CS140: Algorithms

Fall 2001

Solutions to Assignment 7

1. See lecture notes for examples of this type of problem.

2. Problem 27-1 on page 625 of the text.

   To impose vertex flow constraints we can replace a single node v with two nodes v-in and v-out. The in-edges of v become in-edges to v-in and the out-edges of v become out-edges of v-out. In addition we add an edge from v-in to v-out with capacity equal to the vertex flow capacity of v.

   To solve the escape problem we design a flow network with both edge and vertex flow constraints. We replace the edges of the grid with bi-directional edges, each with capacity 1. We add a source and an edge with capacity 1 to every starting point in the grid. We add a sink and an edge with capacity 1 from every point on the boundary of the grid. In addition we require that each vertex have capacity 1.

   The maximum flow of the network is m iff there are disjoint paths from the starting vertices to boundary points.

3. Problem 27-2 on page 626 of the text.

   Note that minimizing the number of paths in a path cover is equivalent to maximizing the number of edges in the path cover. The maximum flow in the network described in the text is exactly the maximum number of edges that can exist in any path cover.

4. Problem 31.2-6 of the text.

   Compute the products ac, bd, and $(a + b)(c + d)$. The real part is $ac - bd$ and the imaginary part is $(a + b)(c + d) - ac - bd$.