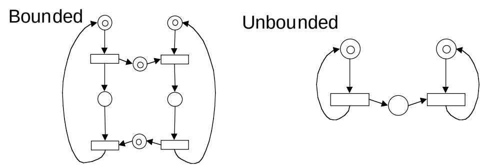
Boundedness of Petri Nets

- Def: A Petri Net is ***bounded*** wrt an initial state if the set of states reachable from the initial state is finite.

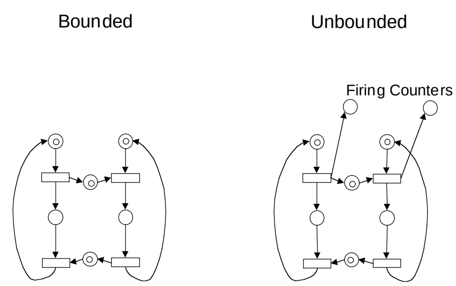


Boundedness

- In general, boundedness is a “good thing”.
- It is essential if the system is to be realized with finite memory.
- It allows the system to be analyzed as a finite-state machine.
- However, a type of unboundedness can be useful for mathematical *analysis*, in the form of ***firing counters***.

Firing Counters

should be unbounded as a *necessary* condition to be free of deadlock



Boundedness of Individual Places

- Def: A set of *places* (circular nodes) in a Petri net is **simultaneously unbounded** w.r.t. an initial state if ($\forall n \in N$), there is a reachable state in which each place has $\geq$ n tokens.

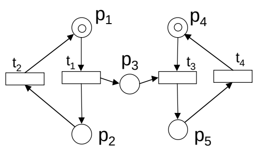
Theorem (Karp & Miller, 1966)

- There is an algorithm for deciding whether any given place in a Petri net is unbounded.

Reachability-Tree Algorithm

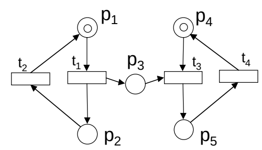
- Construct a tree with the initial state as root.
- Construct successive nodes for each firable transition, as if constructing a state diagram.
- Whenever a node is added that has a predecessor which is pointwise $\leq$ this node, set to ∞ any place that is < in the predecessor. If the result is a repeat, that branch ends.
- This process will terminate. Sets of places with ∞ are simultaneously unbounded.

Reachability-Tree Algorithm

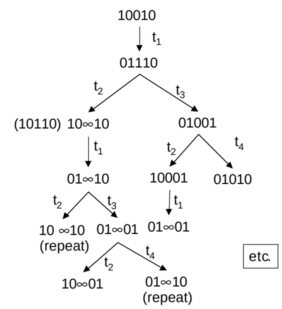


Initial state
10010

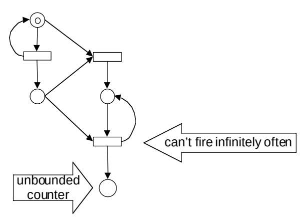
Reachability-Tree Algorithm



Initial state
10010



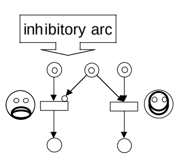
Unboundedness of a Counter for a Transition is a Necessary, but not Sufficient, Condition for the Transition to have an Infinite Firing Sequence



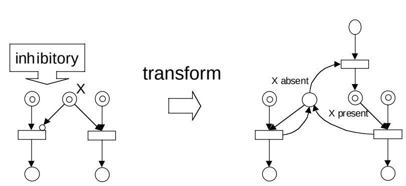
Generalizing Petri Nets

- Adding inhibitory arcs:
  - For finite-state systems: ok, can simulate without inhibitory arcs anyway.
  - For unbounded systems: can destroy essential decidability properties (now can simulate a Turing machine)
- “Colored” tokens (see Kurt Jensen, 3 vols., Springer, 1997)
- Program variables
- Enabling predicates on transitions

Inhibition

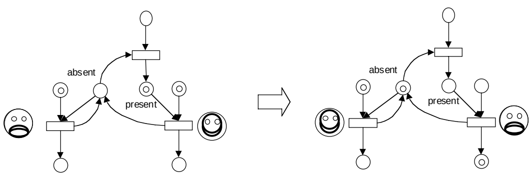


Simulating Inhibitory Arcs



This transformation only works if the place X in question is bounded by 1.

Simulating Inhibitory Arcs



Adding *Time* to Petri Nets

- Variation 1: Transitions have a delay time; firing takes a non-zero time from enabling. Time may be bounded from above or below.
- Variation 2: Places have a delay time: A token must dwell on a place a certain amount of time (determined by the place) before becoming usable in firing.
- Variation 3: Like 2, but tokens have a delay time.

