CS140: Algorithms

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Lecture 3

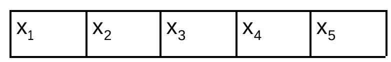
Data Structures

- Elementary data structures
- Heaps
- Binary Search Trees
- Treaps

Elementary data structures

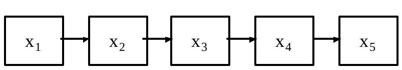
- Arrays and linked lists
- Stacks and queues
- Graphs
- Rooted trees

Arrays



- Read i^{th} cell • O(___)
- Write i^{th} cell • O(___)
- Insert i^{th} cell • O(___)
- Delete i^{th} cell • O(___)

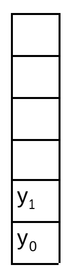
Linked List



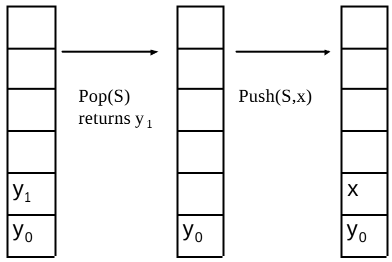
- Read i^{th} cell • O(___)
- Write i^{th} cell • O(___)
- Insert i^{th} cell • O(___)
- Delete i^{th} cell • O(___)

Stack

- Push(x,S)
- x=Pop(S)



Stack

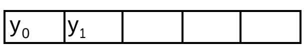


Stack

Implement with linked lists or arrays to get O(__) per operation:

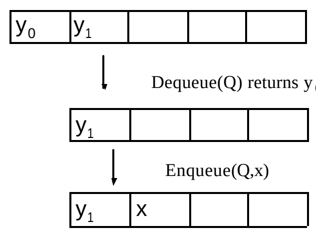
Push(x,S)
x=Pop(S)

Queue



- Enqueue(x,Q)
- x=Dequeue(Q)

Queue



Queue

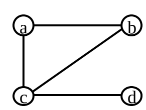
Implement with linked list or (circular) array to get O(__) time per operation:

- Enqueue(x,Q)
- x=Dequeue(Q)

Graph

V={a,b,c,d}

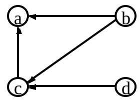
E={(a,b),(a,c),(b,c),(c,d)}



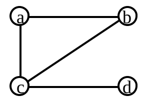
Directed Graph

V={a,b,c,d}

E={<b,a>,<b,c>,<b,c>,<d,c>}



Graph – adjacency list



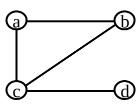
a: b,c

b: a,c

c: a,b,d

d: c

Graph – adjacency matrix



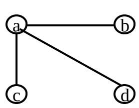
	a	b	c	d
a	0	1	1	0
b	1	0	1	0
c	1	1	0	1
d	0	0	1	0

Graphs
(n vertices, m edges)

- Is (u,v) an edge of G?
  - Adjacency list: O(___)
  - Adjacency matrix: O(___)
- What are the neighbors of v in G?
  - Adjacency list: O(___)
  - Adjacency matrix: O(___)

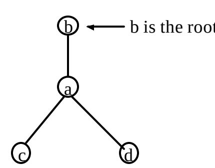
Trees

- A tree is a connected, acyclic graph.



Rooted Trees

- A rooted tree is a connected, acyclic graph with one vertex designated as the root.



Rooted Trees
Implement with pointers

- What is the root of T? O(_____)
- What is the parent of v? O(____)
- What are the children of v? O(___)

Heap

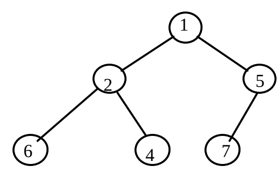
- Data structure for a set of integers to facilitate
  ____

Heaps

A heap is a data-structure for storing integer that supports:

1. Build-heap(S): Return a heap on the integers in the set S.
2. Insert(x,H): Insert the integer x into the heap H.
3. Find-min(H): Return the smallest integer in the heap H.
4. Extract-min(H): Remove the smallest integer from the heap H and return it.

