CS140: Algorithms

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Lecture 19

NP-completeness

Problem A is NP-Complete if

- A is in NP
- A is NP-hard

Last Time

- Reduction by Restriction

New Problems

- Minimum Cover
- Hitting Set
- Subgraph Isomorphism
- Bounded Degree Spanning Tree
- Knapsack
- Multiprocessor Scheduling

Some Basics

- 3SAT
- 3DM
- X3C
- Partition
- HC
- HP
- Longest Path

- VC
- Clique
- Independent Set
- 3 Colorability
- 4 Colorability
- K Colorability

Minimum Cover

- Input: Collection C of subsets of the set S and an integer K
- Question: Does C contain K or fewer sets whose union is S?

- Reduction from Vertex Cover G=(V,E), K'
- S=E, C=Each adjacency list is a subset, K=K'

Hitting Set

- Input: Collection C of subsets of a set S and an integer K
- Question: Is there a subset $W \subseteq S$ containing K or fewer elements such that every set in C has an element in W?
- Reduction from Vertex Cover G=(V,E),K'
- S=V, C=E, K=K'

Subgraph Isomorphism

- Input: Graphs G and H
- Question: Does G contain a subgraph isomorphic to H?
- Reduction from Clique: G', K
- G'=G, H is a fully connected graph on K vertices

Bounded Degree Spanning Tree

- Input: Graph G and integer K
- Question: Does G have a spanning tree in which each vertex is incident to K or fewer edges.
- Reduction from Hamiltonian Path: G'
- G=G; K=2

Knapsack

- Input: A set of "items" each having an integer "weight" w and an integer "value" v, plus integers B and K
- Question: Is there a subset of items that weighs at most B and has value at least K?
- Reduction from Partition $S=\{s_1, \ldots, s_n\}$
- Item i has weight=value=s_i, $B=K=(\Sigma s_i)/2$

Multiprocessor Scheduling

- Input: A set of computational tasks where task i requires t_i seconds to process, an integer K representing the number of processors available, and an integer D representing the amount of seconds available for processing.
- Question: Can the tasks be scheduled on the processors so that all finish in D seconds?

  (A task must be completed on a single processor without interruption. You can assume t_i is an integer.)

Multiprocessor Scheduling

- Input: A set of computational tasks where task i requires t_i seconds to process, K (number processors), D (processing time)
- Question: Can the tasks be scheduled on the processors so that all finish in D seconds?
- Reduction from Partition: $S=(s_1,\ldots,s_n)$
- Task i takes time s_i, K=2, $D=(\Sigma s_i)/2$

Set Packing

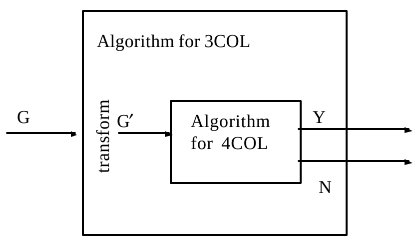
- Input: Collection C of sets and an integer K
- Question: Does C contain K or more disjoint sets?

- Reduction from X3C: S, C’ where ||S||=3q
- C’=C, K=q

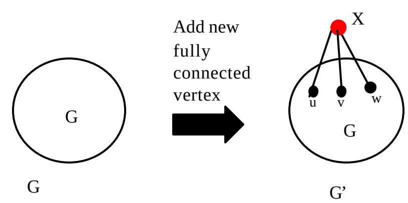
Some New (and Harder) Reductions

- 4 COL
- Partition into Triangles
- Minimum Test Collection
- Feedback Vertex Set
- Exact Cover by 4-Sets

3COL $\propto_p$ 4Col



Transform



Claim

- G is 3 Colorable iff G’ is 4 Colorable

Some New (and Harder) Reductions

NEW
- Exact Cover by 4-Sets
- Partition into Triangles
- Minimum Test Collection
- Feedback Vertex Set

OLD
- Vertex Cover
- 3DM
- 3XC

Exact Cover by 4-Sets

- Input: Collection C of subsets of a set S. Each set in C has cardinality 4.
- Question: Are there ||S||/4 sets in C that partition S?

Partition into Triangles

- Input: Graph G
- Question: Can the vertices of G be partitioned into sets of size three so that each set is fully connected in G?

Feedback Arc Set

- Input: Digraph G and integer K
- Question: Is there a subset W of K or fewer vertices of G such that every (directed) cycle of G passes through at least one vertex of W.