PRAM Computations (cont'd)

- So far have seen:
  - Binary-tree expressions (max, etc.)
  - Prefix sum
  - Vector compression and expansion (uses prefix sum)
  - Max-finding in O(1) (using CRCW-arb)

PRAM Computations (cont'd)

- Today:
  - Parallel merging
  - Pointer techniques
  - Tree-traversal
  - Quicksort (1-version)

Parallel Merging

- How can we **merge** two ordered arrays in parallel on a PRAM?
- One means is to compute the *indices* of where the elements of one array are inserted into the other array.
- We can then use something similar to array compression to do the actual insertion.

Computing indices for Parallel Merging

- A single index can be computed with **binary search**: Find the position in the other array at which the element would be inserted.
- Add the element's current index to it to get the net final position.
- Do all binary searches in parallel.
- Each array computes the final indexes of its elements in the merged array
- Each processor stores its elements simultaneously

Parallel Merging

- Cost: n binary searches in parallel
- O(log n) time (on CREW PRAM)

Exercise

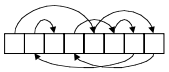
- What is an upper bound for sorting using parallel merging?

Using Pointers in a PRAM

- "Pointer Jumping" technique
- As with prefix sum, this has many uses.
- Basic idea: in a chain of pointers stored in the common memory, the extremities of a chain can be determined in a way that **doubles** the length of the chain at each step:
  - If a location points "N hops away" now, it will point "2N hops away" on the next step.
  - This is because concatenating two N-hop chains gives a 2N-hop chain.

List-Ranking Problem

- A pointer-jumping application
- Given a chain of pointers, determine the rank of each element in the chain



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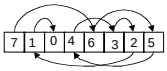
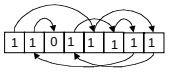


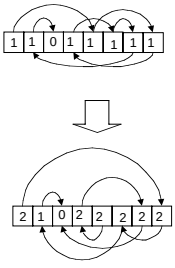
List-Ranking Step-by-Step

Step 0



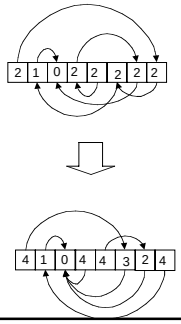
List-Ranking Step-by-Step

Step 1



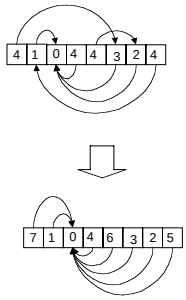
List-Ranking Step-by-Step

Step 2



List-Ranking Step-by-Step

Step 3



Summary

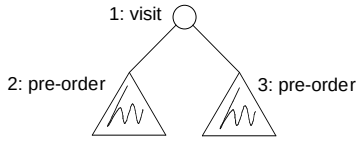
- A list can be ranked in O(log n) time on a PRAM.

Exercises

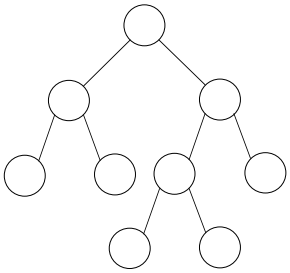
- Construct the PRAM code for list ranking.
- Show that a *list* can be prefixed-summed in O(log n).

Pre-Ordering a Tree

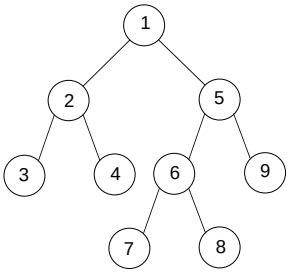
- Recall: Pre-Order:
  - Visit the root
  - Visit recursively the left sub-tree in pre-order
  - Visit recursively the right sub-tree in pre-order



Pre-Order Example



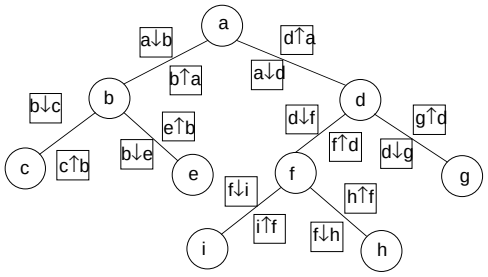
Pre-Order Example



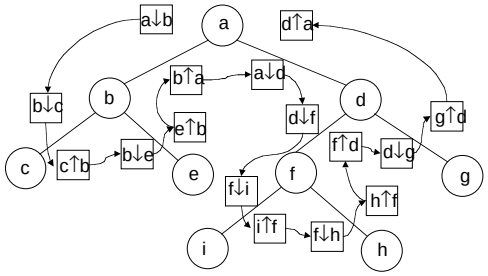
How to Construct a Pre-Order on a PRAM in O(log n)?

