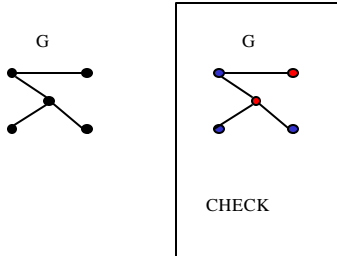
- Let $G=(V,E)$ be a graph.
- G is bipartite if V can be partitioned into V_1 and V_2 such that no pair of vertices in V_i ($i=1,2$) have an edge.

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Bipartite Example



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Bipartite Matching

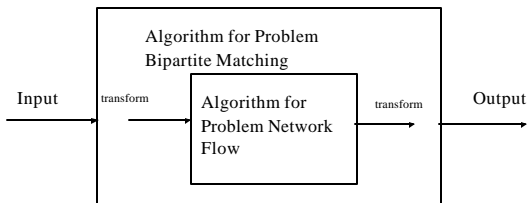
- Input: Bipartite graph G
- Output: A largest matching of G

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Bipartite Matching ∞ Network Flow



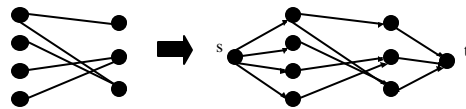
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Bipartite Matching ∞ Network Flow

- Transform Input – Step 1



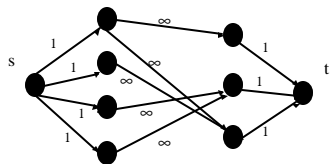
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Bipartite Matching ∞ Network Flow

- Transform Input – Step 2

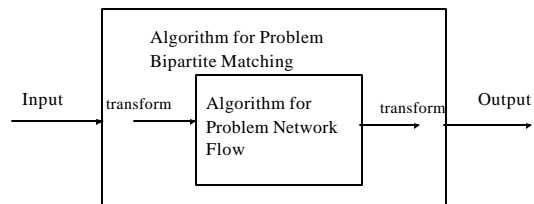


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Bipartite Matching ∞ Network Flow



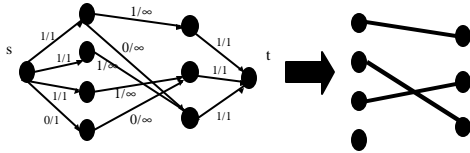
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Bipartite Matching ∞ Network Flow

- Transform output



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Reduction

- Is it correct?
- Is it efficient?

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Integrality theorem

- If the capacities in a network are integral, then the max flow can be achieved with integral flows on each edge.
- Further the Ford-Fulkerson method yields an integral solution.

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Proof of correctness

There is a 1-1 correspondence between 0/1 flows in the network and matchings in the input graph.

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Reduction

- Is it correct?
- **Is it efficient?**

$$T_{BM}(n) = cn + T_{FF}(n+2)$$

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Reductions to Network Flow Problem

- Bipartite Matching ∞ Network Flow
- **The Gee-ball Problem μ Network Flow**

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The Gee-ball Problem

- The southwestern conference of the gee-ball league consists of $n+1$ teams. Team $n+1$ is from HMC.
- We want to know whether it is possible for HMC to win more games this season than any other team in the conference.
- No ties allowed.

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Example

- The teams are Pitzer, CMC, Pomona, and HMC
- Games won so far:
 - Pitzer 4, CMC 3, Pomona 2, HMC 2
- Games to play:
 - 1 game: Pitzer vs. HMC
 - 2 games: Pomona vs. HMC

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The Gee-ball Problem

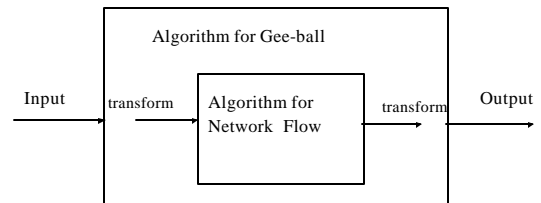
- Teams $t_1, t_2, \dots, t_n, t_{n+1}$
- So far this year team i has won w_i games.
- Teams i and j will play each other g_{ij} more times this season ($g_{ij} = g_{ji}$).

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Gee-ball ∞ Network Flow

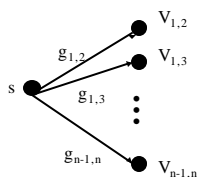


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Transform Input



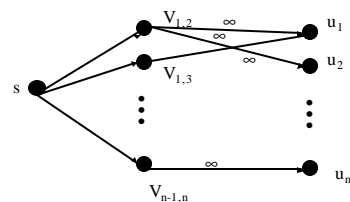
1. Create a source s .
2. Create vertex v_{ij} for $1 \leq i < j \leq n$
3. Create edge from s to v_{ij} with capacity g_{ij}

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Transform Input



4. Create vertices $u_i, 1 \leq i \leq n$
5. Create edge from v_{ij} to u_i and u_j with infinite capacity.

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Transform Input

5. Create a sink t .

6. Create edge from u_i to t with capacity W_i .

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Transform Input

Let $W = w_{n+1} + \sum_{i=1 \dots n} g_{n+1,i}$

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Gee-ball ∞ Network Flow

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Transform Output

There is a way for HMC to win the season if and only if _____.

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Decision Problems

Decision problems are computational problems for which the output is “yes” or “no”

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Search: Vertex Cover

- Input: Graph G
- Output: A smallest subset of vertices W such that every edge of G is incident to some vertex in W .

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Decision: Vertex Cover

- Input: Graph G and integer K
- Question: Is there a subset W of K or fewer vertices of G such that every edge is incident to some vertex in W ?

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Polynomial Time Reductions

- Decision-VC \leq_p Search-VC
- Search-VC \leq_p Decision-VC

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Independent Set

- Input: Graph G and integer K
- Question: Is there a subset W of K or more vertices of G such that no two vertices of W are adjacent?
- Does Search-IS \leq_p Decision-IS?

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3-Colorability

- Input: Graph G
- Question: Can the vertices of G be colored with Red, Green, and Blue so that no adjacent vertices have the same color?
- Does Search-3Col \leq_p Decision-3Col?

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