Theorem (Liu & Layland, 1973)

- A *sufficient* condition for a set of n tasks to be rate-monotonically schedulable on 1 processor, regardless of phasing is:

  total utilization $\leq n*(2^{1/n} - 1)$

- where *total utilization* is defined as sum(C_i/P_i, i = 1 to n)
- The condition is sufficient, but not necessary.

Numeric Values for Liu & Layland Rule

tasks	bound on total utilization
1	1
2	0.828427
3	0.779763
4	0.756828
8	0.724062
16	0.707472
32	0.700709
64	0.696914
…	
∞	ln(2) ≈ 0.693147

Rationale behind Liu & Layland

- Assume all phases are 0.
- Case of 1 task, T_1, period P_1, time C_1.
  - Obviously this will be schedulable iff
    $C_1 \leq P_1$,
    which is the same as
    sum(C_i/P_i, i = 1 to 1) = 1.

RMA Rationale

- Case of n tasks, $T_1, \ldots, T_n$, periods $P_1 \leq \ldots \leq P_n$, times $C_1, \ldots, C_n$.
  - Consider an interval of time [0, t].
  - During the interval each T_i must execute $\lfloor t/P_i \rfloor$ times.
  - The total time used for T_i during the interval will be $C_i * \lfloor t/P_i \rfloor$.
  - In order for each T_i to execute the appropriate number of times in the interval, we need ($\forall t' < t$) $t' \geq$ sum($C_i * \lfloor t/P_i \rfloor$).
  - If we can find a $t \leq$ lcm($P_1, \ldots, P_n$) having this property, then all tasks can be scheduled.

Observation

- ($\forall t' < t$) $t' \geq$ sum($C_i * \lfloor t/P_i \rfloor$)
  iff
  ($\forall t' < t$) t' is a time corresponding to the end of some task's period $\Rightarrow t' \geq$ sum($C_i * \lfloor t/P_i \rfloor$).

Example

- $P_1 = 100$, $P_2 = 150$,
  $C_1 = 20$, $C_2 = 30$.
- Consider an interval of time [0, 300], during which T_1 must complete $\lceil 300/100 \rceil = 3$ times and T_2 must complete 2 times.
- The total time required is 3*20 + 2*30 = 120 $\leq$ 300.
- What schedule realizes the requirements?

task	T_1	T_2	T_1	T_2	T_1
start	0	20	100	150	200
end	20	50	120	180	220

Example

(L&L rule not necessary)

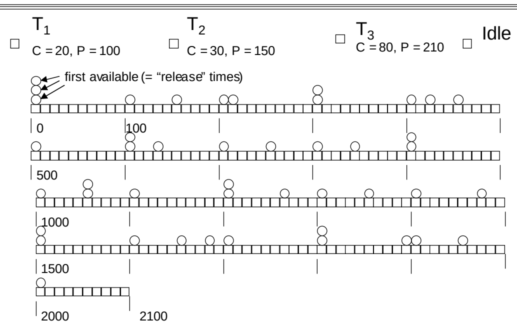
- Consider adding a third task:
  $P_1 = 100$, $P_2 = 150$, $P_3 = 210$,
  $C_1 = 20$, $C_2 = 30$, $C_3 = 80$.
- The Liu and Layland rule computes total utilization: 20/100 + 30/150 + 80/210 = 0.780952 which is not realizable for 3 tasks (0.7796).
- Since the Liu and Layland rule is sufficient, but not necessary, there still might be a schedule.
- Can you construct one?

Example

- $P_1 = 100$, $P_2 = 150$, $P_3 = 210$,
  $C_1 = 20$, $C_2 = 30$, $C_3 = 80$.
- Consider an interval of time [0, 2100], (2100 is the lcm of 100, 150, and 210) during which T_1 must complete 21 times, T_2 14 times, and T_3 10 times.
- The total time required is 21*20 + 14*30 + 10*80 = 420+720+800 = 1940 $\leq$ 2100. So it is at least *plausible* that there is a schedule.
- On the next page, we construct a schedule.

A Rate-Monotone Schedule

(each box = 10 time units)



We didn't have to compute that whole thing:

Lehoczky, Sha, and Ding Theorem (1987)

- Using RM scheduling, if each task meets its **first** deadline, then all deadlines will be met.

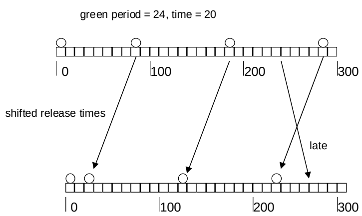
Example

- Consider adding a fourth task:
  $P_1 = 100$, $P_2 = 150$, $P_3 = 210$, $P_4 = 400$,
  $C_1 = 20$, $C_2 = 30$, $C_3 = 80$, $C_4 = 100$.
- Total utilization: 20/100 + 30/150 + 80/210 + 100/400 = 1.03095 > 1, so this set is not realizable.

2nd Theorem of Liu & Layland

- If a set of tasks is schedulable under any fixed priority scheme, then it is schedulable using rate-monotonic scheduling.
- In other words, rate-monotonic is optimal among fixed priority schemes.

