CS140: Algorithms

Fall 2001

Assignment 10

This assignment is due by the end of the day Wed., Nov. 21, 2001. You may collaborate with your classmates to complete this assignment but you must reference your collaborators. In addition, you must do your own write-up. This assignment is worth 200 points (100 points if handwritten).

1. In the study of NP-completeness we focus on "decision problems," which differ from standard search problems in that we are only required to answer a yes/no question about the input. In practice, of course, we usually want to solve the search variations of these decision problems. In this exercise you'll show that the search variations of several decision problems are really no harder than the decision version. You'll do this by reducing the search problem to the decision problem. Your reductions should be polynomial-time.

   Dominating Set: A dominating set of a graph G is a subset of vertices W such that every vertex of G is either in W or is adjacent to a vertex in W.

   Input: A graph G and integer k

   Question: Does G have a dominating set of size at most $k/$

   Input: A graph G

   Search Output: A smallest dominating set of G

   Partition: AKA the grocery bag problem. You might ask yourself whether or not our algorithm for the grocery bag problem means we've proven $P = NP$?

   Input: A set of integers S

   Question: Can S be partitioned into two sets, S_1 and S_2, such that $\sum_{s \in S_1} s = \sum_{s \in S_2} s$?

   Input: A set of integers S

   Search Output: A partition of S into two sets, S_1 and S_2, such that $|\sum_{s \in S_1} s - \sum_{s \in S_2} s|$ is minimized.

   Clique: A clique in a graph G is a subset of vertices W such that each pair of vertices in W is connected by an edge in G.

   Input: A graph G and an integer k

   Question: Does G have a clique of size at least k?

   Input: A graph G

   Search Output: A largest clique of G

   Hamiltonian Cycle: A Hamiltonian cycle of a graph G is a simple cycle that includes every vertex.

Input: A graph G
Question: Does G contain a Hamiltonian cycle?

Input: A graph G
Search Output: A Hamiltonian cycle of G if one exists and otherwise "No Hamiltonian cycle"

3-coloring: A 3-coloring of graph G is an assignment of a unique color in {red, blue, green } to each vertex of G so that the endpoints of each edge are colored differently.

Input: A graph G
Question: Can G be 3-colored?

Input: A graph G
Search Output: A 3-coloring of G if one exists and otherwise "No 3-coloring"

3 Dimensional Matching: Let U, V, and W be disjoint sets such that $||U|| = ||V|| = ||W|| = q$. Let C be a subset of $U \times V \times W$; in other words C is a collection of triples (u, v, w), where $u \in U$, $v \in V$, $w \in W$. A 3 dimensional matching is a subset of C such that each element of U, V and W occurs in exactly one triple of C.

Input: Disjoint sets U, V, and W, where $||U|| = ||V|| = ||W|| = q$ and a subset C of $U \times V \times W$.
Question: Does C contain a three-dimensional matching?

Input: Disjoint sets U, V, and W, where $||U|| = ||V|| = ||W|| = q$ and a subset C of $U \times V \times W$.
Search Output: A three-dimensional matching from C, if one exists, and otherwise a statement that one does not exit.

For each decision problem above do the following:

(a) Give a yes-instance.
(b) Give a no-instance.
(c) Show the problem is in NP.

2. The Largest Common Subgraph problem is defined as follows.

Input: Graphs G_1 and G_2 and integer k,

Question: Is there a graph G containing at least k edges that is a subgraph of both G_1 and G_2.

(a) Give a Yes-instance of LCS.
(b) Give a No-instance of LCS.
(c) Cast this problem as a search problem. Show that the search problem is polynomial-time equivalent to the decision problem.
(d) Show that LCS is in NP.

(e) Show that LCS is NP-complete by giving a reduction from one of the NP-complete problems described in problem 1.

3. Repeat the above exercise for the Minimum Sum of Squares problem defined below.

   Input: A set A of integers and two additional integers K and J.

   Question: Can the elements of A be partitioned into K disjoint sets $A_1, A_2, \ldots, A_K$ such that $\sum_{i=1}^{K}(\sum_{s \in A_i} s)^2 \leq J$?

4. Repeat the previous exercise for the Steiner Tree in Graphs problem defined below. (Hint: Reduce from Vertex Cover.)

   Input: A graph G, a subset R of the vertices of G, and an integer K.

   Question: Is there a subtree (not necessarily a spanning tree) of G that includes the vertices R and at most K edges?