Heap: {7,1,5,4,2,6}



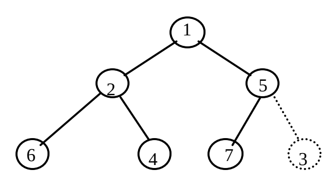
1. Rooted, binary tree, filled level by level from the left.
2. (Min) Heap property: the integer stored at a node is no larger than those of its descendents.

Heap

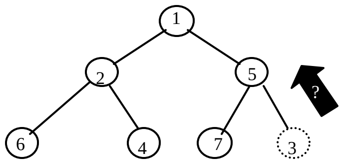
A heap is a data-structure for storing integer that supports:

1. Build-heap(S): Return a heap on the integers in the set S.
2. Insert(x,H): Insert the integer x into the heap H.
3. O(1) Find-min(H): Return the smallest integer in the heap H.
4. Extract-min(H): Remove the smallest integer from the heap H and return it.

Insert(3,H) – Step 1 (add)



Insert(3,H) – Step 2 (bubble up)



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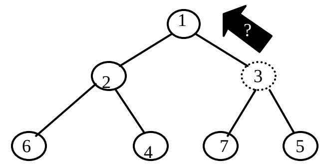
Insert(3,H) – Step 2 (bubble up)



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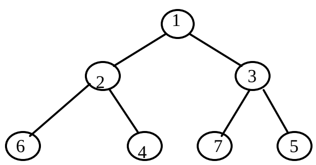
Insert(3,H) – return



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Heap

A heap is a data-structure for storing integer that supports:

1. Build-heap(S): Return a heap on the integers in the set S.
2. Insert(x,H): Insert the integer x into the heap H. O(lg n)
3. Find-min(H): Return the smallest integer in the heap H. O(1)
4. Extract-min(H): Remove the smallest integer from the heap H and return it.

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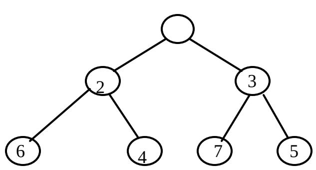
Extract-min(H) – Step 1 (remove)



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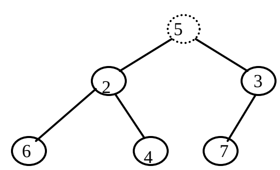
Extract-min(H) – Step 2 (move last to root)



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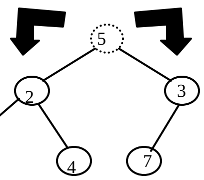
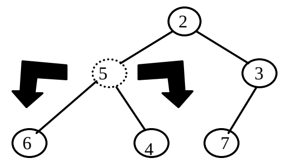
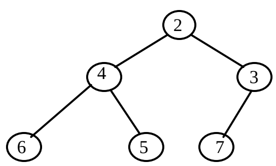
Extract-min(H) – Step 3 (bubble down)



Extract-min(H) – Step 3 (bubble down)



Extract-min(H) – return



Heap

A heap is a data-structure for storing integer that supports:

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2. O(lg n) Insert(x,H): Insert the integer x into the heap H.
3. O(1) Find-min(H): Return the smallest integer in the heap H.
4. O(lg n) Extract-min(H): Remove the smallest integer from the heap H and return it.

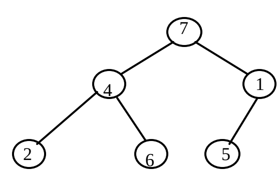
Heap

A heap is a data-structure for storing integer that supports:

1. O(n) Build-heap(S): Return a heap on the integers in the set S.
2. O(lg n) Insert(x,H): Insert the integer x into the heap H.
3. O(1) Find-min(H): Return the smallest integer in the heap H.
4. O(lg n) Extract-min(H): Remove the smallest integer from the heap H and return it.

