CS140: Algorithms

Fall 2001

Assignment 3

This assignment is due at the start of class on Wednesday, Sept. 19. You may collaborate in small groups with your classmates to complete this assignment but you should reference your collaborators. You also must thoroughly understand any solutions you submit and must do your own write-up of those solutions. You may not refer to solutions to assignments given in previous semesters of CS140. This assignment is worth 50 points (25 points if handwritten).

1. Prove that the expected running time of Quicksort is $O(n \lg n)$. Note: we did much of the proof in lecture but you should reiterate those steps with your own explanation.

   Recall there was an error in my lecture notes where I dropped a factor of 2. As a result, one cannot complete the proof correctly. The result is certainly true but one needs a slightly tighter analysis to prove it. Refer to the text for a proof of the result. Everyone will receive credit for this problem. Those that recognized the problem and actually submitted correct proofs will receive bonus points.

2. Here is an inductive "proof" that $n^2 = O(n)$.

   Clearly $1^2 = O(1)$. Now suppose that $(n-1)^2 = O(n-1)$. Then

   $$n^2 = (n-1)^2 + 2n - 1 = O(n-1) + 2n - 1 = O(n).$$

   What is wrong with this "proof." Hint: try to replicate the proof using the defintion of $O(n)$ explicitly.

   Using the definition of big-O, our proof shows that for every n there is a constant c such that $n^2 \leq cn$. This in fact is true; i.e. let $c = n$. It is not, however, what we want to prove.

   Technically the problem is with the base case. We want to claim that for $n = 1$, statement X holds for all n. But all we really can claim is that for $n = 1$ statement X holds.