CS140: Algorithms

Fall 2001

Assignment 1

This assignment is due at the start of class on Wed., Sept. 12. You may collaborate in small groups (< 5) to complete this assignment but you must reference your collaborators. You may not consult solutions to old assignments or exams for CS140 to complete this assignment. You must do your own write-up. This assignment is worth 50 points (35 points if handwritten).

1. For every positive integer n, algorithm A takes $3n^2$ steps on some input of size n. State whether or not this statement implies the following.

   (a) Algorithm A is $O(n^3)$.
   Ans. No

   (b) Algorithm A is $\theta(n^2)$.
   Ans. No

   (c) Algorithm A is $\Omega(n^2)$.
   Ans. Yes

2. Fill in the table with Y/N to the question $f(n) = \mathcal{X}(g(n))$ where $\mathcal{X} = O, \Omega, \Theta, o, \omega$. Assume that ϵ, k and c are constants satisfying the following: $\epsilon > 0$, $k > 1$, and $c > 1$.

$f(n)$	$g(n)$	O	Ω	Θ	o	ω
n^ϵ	$\log^k n$	N	Y	N	N	Y
n^k	c^n	Y	N	N	Y	N
$n^{\lg c}$	$c^{\lg n}$	Y	Y	Y	N	N
n^k	n^{2k}	Y	N	N	Y	N
n^k	$(n/c)^k$	Y	Y	Y	N	N
c^n	c^{n+k}	Y	Y	Y	N	N

3. Which of the functions in the previous table are polynomially bounded?
   Ans. n^ϵ, $\log^k n$, n^k, $n^{\lg c}$, $c^{\lg n}$, n^k, n^{2k}, $(n/c)^k$.
   For which does $f(2n) = O(f(n))$?
   Ans. Same as in previous part.

4. Compute the running time of the following algorithm.

```
algorithm what(n)
        for i = 1 to ⌊√n⌋ do
           for j = ⌊√n⌋ downto 1 do
              for k = 1 to j do
                 { O(1) time sequence of statements }
              end
           end
        end
end algorithm
```

   Ans. $O(n^{3/2})$