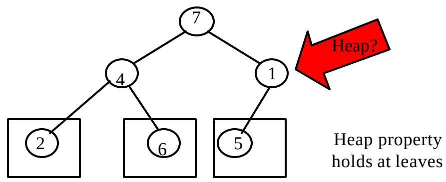
Build-heap{7,1,5,4,2,6}

Build rooted tree

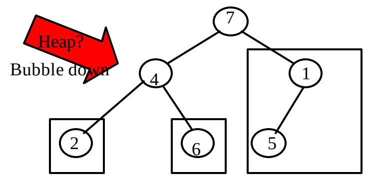


Build-heap{7,1,5,4,2,6}

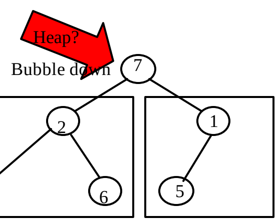
Fix subtrees



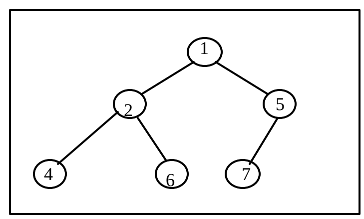
Build-heap{7,1,5,4,2,6}



Build-heap{7,1,5,4,2,6}



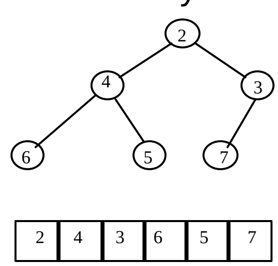
Build-heap{7,1,5,4,2,6}



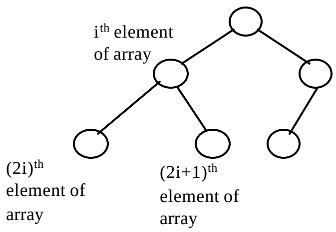
Running Time

- The leaves are at height 0. Consider the nodes at height i.
  - How long does it take to fix a subtree rooted at height i assuming its children root heaps?
  - How many nodes are at height i?
  - What is the running time of Build-heap?

Implementing a heap in an array



Array Indexing



Heap-sort(S)

H=Build-heap(S) → O(n)
For i=1 to n
S(i) = Extract-min(H) → O(lg n)
Return

Heap-sort is O(n lg n)

Dictionary Data Structure

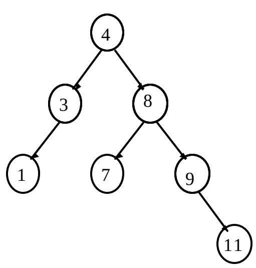
Data structure that supports add, delete, find for set of keyed records.

- Binary search tree
  - Balanced binary search tree
- General search tree
- Hash Table

Binary Search Tree for S

- T is a rooted, binary tree
- Each node in T is assigned a record in S (one-to-one)
- BST Property: For any node X in T
  - If node Y is in the left subtree of X then Y.key ≤ X.key
  - If node Y in the right subtree of X thenY.key ≥ X.key

BST

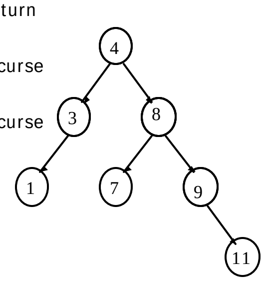


BST – Find (x)

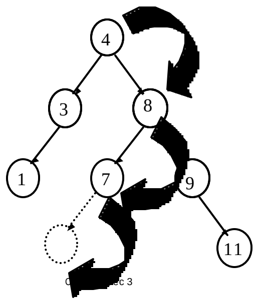
If root.key=x return root

If x<root.key recurse on left subtree

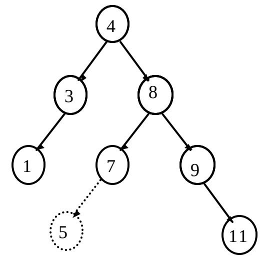
If x>root.key recurse on right subtree



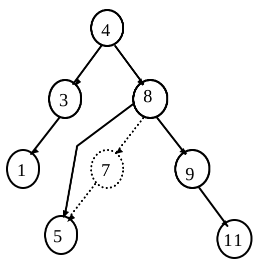
Insert (5)



