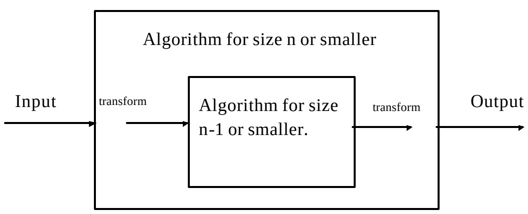
CS140: Algorithms

Z Sweedyk
Lecture 7

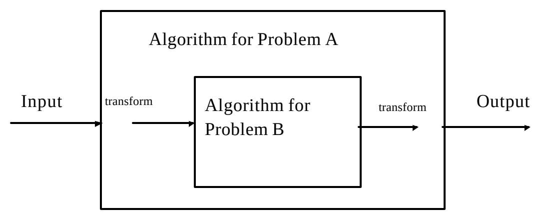
Algorithm Design Techniques

- Induction
- **Reduction**

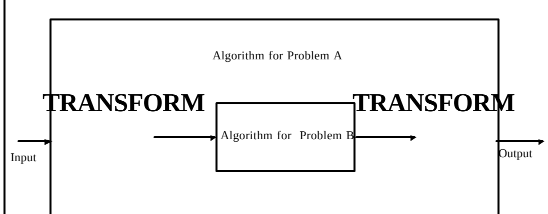
Self-Reduction



Reduction: $A \propto B$



Reduction: $A \propto B$



Some reductions we've seen

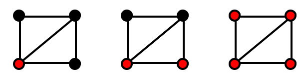
- Sorting $\propto$ Find-max
- General Selection $\propto$ Find-median

Inductive Design to solve A

- One Stage
  - Self-Reduction
- Two Stage
  - Define B ← Strengthening the inductive hypothesis
  - Reduce A to B
  - Solve B using self-reduction

Dominating Set

- A dominating set of a graph G is a subset W of the vertices of G such that every vertex in G is either in W or adjacent to a vertex in W.
- Examples

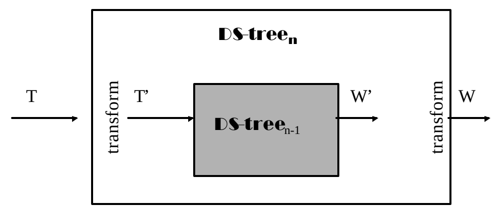


Dominating Set

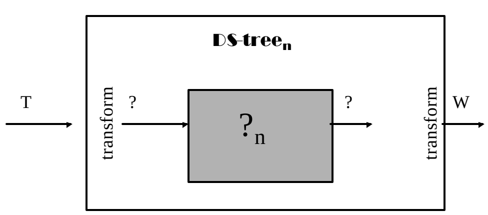
- Input: A graph G
- Output: The smallest dominating set of G

Dominating Set in Trees with n or fewer nodes

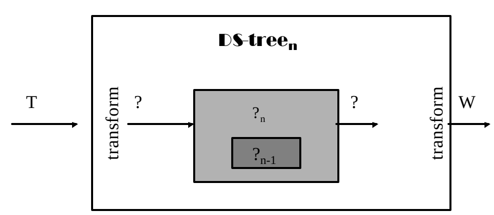
Doesn't seem to work



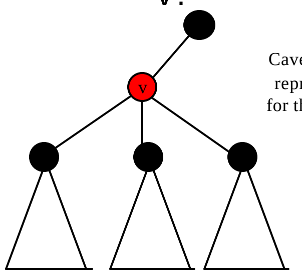
Dominating Set in Trees with n or fewer nodes: Define new problem



Dominating Set in Trees with n or fewer nodes: Define new problem **that is self-reducible**

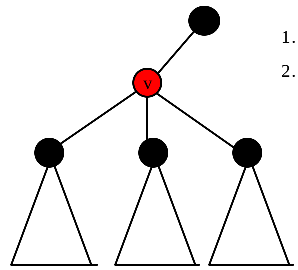


What do you want to know about the subtrees of node v?



Caveat: You have to reproduce that info for the subtree rooted at v.

Visualize a solution …



1. v is in the DS
2. v is not in the DS
   A) a child of v is in the DS
   B) the parent of v is in the DS

What do you want to know about a child w?

1. Smallest dominating set that includes w.
2. Smallest dominating set that does not include w.
3. Smallest dominating set on the subtrees rooted at the children of w. (Note: w need not be covered.)

Definitions

1. I(w): Smallest dominating set of the subtree rooted at w that includes w.
2. E(w): Smallest dominating set of the subtree rooted at w that does not include w.
3. C(w): Smallest dominating set on the subtrees rooted at the children of w. (Note: w need not be covered.)