- Create a list of nodes, two per arc of the original graph:
  - One node for the arc in the normal (downward) direction
  - A second for the arc in the other direction
- Add a parity-indicator
- Connect the nodes to represent the original arcs

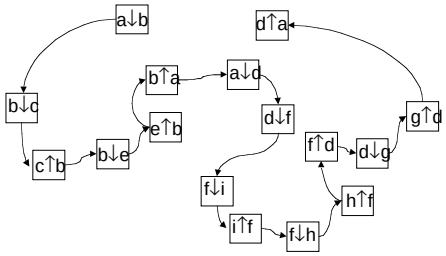
Pre-Order Example



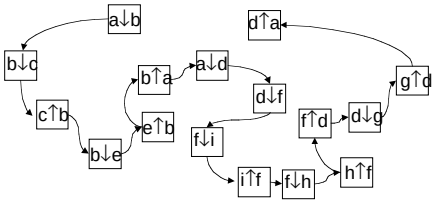
Pre-Order Example



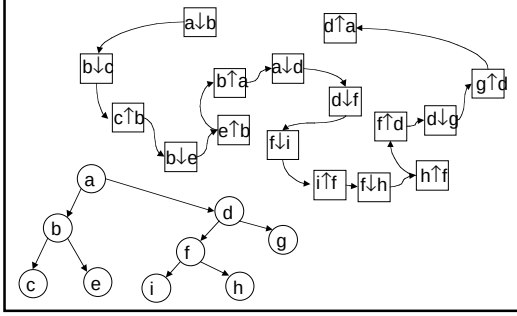
We now have an alternate representation from which we can recover the original tree



Try it

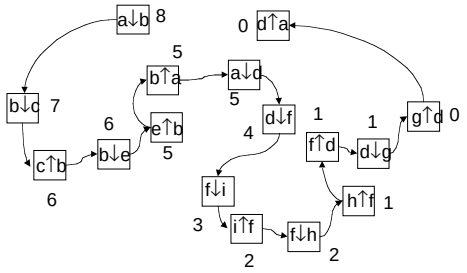


Try it

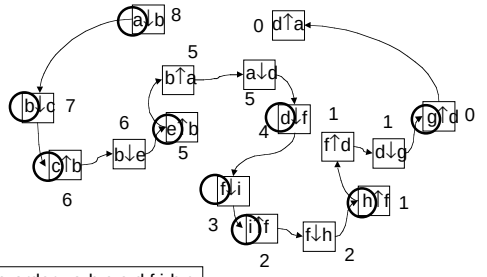


What now?

List ranking variation
Count *downward* nodes only



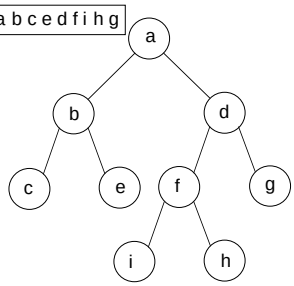
Pre-order of original is reverse of this order
using first component of the root and the *targets* of the ↓ nodes only



Pre-order: a b c e d f i h g

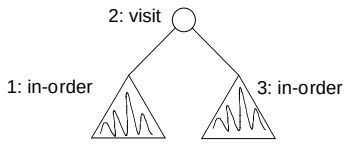
Check with Original

Pre-order: a b c e d f i h g



Exercise

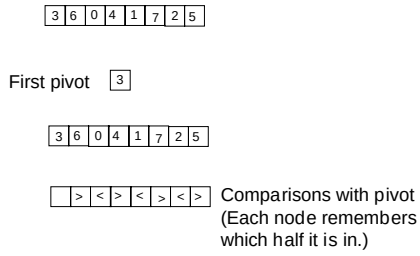
- We just showed that a pre-order traversal can be done in time O(log n). Do the same for an *in-order* traversal.