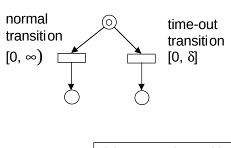
Variation 1 is Prevalent

- Ramchandani, MIT PhD Thesis, 1974.
- Richard Merlin, UCI Thesis, 1974.
- Barthomieu & Diaz, IEEE TSE, **3**, 1991.

Barthomieu & Diaz

- A Time Petri Net is like a Petri Net with a time interval on each transition:
  $[t_1, t_2]$ or $[t_1, \infty)$
- From the time the transition is enabled, it cannot fire before t_1 and *must* fire by t_2 (unless disabled by firing another transition).

Example: Representing Time-out

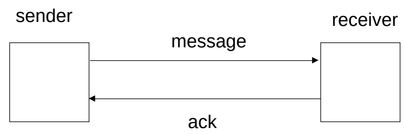


If the normal transition has not fired by time δ after being enabled, then the time-out transition will fire and the normal transition will be disabled.

Barthomieu & Diaz

- Showed that boundedness is decidable for Time Petri Nets with rational time bounds.

Example: Sending messages between two sites.



Problem

- A message sent by a sender could get lost or be garbled.
- Thus the receiver must ack each message.
- If the ack is not received in a specified time, the transmission is regarded as having timed out and the sender must resend.
- The ack could also get lost or be garbled.

Problem, continued

- The sender might decide to resend although the original message has just been delayed, not lost.
- How can the receiver tell whether an incoming message is new or just a retransmission of an earlier message?

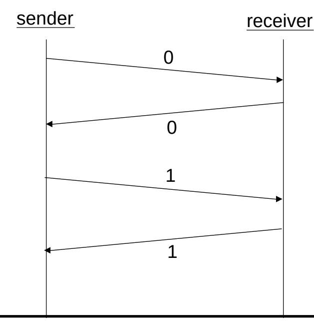
A Solution

- Each message is uniquely timestamped by a sequence number.
- The receiver only accepts and acknowledges the next number in sequence, not the replay of an earlier number.
- The acknowledgment indicates the number of the message being acknowledged.

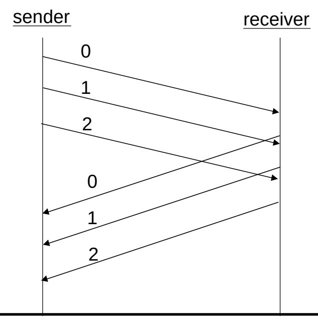
Problem & Fix

- The set of timestamps is not bounded.
- Alternating bit protocol (ABP):
  - Use only 0 and 1 as numbers.
  - Sender sends 1 only when 0 has been successfully acknowledged.
  - Sender sends 0 only when 1 has been successfully acknowledged.

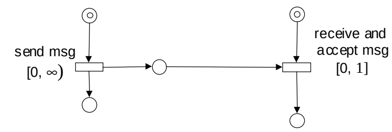
Drawback: Only one message can be in transit at a time.



More general possibilities (not ABP)

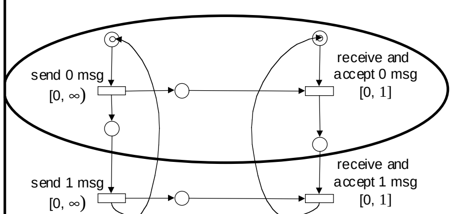


Example: Representing ABP in TPN: Basic

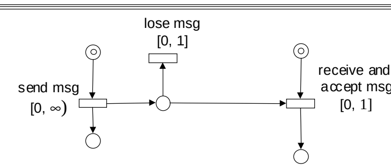


If the message is sent, it will be received by time 1.

We will develop only the 0 half of the protocol. The 1 half fits symmetrically.

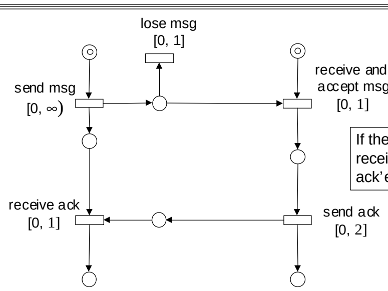


ABP: Add possible loss



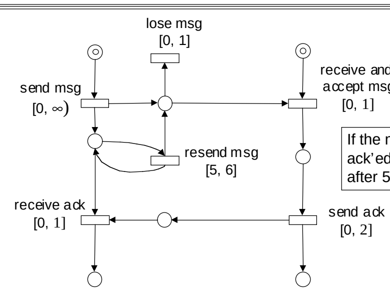
If the message is sent, it will be received or lost by time 1.

