CS140: Algorithms

Fall 2001

Assignment 8

This assignment is due by the start of class on Monday 10/29/01. You may collaborate with your classmates to complete this assignment but you must reference your collaborators. In addition, you must do your own write-up. This assignment is worth 100 points (65 points if handwritten).

1. Give an algorithm for reducing the key of a record in a heap that runs in time $O(h)$, where h is the height of the heap.
2. A Bottleneck Spanning Tree (BST) of a connected graph G is a spanning tree whose maximum weight edge is a small as possible. In other words, the cost of a spanning tree for this problem is the weight of its maximum weight edge. The problem is to find a spanning tree with minimum cost. (Assume G has n vertices and m edges.)
   (a) Suppose T is an MST G. Show that T is also a BST of G.
   (b) Give a weighted, connected graph G and a BST, T, of G such that T is not an MST of G.
   (c) By part (a) we can clearly reduce BST to MST. But by part (b) we might guess that there is a more efficient algorithm for finding a BST of G.
      i. Suppose I tell you that the cost of a BST of G is W. Give an $O(m)$ algorithm that can then find a BST of G.
      ii. Suppose I simply want to know if the weight of a BST of G is less than or equal to W. Give an $O(m)$ algorithm that answers this question.
      iii. Now design an $O(m)$ algorithm that finds a BST of G. Hint 1: Use divide and conquer. Hint 2: You can check if a graph is connected in time $O(n + m)$ using Depth First Search.