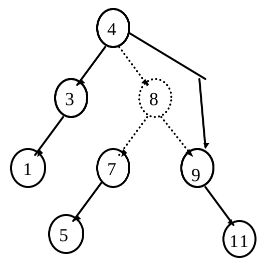
Delete(5)
(Leaf is easy)



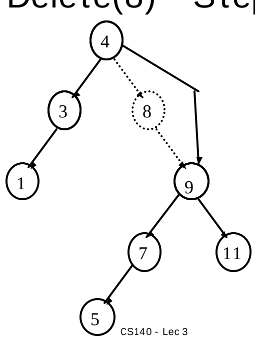
Delete(7)
(Node with 1 child is easy)



Delete(8) - Step 1



Delete(8) – Step 2



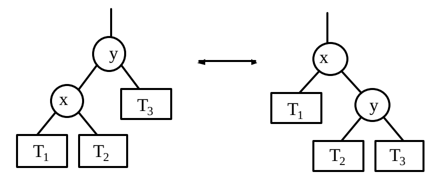
Operation Run Time

- Search(x) – O(h)
- Insert(x) – O(h)
- Delete(x) – O(h)

Keeping a good balance …

- Search trees: O(h) time per operation
- “Balanced trees” insure O(log n) time per operation.
  - How to balance
  - When and where to balance

How: Rotations



Keeping a good balance …

- Search trees: O(h) time per operation
- “Balanced trees” insure O(log n) time per operation.
  - How to balance
  - When and where to balance

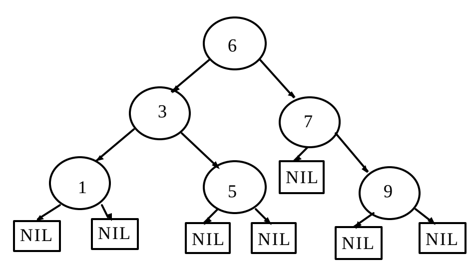
When to balance?

- Red/black trees
- 2-3 trees
- AVL trees
- Treaps ← Makes the when & where question easy to answer!

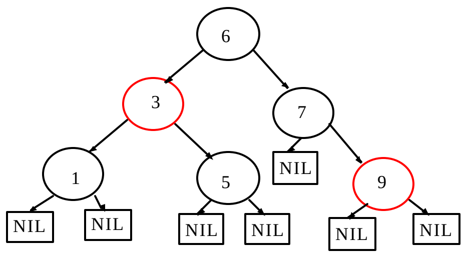
Red-Black Trees

- Binary Search Tree that satisfies the following
  - Each node is colored either red or black
  - Every leaf is NIL and black
  - If a node is red each of its children are black
  - For a node x, every simple path from x to a leaf has the same number of black nodes

Color this to be a Red-Black Tree



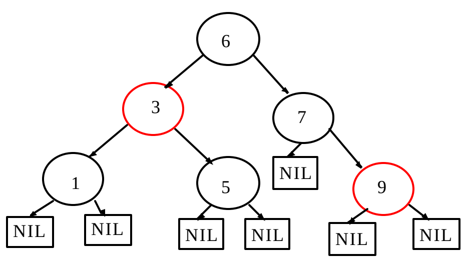
Red-Black Tree



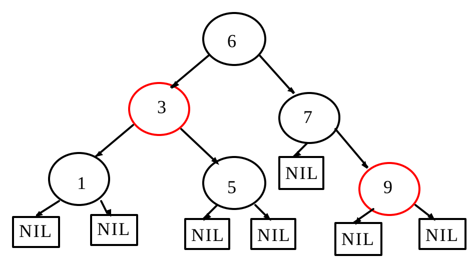
Claim

A Red-Black tree with n internal nodes has height at most 2 lg (n+1).

