CS140: Algorithms

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Lecture 20

NP-completeness

Problem A is NP-Complete if

- A is in NP
- A is NP-hard

Last Time – Harder Reductions

- Next homework:
  - Partition into triangle (with hints)
  - Minimum test collection (with hints)

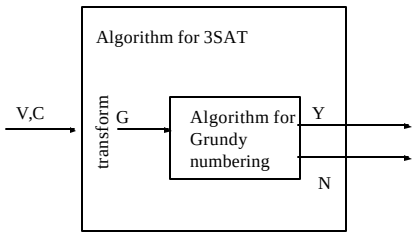
Today – Killer Reductions

- Grundy numbering
- Partition into paths of length 2

Grundy numbering

- Input: Directed graph G=(V,E)
- Question: Can the vertices of G be labeled with integers such that, for every vertex v, the label of v, L(v) is the smallest non-negative integer that is NOT in the set {L(u): <v,u> is in E}

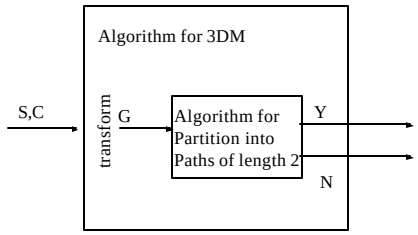
3SAT $\propto_p$ Grundy



Partition into Paths of Length 2

- Input: Graph G=(V,E) where ||V||=3q for some integer q
- Question: Can V be partitioned into q disjoint sets $V_1, \ldots, V_q$, where each set contains three vertices, so that the vertices x,y,z of V_i comprise a path of length 2 in G.

3DM $\propto_p$ Paritition into Paths of Length 2



Approach

- Understand the problem
- Build some gadgets
- Combine gadges

Approach: Grundy numbering

- Understand the problem
  - Build some simple graphs that have grundy numberings
  - Build some that don't
- Build some gadgets
  - Build a gadget that enforces the truth assignment condition
  - Build a gadget that enforces a satisfiability condition
- Combine gadgets

Truth Assignment

- For each variable x exactly one of x and its complement is set to true and the other is set to false

Satisfiability

- For each clause c, at least one literal in c evaluates to true.