How release time of a higher-priority task can affect the response of a task in RM scheduling



Liu & Layland Idea

- A *critical instant* of a task is a time at which the task becomes available and all higher priority tasks also become available.
- If task scheduling can be determined to be feasible at critical instants, then it is feasible in general.

Priorities Revisited

- RMA is an example of a **fixed priority** scheme: priority is determined in order opposite periods.
- Dynamic priority schemes are possible.

Dynamic Priority

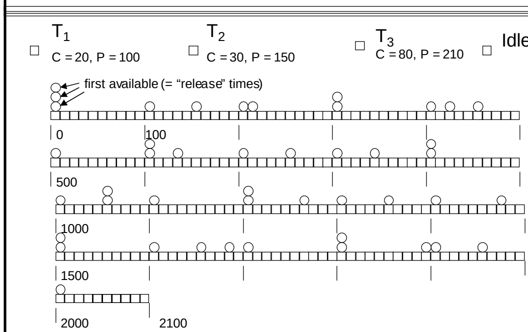
- Recall Horn's rule: Preemptive EDF (Earliest Deadline First)
- It works for *arbitrary* arrivals.
- Therefore it will work for periodic tasks as well.

Dynamic Priority

- EDF is *dynamic* because the relative priority will depend on what is being executed when a task arrives. This could be:
  - A task with a longer period but nearer deadline.
  - A task with a shorter period but more distant deadline.

A Dynamic-Priority Schedule

(each box = 10 time units)



Exercise

- Devise a system of two periodic tasks such that:
  - There is no rate-monotonic schedule
  - There is an EDF schedule

Hint

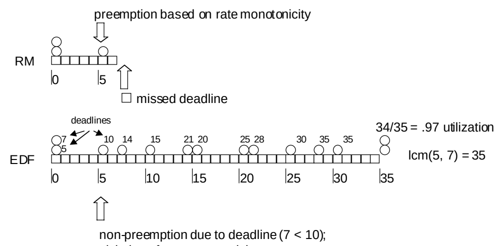
- Make the longer-period task have a relatively high utilization, so that deferring it will make it miss its deadline.

Hint

- $P_1 = 5$, $C_1 = 2$
- $P_2 = 7$, $C_2 = 4$

Example

- $P_1 = 5$, $C_1 = 2$
- $P_2 = 7$, $C_2 = 4$



Utilization Bounds for Dynamic Priority Assignment

- For dynamic priority assignment, any system with a total utilization ≤ 1 can be scheduled.
- EDF is adequate for constructing such a schedule.

Generalization of Periodic Tasks

- So far,
  deadline of a periodic task
  = end of period
- Generalization:
  periodic tasks with fixed deadlines ***relative*** to start of period
  (deadlines possibly *sooner* than end of period).

Deadline-Monotonic (DM) Scheduling

- Assume relative deadlines are *constant*.
- Assign priorities in order of nearest relative deadline.
- Since the relative deadlines are constant, this is a *static* priority assignment.
- DM has been shown *optimal* for this more general case (Leung and Whitehead, 1982).

If relative deadlines are $\leq$ period and not constant

- EDF again works
- Schedulability can be checked using "processor demand" approach:
  - Processor demand in an interval [0, L] is the computation time required in order for all tasks to complete by their deadlines in that interval.
  - Check that processor demand <= length of interval.
  - Need only check at release times between 0 and lcm(periods).

Summary of Applicability

	Deadline = Period	Deadline $\leq$ Period
Static Priority	Rate Monotonic	Deadline Monotonic
Dynamic Priority	EDF	EDF

References

- C. L. Liu and J. W. Layland. *Scheduling Algorithms for Multiprogramming in a Hard Real Time Environment*. Journal of the Association of Computing Machinery. January 1973. pp. 46-61.
- John Lehoczky, Lui Sha, and Ye Ding. *The Rate Monotonic Scheduling Algorithm: Exact Characterization and Average Case Behavior*. IEEE Real-Time Systems Symposium, 1989. pp.166-171.
- S.K. Baruah, L.E. Rosier, and R.R. Howell. *Algorithms and complexity concerning the preemptive scheduling of periodic, real-time tasks on one processor*. Journal of Real-Time Systems, **2**, 1990.

Resource Access

- Semaphores can be used to control access to resources in a real-time system.
- Some interesting issues associated with priority arise.

Priority Inversion

- Consider two tasks, one high priority, one low, that share a resource protected by a critical section.
- If the high-priority task becomes schedulable during the time the low-priority one is in its critical section:
  The high-priority task is **blocked** by the low priority one.

Push-Through Blocking

- The low-priority task, with the resource locked, could be preempted by a medium priority task that doesn't necessarily use the resource.
- This task could go on indefinitely, in-effect blocking the higher-priority task through the lower-priority one.

Possible Resolutions of Priority Inversion

- Abort the low-priority task:
  - Messy, since this could leave the system in an inconsistent state.
- Priority Inheritance Protocol
- Priority Ceiling Protocol

Priority Inheritance Protocol (PIP)

- During the time the low-priority task is in its critical section, **and** while the high-priority task is blocked because of this, the low-priority task **inherits** the priority of the high-priority task.