ABP (continued)



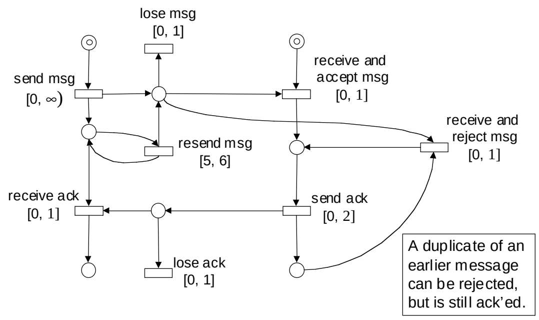
If the message is received, it will be ack'ed within 2.

ABP (continued)



If the message is not ack'ed, it will be resent after 5 and by 6.

ABP (continued)



Summary

- A message can be sent at an arbitrary time.
- Once sent, the message can be received or lost, within a bounded time.
- If the message is received, an ack is sent.
- The ack can be received or lost.
- If an ack is not received in a bounded time, the message is resent.
- If a resent message is received, it is ack'ed.

Homework, Part 2

- Construct a Time Petri Net model for the home alarm system described earlier.

Temporal Constraint Networks

Temporal Constraint Networks

- TCN's are a graphical model used to reason about temporal systems.
- They have also been used as a form of control specification for real-time systems.

Temporal Constraint Networks: Basic Idea

- Nodes are "timepoints"; they represent points in time.
  - Timepoints are not necessarily bound to a specific time; they may be "floating".
  - A timepoint becomes "grounded" when it is associated with a specific time.
- Arcs represent temporal constraints between timepoints.
  - Each arc is labeled with a minimum and maximum time between the timepoints it connects.

Temporal Constraints between Timepoints

$$a \xrightarrow{\text{u to v}} b$$

means that between the time at which a is grounded and the time at which b is grounded, there is at least u units of time and at most v units of time:

$$t(a) + u \leq t(b) \leq t(a) + v$$

u or v can be negative: u units before is the same as -u units after.

$$a \xleftarrow{\text{-v to -u}} b$$

Other Ways of Writing

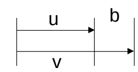
$$a \xrightarrow{\text{u to v}} b$$

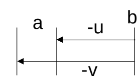
$$t(a) + u \leq t(b) \leq t(a) + v$$

$$u \leq t(b) - t(a) \leq v$$

$$-u \geq t(a) - t(b) \geq -v$$

$$-v \leq t(a) - t(b) \leq -u$$





$$a \xleftarrow{\text{-v to -u}} b$$

Temporal Constraint Example

$$\text{launch} \xrightarrow{\text{5 to 20}} \text{orbit}$$

means that orbit can occur no sooner than 5 time units, nor no later than 20 time units, after launch.

Absolute Time

$$\text{reference} \xrightarrow{\text{5 to 10 seconds}} \text{event}$$

supposing that reference is a fixed reference time, such as Jan. 1, 2000, this says that event can occur not before 5 seconds and not later than 10 seconds into the year.

$$\text{reference} \xrightarrow{\text{10 to 10 seconds}} \text{event}$$

says event must occur exactly 10 seconds into the year.

Reasoning with TCN's

$$a \xrightarrow{\text{u to v}} b \xrightarrow{\text{w to x}} c$$

$\Downarrow$ implies

$$a \xrightarrow{\text{(u+w) to (v+x)}} c$$

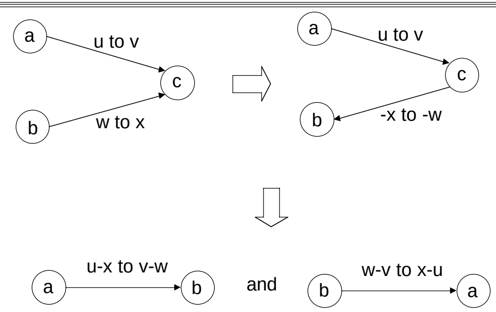
Reasoning with TCN's

$$a \xrightarrow{\text{u to v}} c \qquad b \xrightarrow{\text{w to x}} c$$

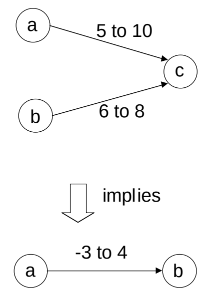
$\Downarrow$ implies

$$a \xrightarrow{\text{??}} b$$

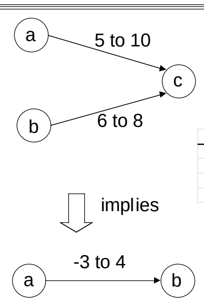
Reasoning with TCN’s



Example



Check Extreme Cases



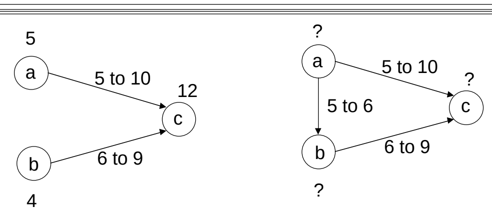
a	c	b	b-a
0	5	-3	-3
0	5	-1	-1
0	10	2	2
0	10	4	4

