CS140: Algorithms

Z Sweedyk
Lecture 13

Algorithm Design Techniques

- Induction (Self-Reduction)
  - Divide and Conquer
  - **Dynamic Programming continued**

Outline

- Hum-Soc reading problem
- String edit problem
- Neat printing problem

Hum-Soc reading

- You are assigned books $b_1, \ldots, b_n$ to read in K days (typically K<n)
  - The books have to be read in order
  - You must finish any book you start on the same day
  - Book b_i has p_i pages
- Objective: minimize the maximum number of pages you need to read in any day

Hum-soc reading

Definition:

Let M(i,j) denote the maximum number of pages you have to read in any day in an optimal solution to the problem of reading books $b_1, \ldots, b_i$ in j days

Hum-soc reading

- Base Case: j=1, i=0…n

  $M(i,j)=\Sigma_{m=1\ldots i} p_m$
- Inductive Step: j=2…k, i=0…n

  $M(i,j)=\min_{m=0\ldots i} \max(M(m,j-1),\Sigma_{l=m+1\ldots i} p_l)$

Hum-soc reading

- Analysis
  - Compute M(i,j) for i=0…n, j=1…K
  - Each computation can be done in O(n)
  - Overall running time is $O(n^2K)$

10/31/01 CS140 Lec 13 7

String Edit

- Input: Strings $X=x_1x_2\ldots x_m$ and $Y=y_1y_2\ldots y_n$
- Output: Edit distance between X and Y
- Edit operations are insert/delete, match and substitute. Insert/deletes cost c_1 and substitutes cost c_2. Matches are free.
- Edit distance is the minimum cost of edits needed to transform X into Y.

10/31/01 CS140 Lec 13 8

String Edit

- Let D(i,j) denote the edit distance between $x_1\ldots x_i$ and $y_1\ldots y_j$.

10/31/01 CS140 Lec 13 9

String Edit

- Base case: D(i,j) where i=0 or j=0
  - $D(0,j) = c_1 j$
  - $D(i,0) = c_1 i$
- Inductive step: D(i,j) where i>0 and j>0
  - If $x_i=y_j$:
    $D(i,j) = \min(c_1+D(i-1,j), c_1 + D(i,j-1), D(i-1,j-1))$
  - Else:
    $D(i,j) = \min(c_1+D(i-1,j), c_1 + D(i,j-1), c_2+ D(i-1,j-1))$

10/31/01 CS140 Lec 13 10

String Edit

Analysis:

D(i,j) takes constant time to compute given D(i-1,j), D(i,j-1) and D(i-1,j-1).

D(i,j) is computed for i=0…n, j=0…m

The running time is O(nm)

10/31/01 CS140 Lec 13 11

Neat Printing

- We want to print words $w_1, \ldots, w_n$
- Word w_i is c_i characters long
- Adjacent words on a line must be separated by a blank
- M characters (including blanks) can be written on a line
- No hyphenation is allowed
- We want to minimize the sum of the squares of the trailing blanks on the printed lines.

10/31/01 CS140 Lec 13 12

Neat Printing

- Definition 1:
  - For $j>i$ let $B(i,j)$ denote the number of trailing blanks when words w_i through w_j are printed on a single line provided the words fit on a single line. If the words don't fit then $B(i,j)=\infty$.
  - Note: if $B(i,j)$ is finite then $j-i+1 \le M/2$, which is constant.
  - We can compute $B(i,j)$ for i,j such that $j-i+1 \le M/2$ in $O(n)$ time.

Neat Printing

- Definition 2:
  - Let $C(i)$ denote the minimum cost of printing words $w_1 \ldots w_i$.

Neat Printing

- Base Case: $i=0$
  - $C(0)=0$
- Inductive Step: $i>0$
  - $C(i) = \min_{j<i,\ j>0,\ j\ge i-M/2} C(j) + B(j+1,i)^2$

Neat Printing

- Analysis:
  - $C(i)$ takes $O(1)$ time to compute given $B(i,j)$ for $j-i+1 \le M/2$
  - We need to compute $C(i)$ for $i=0,\ldots n$
  - The running time is $O(n)$