Caveat

Compute I(v), E(v) and C(v) if we have I(w), E(w) and C(w) for each child w of v?

I(v)=

E(v)=

C(v)=

Base Case

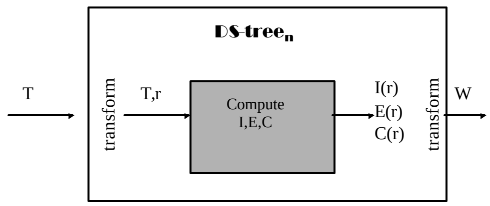
v is a leaf:

I(v)=

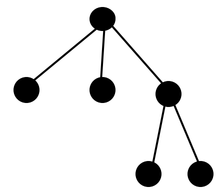
E(v)=

C(v)=

Dominating Set in Trees with n or fewer nodes



Example



DS-tree algorithm

- Is it correct?
- Is it efficient?

Longest Increasing Subsequence

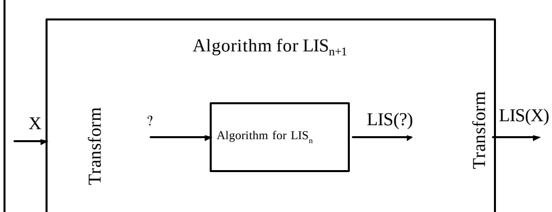
- Input: Sequence of integers X: $x_1, x_2, \ldots, x_n$
- Output: Longest increasing subsequence of X; i.e. a subsequence Z: $z_1, z_2, \ldots, z_k$ such that $z_i < z_{i+1}$ for each i:1…k-1.

Example

- 1, -3, 2, 10, 8, 23, -2, 17, 5

$LIS_{n+1} \propto LIS_n$

Don't know how to do it!!!



To solve A

- **Define B** (Strengthen the inductive hypothesis)
- Reduce A to B
- Solve B using self-reduction

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LIS and Modified LIS

- Input: Sequence of integers X: $x_1, x_2, \ldots, x_n$
- Output: Longest increasing subsequence

- Input: Sequence of integers X: $x_1, x_2, \ldots, x_n$
- Output: For each i:1…n, a longest increasing subsequence of $x_1,\ldots,x_i$ that ends in x_i.

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MLIS($x_1,\ldots,x_n$)

MLIS($x_1,\ldots,x_n$) =

1. LIS of x_1 that ends in x_1
2. LIS of x_1, x_2 that ends in x_2

⋮

n-1. LIS of x_1, … , x_{n-1} that ends in x_{n-1}

n. LIS of x_1, … , x_n that ends in x_n

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Example

- 1, -3, 2, 10, 8, 23, -2, 17, 5

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To solve A

- Define B
- **Reduce A to B**
- Solve B using self-reduction

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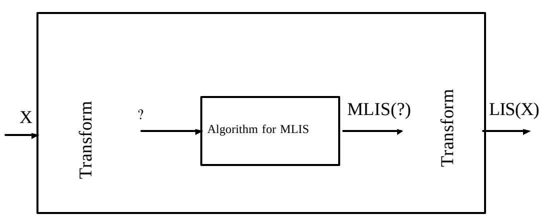
LIS ∝ MLIS



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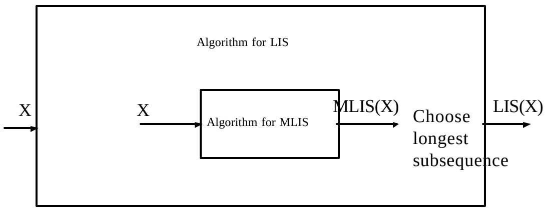
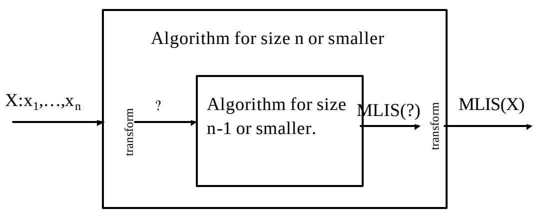
LIS ∝ MLIS



To solve A

- Define B
- Reduce A to B
- **Solve B using self-reduction**

MLIS Self-Reduction



MLIS(x_1,…,x_n)

MLIS(x_1,…,x_n) =

1. LIS of x_1 that ends in x_1
2. LIS of x_1, x_2 that ends in x_2

⋮

n-1. LIS of x_1, … , x_{n-1} that ends in x_{n-1}

n. LIS of x_1, … , x_n that ends in x_n

MLIS(x_1,…,x_n)

MLIS(x_1,…,x_n) = MLIS(x_1, … , x_{n-1})

1. LIS of x_1 that ends in x_1
2. LIS of x_1, x_2 that ends in x_2

⋮

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MLIS(x_1,…,x_n)

MLIS(x_1,…,x_n) =

1. LIS of x_1 that ends in x_1
2. LIS of x_1, x_2 that ends in x_2

⋮

n-1. LIS of x_1, … , x_{n-1} that ends in x_{n-1}

n. LIS of x_1, … , x_n that ends in x_n

How can we produce this?

