NP-completeness
Problem A is NP-Complete if
• A is in NP
• A is NP-hard

Last Time – Harder Reductions
• Next homework:
  – Partition into triangle (with hints)
  – Minimum test collection (with hints)

Today – Killer Reductions
• Grundy numbering
• Partition into paths of length 2

Grundy numbering
• Input: Directed graph G=(V,E)
• Question: Can the vertices of G be labeled with integers such that, for every vertex v, the label of v, L(v) is the smallest non-negative integer that is NOT in the set {L(u): <v,u> is in E}

3SAT $\preceq_p$ Grundy
Partition into Paths of Length 2

- **Input:** Graph $G=(V,E)$ where $|V|=3q$ for some integer $q$
- **Question:** Can $V$ be partitioned into $q$ disjoint sets $V_1, \ldots, V_q$ where each set contains three vertices, so that the vertices $x,y,z$ of $V_i$ comprise a path of length 2 in $G$.

3DM $\propto_p$ Partition into Paths of Length 2

Algorithm for 3DM

<table>
<thead>
<tr>
<th>S.C</th>
<th>G</th>
<th>Algorithm for Partition into Paths of length 2</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Y</td>
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Approach

- Understand the problem
- Build some gadgets
- Combine gadgets

Approach: Grundy numbering

- Understand the problem
  - Build some simple graphs that have Grundy numberings
  - Build some that don’t
- Build some gadgets
  - Build a gadget that enforces the truth assignment condition
  - Build a gadget that enforces a satisfiability condition
- Combine gadgets

Truth Assignment

- For each variable $x$ exactly one of $x$ and its complement is set to true and the other is set to false

Satisfiability

- For each clause $c$, at least one literal in $c$ evaluates to true.