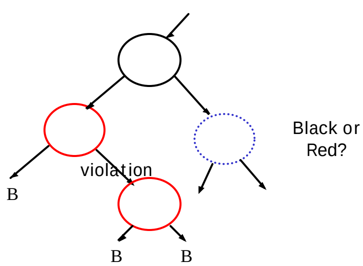
Red-Black Tree: Insert 2



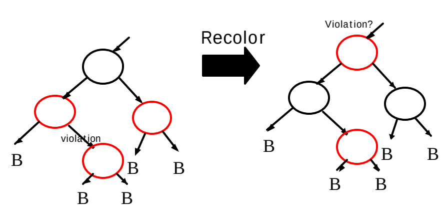
Red-Black Tree: Insert 10



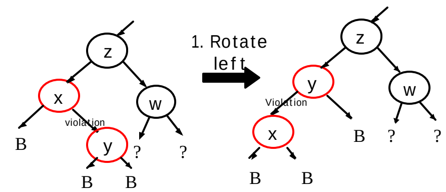
Red-Black Fix



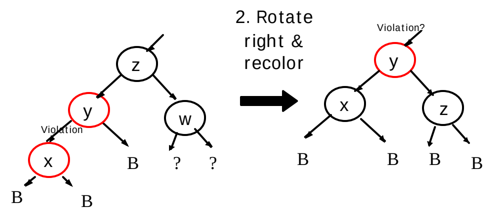
Red-Black Fix



Red-Black Fix



Red-Black Fix



YUCK

Treaps: Step 1

- S={1,3,5,7,9}
- Each element of S is assigned a unique "heap key" :

  T=(1,15), (3,10), (5,30), (7,0), (9,25)

Treaps: Step 2

- (1,15), (3,10), (5,30), (7,0), (9,25)
- Build tree where
  - S-key satisfy BST Property
  - H-key satisfy Heap Property

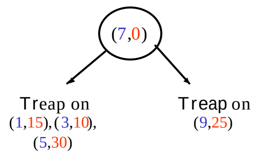
Treap

T=(1,15), (3,10), (5,30), (7,0), (9,25)

- Root:
- Left subtree:
- Right subtree:

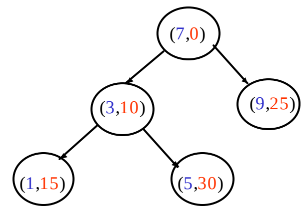
Treap

T=(1,15), (3,10), (5,30), (7,0), (9,25)



Treap

T=(1,15), (3,10), (5,30), (7,0), (9,25)



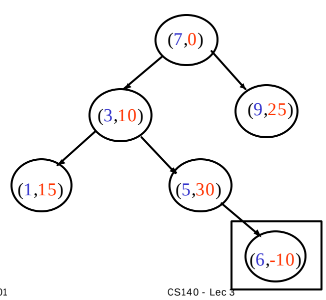
Treaps

- Claim: If the heap keys are unique then the treap is unique.
- Proof:

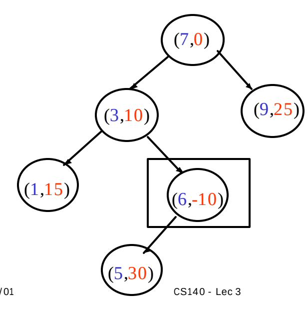
Binary Tree Operations

- Insert
- Delete

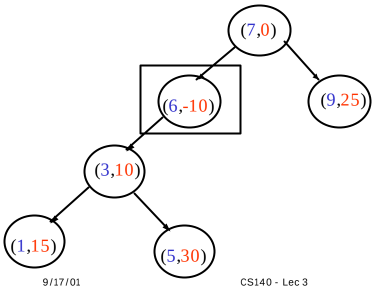
Treap: Insert (6,-10)



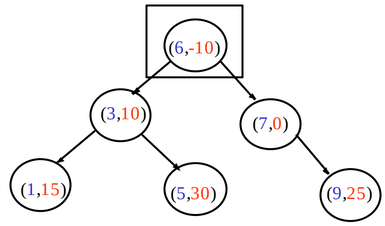
Treap: Rotate



Treap: Rotate



Treap: Rotate



Treaps

Claim: If the heap keys are chosen uniformly at random from [-B,B], where B>>n, then

1. With high probability the keys will be unique.
2. The expected height of the treap is O(lg(n)).