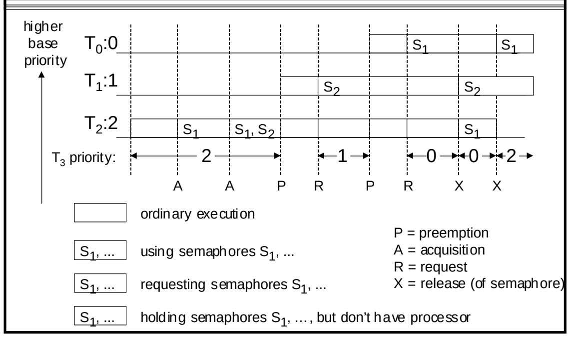
Possible Resolution of Blocking

- **Priority Inheritance Protocol**:
  - More precisely: A task locking a shared resource inherits the priority of the highest-priority task blocked because of locking.
  - The locking task's priority is *recomputed* whenever:
    - Other tasks request or release the shared resource, or
    - the locking task leaves the critical section, in which case the highest-priority blocked task is awakened.

Transitivity

- **Priority Inheritance must be made Transitive**:
  - If T_1 blocks T_2, and T_2 blocks T_3, then T_1 inherits the priority of T_3.
- Note that transitive inheritance occurs only with nested critical sections (different semaphores). If there were only *one* semaphore, then T_1 would be blocking T_3 directly.

PIP Example



Implementation Note

- Implementing priority inheritance in semaphores requires added sophistication beyond the basic semaphore mechanism.

Computing Blocking Time

- If the computation time (without block) within critical sections is known, then the blocking time due to priority inheritance can be computed.
- This can be used in determining schedulability of a system of tasks with shared resources.

Example: Solaris Operating System

- Solaris kernel schedules based on LWP's (Light-Weight Processes).
- Regard these as schedulable units of processor time.
- A new LWP can be created as an optional aspect of creating a new thread.

Example: Solaris Operating System

- Each LWP has a priority, in one of three coarse classes:
  - RT (real-time) is highest.
  - System class is middle (not used by user processes).
  - TS (time-share) is lowest.

Example: Solaris Operating System

- For the time-sharing class, the dispatch priority is calculated from
  - amount of CPU used since last I/O (less $\Rightarrow$ higher-priority)
  - its *nice* level (set by the user)
- For the real-time class,
  - the highest-priority LWP runs until it blocks, terminates, reaches end of time-slice, or is preempted.

Example: Solaris Operating System

- When a process is created, its LWP gets the scheduling class and priority of the parent process.
- A **thread** can be either:
  - bound to a specific LWP
  - unbound (multiplexed among various LWP's)
- All unbound threads in a process have the same class and priority.
- Bound threads have the class and priority of the LWP to which they are bound.

Example: Solaris Operating System Priority Inheritance

- Solaris implements basic priority inheritance protocol:
  - When a higher-priority thread is blocked, its priority is given to the lower-priority thread blocking it.
  - When the lower-priority thread ceases to block a higher priority one, its priority is set back to the original priority.
- Source: Ben Catanzaro, *Multiprocessor System Architectures*, Sun Microsystems, 1994.

Priority Ceiling Protocol (PCP)

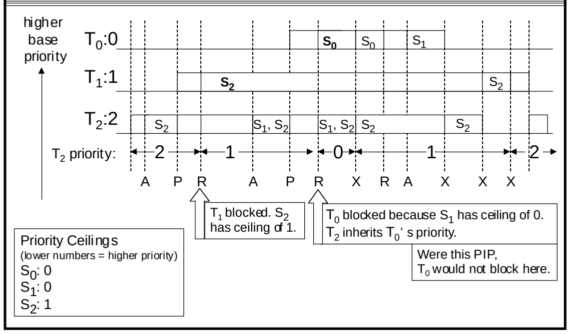
- A more involved form of priority inheritance.
- The PIP does not prevent a high priority task from being blocked multiple times by lower priority ones.
- The PCP does: Once a task gains entry to a critical section, it cannot be blocked by a lower priority task.

Priority Ceiling Protocol (PCP)

- Each semaphore has a (static) priority **ceiling** equal to the priority of the highest-priority task that could possibly lock it.
- A task is allowed to enter a critical section only if its priority is *higher* than the priority ceilings of all semaphores *currently* locked by other tasks.
- If a task is blocked on a semaphore, the priority of the blocked task is inherited by the task locking that semaphore.

PCP Example

(from Buttazzo, Hard Real-Time Computing Systems, Kluwer, 1997)



Comparison: PIP vs. PCP

- PCP is more efficient at run-time, in that a high priority task cannot be blocked as many times.
- PIP is less demanding, in that it does not require a thorough analysis of a task's behavior (in terms of which semaphores in might request).

References

- L. Sha, R. Rajkumar, J. Lehoczky, *Priority Inheritance Protocols*, IEEE Trans. Computers, **20**, 9, pp. 1175-1185, Sep. 1990.