NP-completeness

Problem A is NP-Complete if
• A is in NP
• A is NP-hard

Last Time
• Reduction by Restriction

New Problems
• Minimum Cover
• Hitting Set
• Subgraph Isomorphism
• Bounded Degree Spanning Tree
• Knapsack
• Multiprocessor Scheduling

Some Basics
• 3SAT
• 3DM
• X3C
• Partition
• HC
• HP
• Longest Path
• VC
• Clique
• Independent Set
• 3 Colorability
• 4 Colorability
• K Colorability

Minimum Cover
• Input: Collection C of subsets of the set S and an integer K
• Question: Does C contain K or fewer sets whose union is S?
• Reduction from Vertex Cover G=(V,E), K’
• S=E, C=Each adjacency list is a subset, K=K’
Hitting Set

- Input: Collection $C$ of subsets of a set $S$ and an integer $K$
- Question: Is there a subset $W \subseteq S$ containing $K$ or fewer elements such that every set in $C$ has an element in $W$?
- Reduction from Vertex Cover $G=(V,E), K'$
  - $S=V$, $C=E$, $K=K'$

Subgraph Isomorphism

- Input: Graphs $G$ and $H$
- Question: Does $G$ contain a subgraph isomorphic to $H$?
- Reduction from Clique: $G'$, $K$
  - $G'=G$, $H$ is a fully connected graph on $K$ vertices

Bounded Degree Spanning Tree

- Input: Graph $G$ and integer $K$
- Question: Does $G$ have a spanning tree in which each vertex is incident to $K$ or fewer edges.
- Reduction from Hamiltonian Path: $G'$
  - $G=G'$, $K=2$

Knapsack

- Input: A set of “items” each having an integer “weight” $w$ and an integer “value” $v$, plus integers $B$ and $K$
- Question: Is there a subset of items that weighs at most $B$ and has value at least $K$?
- Reduction from Partition $S=\{s_1, \ldots, s_n\}$
  - Item $i$ has weight=value=$s_i$, $B=K=(\Sigma s_i)/2$

Multiprocessor Scheduling

- Input: A set of computational tasks where task $i$ requires $t_i$ seconds to process, an integer $K$ representing the number of processors available, and an integer $D$ representing the amount of seconds available for processing.
- Question: Can the tasks be scheduled on the processors so that all finish in $D$ seconds?
  (A task must be completed on a single processor without interruption. You can assume $t_i$ is an integer.)
- Reduction from Partition: $S=\{s_1, \ldots, s_n\}$
  - Task $i$ takes time $s_i$, $K=2$, $D=(\Sigma s_i)/2$
Set Packing

• Input: Collection C of sets and an integer K
• Question: Does C contain K or more disjoint sets?
• Reduction from X3C: S, C' where ||S||=3q
• C'=C, K=q

Some New (and Harder) Reductions

• 4 COL
• Partition into Triangles
• Minimum Test Collection
• Feedback Vertex Set
• Exact Cover by 4-Sets

3COL \leq_p 4Col

Algorithm for 3COL

G

Algorithm for 4COL

G'

X

Add new fully connected vertex

G

G'

Claim

• G is 3 Colorable iff G’ is 4 Colorable

Some New (and Harder) Reductions

NEW
• Exact Cover by 4-Sets
• Partition into Triangles
• Minimum Test Collection
• Feedback Vertex Set

OLD
• Vertex Cover
• 3DM
• 3XC
Exact Cover by 4-Sets

- Input: Collection C of subsets of a set S. Each set in C has cardinality 4.
- Question: Are there ||S||/4 sets in C that partition S?

Partition into Triangles

- Input: Graph G
- Question: Can the vertices of G be partitioned into sets of size three so that each set is fully connected in G?

Feedback Arc Set

- Input: Digraph G and integer K
- Question: Is there a subset W of K or fewer vertices of G such that every (directed) cycle of G passes through at least one vertex of W.