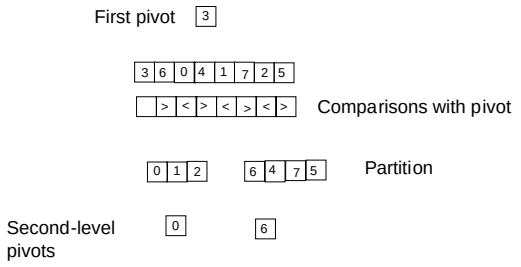
Application of In-Order Traversal

- A PRAM version of Quicksort
- Construct a tree indicating how the nodes partition (without actually moving any data)
- An in-order traversal of the tree gives the nodes in sorted order.

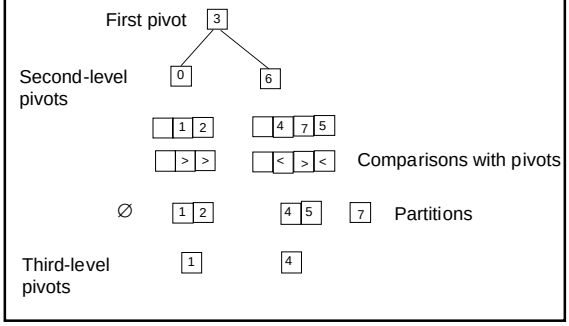
Quicksort Partition Tree



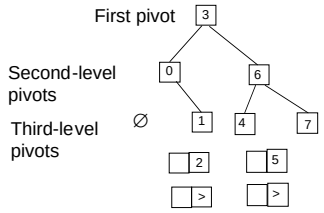
Quicksort Partition Tree



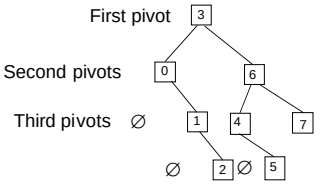
Quicksort Partition Tree



Quicksort Partition Tree



Quicksort Partition Tree



In-order traversal is sorted array

Exercise

- Assuming that the tree splits fairly evenly across each level, what is the time taken to do this version of Quicksort (assuming the asserted bound for in-order traversal).

"Parallel sorting in O(1)"

(from http://www.cs.uku.fi/~penttone/parallel/sort.html, has demo)

```
procedure Sort(modifies A: array 1..n of integer)
for i in 1..n pardo K[i]:=0
for i in 1..n pardo
  for j in 1..n pardo
    if A[i]<=A[j] then K[j]:=K[j]+1
for i in 1..n pardo A[K[i]]:=A[i]
```

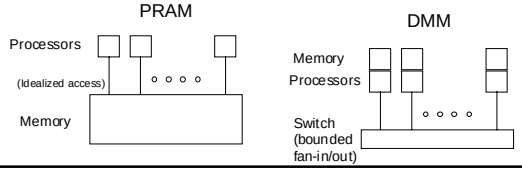
How many processors?
What kind of conflict resolution?
How much effort?

Misc. Notes on PRAM

- Cole's sorting method based on merging: O(log n) on a CREW PRAM (c.f. Gibbons, A.M. & Rytter, W. (1988) Efficient Parallel Algorithms. Cambridge University Press.)
- Parallel recognition of a context-free language: $P(\log^2 n)$ time using n^6 processors.
- Other problems/algorithms are known.

Using PRAM results in real life

- What are some problems of simulating PRAM's on real multiprocessors, say a on a Distributed-Memory Machine (DMM)?



PRAM ->DMM problems

- Memory conflict resolution at word-level
- Memory conflict resolution at memory-module level
- Communication delays

PRAM ->DMM possible resolutions

- Memory conflict resolution at word-level:
  Use only EREW model
- Memory conflict resolution at memory-module level
  Split memory into multiple modules; Use multiple copies of contended data (must provide for reconciling)
- Communication delays
  Use 2-phase, random routing

Efficient Simulation of PRAM

- Karp, et al. STOC 1991
- An n log log(n) log*(n) processor CRCW-arb PRAM is simulated on an n-processor DMM (Distributed memory machine).
- Average slowdown O(log log(n) log*(n)).
- Survey of related results: Tim Harris, ACM Computing Surveys, **26**, 2, June 1994, 187-206.