For b: -3 = 5 -8
-1 = 5 - 6
2 = 10 - 8
4 = 10 - 6

Consistency of TCN’s

- Def: A temporal constraint network is **consistent** if it is possible to assign times to each of the timepoints such that the all constraints are simultaneously satisfied.

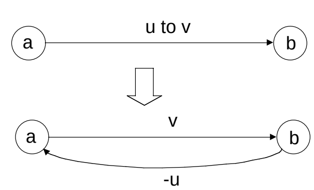
Example of Consistent vs. Inconsistent



Consistency Checking Algorithm

Dechter, R., Meiri, I., & Pearl, J. (1991).
Temporal constraint networks. Artificial Intelligence, 49, 61-95.

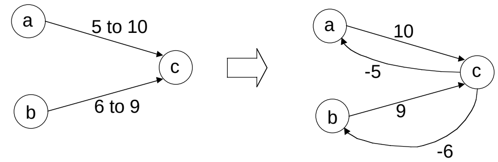
- Convert the TCN to a labeled directed graph with just one distance on each arc, using the following transformation:



Consistency Checking Algorithm

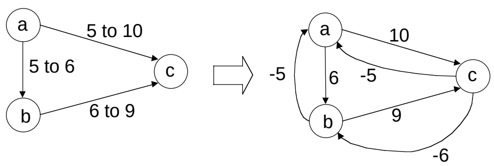
- Test the resulting graph to see if any negative-sum cycles are present (e.g. using Floyd's algorithm).
- The original TCN is consistent iff there is no negative-sum cycle in the transformed graph.

Example



Consistent

Example

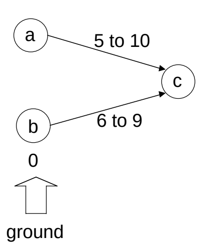


Inconsistent

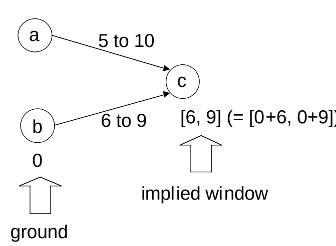
Constraint Propagation

- Suppose a node in a consistent TCN is grounded (is assigned a fixed time).
- Then for each other node in the TCN, a window for possible groundings can be determined from the temporal constraints.

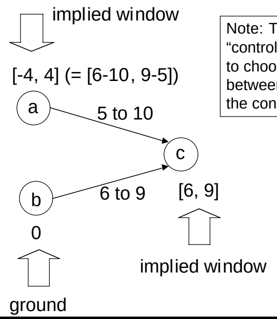
Constraint Propagation Example



Constraint Propagation Example



Constraint Propagation Example

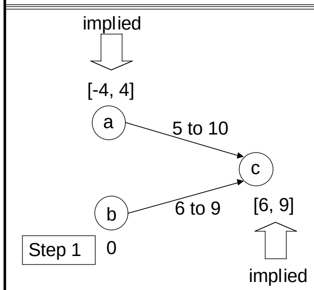


Note: The window idea assumes "controllable" constraints: *We* get to choose the actual distance between timepoints consistent with the constraint.

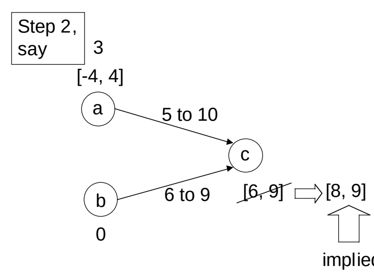
Conjecture

- If a TCN is consistent, then grounding any node within its window, by adding a constraint between a reference timepoint and the node, results in a consistent net.
- The effect of grounding is to narrow other windows.

Sequential Grounding Example



Sequential Grounding Example



So a possible grounding sequence is: b(0), a(3), c(9).

Possible use of TCN's for RT Control

- Windows are maintained with earliest and latest groundable times for all nodes.
- Some reference node fires at "time 0".
- while(unfired nodes exist)
  {
  choose ungrounded node with earliest window;
  ground that node;
  propagate constraints, updating windows;
  }

Difficulties in using TCN's for RT Control

- The constraints have to be controllable in the sense mentioned earlier.
- The check for consistency can be computationally expensive.