Construct MLIS($x_1,\dots,x_n$)

MLIS($x_1,\dots,x_n$) =

1) MLIS($x_1,\dots,x_{n-1}$) plus
2) Choose longest LIS($x_1,\dots,x_j$) ending in x_j ($j<n$) such that $x_j<x_n$. Append x_n.

LIS algorithm

- Is it correct?
- Is it efficient?

Recap: To solve A

- Define B
- Reduce A to B
- Solve B using self-reduction

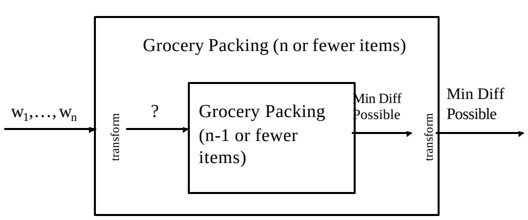
Grocery Bags

How should we pack n items weighing $w_1, w_2,\dots, w_n$ ($w_i \le W$) in two bags so as to minimize the difference in the weights of the bags?

Or even simpler: What is the smallest possible weight difference?

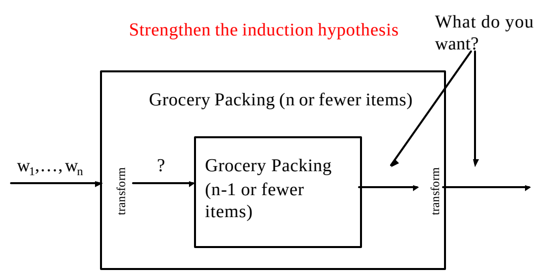
Self-Reduction

I don't know how to make this work!



Self-Reduction

Strengthen the induction hypothesis



Problem B

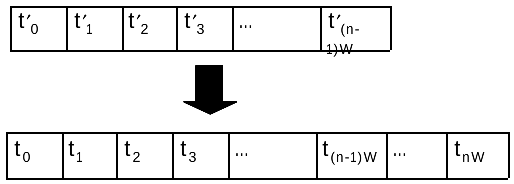
- Input: Weights $w_1, w_2, \ldots, w_n$
- Output: A binary vector T:

  T[i] = 1 if some subset of the weights sum to i

  T[i] = 0 otherwise

  for i=0,…,nW

Transform



Transform

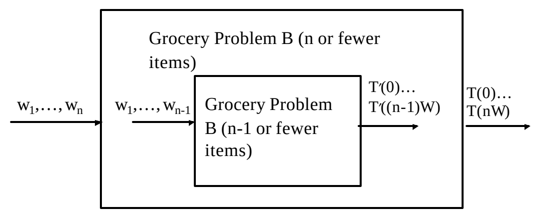
t_0	t_1	t_2	t_3	…	t_i	…	t_{nW}

Set t_i= 1 if _______________ or ___________________

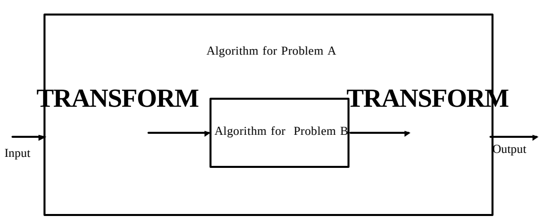
Else t_i=0

Self-Reduction: Problem B

What are the base cases?



Reduction: A ∝ B



Grocery Bag algorithm

- Is it correct?
- Is it efficient?

Algorithm A

Use Algorithm B to compute t[0]…t[nW]

Let $S=\Sigma w_i$

(Note: t[0..S] is symmetric about S/2)

Let j be the closest index to S/2 such that t[j]=1

Return |j-S/2|

Grocery Bags

What about this problem?

How should we pack n items weighing $w_1, w_2, \ldots, w_n$ ($w_i \leq W$) in two bags so as to minimize the difference in the weights of the bags?

Or even simpler: What is the smallest possible weight difference?