CS140: Algorithms

Fall 2001

Assignment 0

This assignment is due at the beginning of class on Mon., Sept. 10. You may collaborate with your classmates to complete this assignment but you must cite your collaborators. In addition, you must do your own write-up. This assignment is worth 20 points (0 points if handwritten).

1. For each of the following functions $f(n)$ and $g(n)$ state the ranges of positive integers for which

   (i) $f(n) < g(n)$

   (ii) $f(n) = g(n)$

   (iii) $f(n) > g(n)$

   For example, if $f(n) = 2n$ and $g(n) = 2^n$ then (i) hold in the range $[3, \infty)$, (ii) holds in the range $[1, 2]$, and (iii) fails to hold for any positive integers.

   (a) $f(n) = 10n$ and $g(n) = 3^n$

   (b) $f(n) = n/10$ and $g(n) = 10 \lg n$ (i.e. $\log_2$)

   (c) $f(n) = sin(2\pi n)$ and $g(n) = cos(2\pi n)$

   (d) In this case the functions are defined recursively as follows:

   $n = 1$**:** $f(n) = 1$, $g(n) = 1$

   $n > 1$ **and even:** $f(n) = f(n-1)$, $g(n) = 2^{f(n-1)}$

   $n > 1$ **and odd:** $f(n) = 2^{g(n-1)}$, $g(n) = g(n-1)$

   (e) $f(n) = \log_3 n^2$, $g(n) = 2 \lg n$

2. Identify the following statements as true or false. You may assume that a, w, k and n are positive real numbers.

   (a) $w = a^{\log_a w}$

   (b) $\log_a^k n = k \log_a n$

   (c) $\log_a n^k = k \log_a n$

   (d) $\log_a n = \log_a b \, * \, \log_b n$

3. Give an example of a function $f : \mathcal{N} \rightarrow \mathcal{N}$ such that $lim_{n \rightarrow \infty} f(n)$ exists.

4. Give an example of a function $f : \mathcal{N} \rightarrow \mathcal{N}$ such that $lim_{n \rightarrow \infty} f(n)$ does not exist.