CS140: Algorithms

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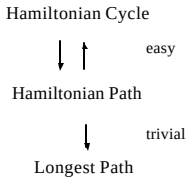
Lecture 18

NP-completeness

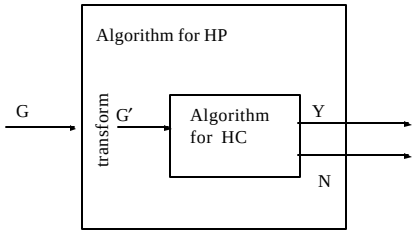
Problem A is NP-Complete if

- A is in NP
- A is NP-hard

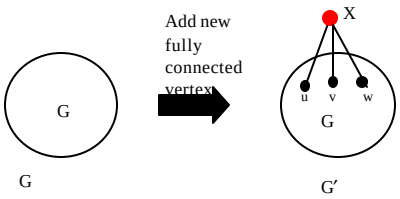
Reduction Map II



Exercise: HP $\propto_p$ HC



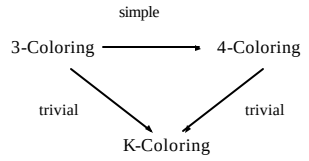
Transform



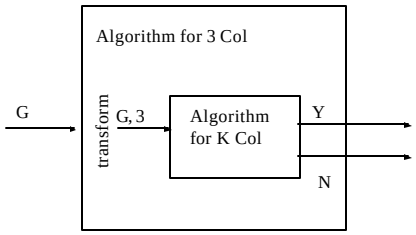
Claim

- G has a Hamiltonian Path iff G' has a Hamiltonian Cycle.

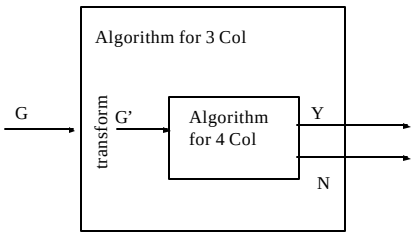
Exercise: Reduction Map III



3 Col $\propto_p$ K Col



Exercise: 3 Col $\propto_p$ 4 Col



Reduction

Construct G' based on G so that

G' is 4 colorable iff G is 3 colorable

Some Basics

- 3SAT
- 3DM
- X3C
- Partition
- HC
- HP
- Longest Path
- VC
- Clique
- Independent Set
- 3 Colorability
- 4 Colorability
- K Colorability

3 SAT

- Input: Boolean formula in conjunctive normal form in which each clause has 3 literals
- Question: Is there a truth assignment to the variables under which the formula evaluates to true?

X3C: Exact Cover by 3-Sets

- Input: A collection of triples over an underlying set U.
- Question: Is there a subset of the triples such that every element in U occurs in exactly one triple of the subset?

Partition

- Input: Set of integers S
- Question: Can S be partitioned into two sets A and B so that $\Sigma_{x \text{ in } A} x = \Sigma_{x \text{ in } B} x$?

Reduction Techniques

- Restriction (easy)
- Local Replacement (medium)
- Component Design (hard)

Restriction

- Problem A is a special case of Problem B

$A \propto_p B$ by restriction

Examples of Restriction

- HP $\propto_p$ Longest Path
- 3-Colorability $\propto_p$ K-Colorability

New Problems

- Minimum Cover
- Hitting Set
- Subgraph Isomorphism
- Bounded Degree Spanning Tree
- Knapsack
- Multiprocessor Scheduling

Minimum Cover

- Input: Collection C of subsets of the set S and an integer K
- Question: Does C contain K or fewer sets whose union is S?

Hitting Set

- Input: Collection C of subsets of a set S and an integer K
- Question: Is there a subset $W \subseteq S$ containing K or fewer elements such that every set in C has an element in W?

Subgraph Isomorphism

- Input: Graphs G and H
- Question: Does G contain a subgraph isomorphic to H?

Bounded Degree Spanning Tree

- Input: Graph G and integer K
- Question: Does G have a spanning tree in which each vertex is incident to K or fewer edges.

Knapsack

- Input: A set of "items" each having an integer "weight" w and an integer "value" v, plus integers B and K
- Question: Is there a subset of items that weighs at most B and has value at least K?

Multiprocessor Scheduling

- Input: A set of computational tasks where task i requires t_i seconds to process, an integer K representing the number of processors available, and an integer D representing the amount of seconds available for processing.
- Question: Can the tasks be scheduled on the processors so that all finish in D seconds?

(A task must be completed on a single processor without interruption. You can assume t_i is an integer.)

Set Packing

- Input: Collection C of sets and an integer K
- Question: Does C contain at K or more disjoint sets?