

## Survival Kit for Project 2

1. Casting Rays
2. Computing Intersection Points
  - Triangle
  - Sphere
3. Computing color
4. Recursive ray tracing

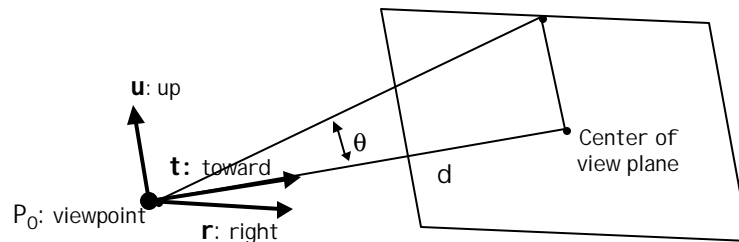
1

## Survival Kit for Project 2

- 1. Casting Rays**
2. Computing Intersection Points
  - Triangle
  - Sphere
3. Computing color (without shadows)

2

## World View



$P_0$  is the viewpoint

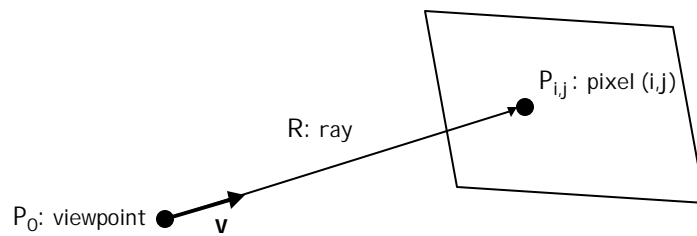
$\mathbf{u}$ ,  $\mathbf{t}$ , and  $\mathbf{r}$  are unit vectors

$d$  is the distance between  $P_0$  and view plane

$\theta$  is the half-height angle of the frustum

3

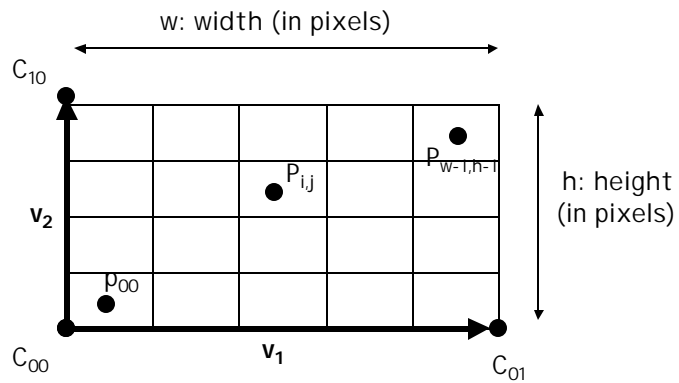
## Casting Rays



Computing  $R$ :  $R = (P_0, \mathbf{v})$  where  $\mathbf{v} = (P_{ij} - P_0) / \|P_{ij} - P_0\|$

4

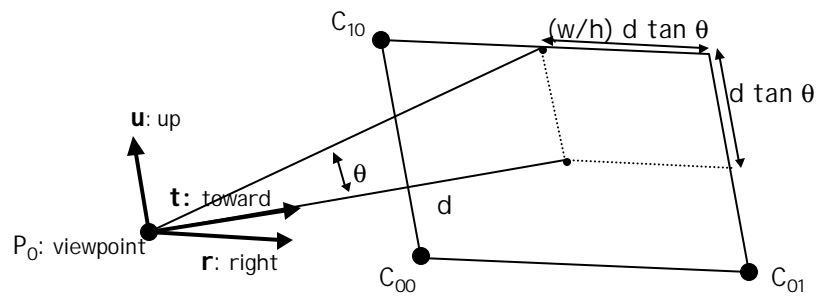
# Casting Rays



Computing  $P_{ij}$  :  $P_{ij} = C_{00} + (i+0.5)/w \mathbf{v}_1 + (j+0.5)/h \mathbf{v}_2$

5

# Casting Rays



Computing  $C_{00}$ ,  $C_{01}$ ,  $C_{10}$  :

$$C_{00} = P_0 + d\mathbf{t} - (d \tan \theta)\mathbf{u} - (w/h)(d \tan \theta) \mathbf{r}$$

$$C_{01} = P_0 + d\mathbf{t} - (d \tan \theta)\mathbf{u} + (w/h) (d \tan \theta) \mathbf{r}$$

$$C_{10} = P_0 + d\mathbf{t} + (d \tan \theta)\mathbf{u} - (w/h)(d \tan \theta) \mathbf{r}$$

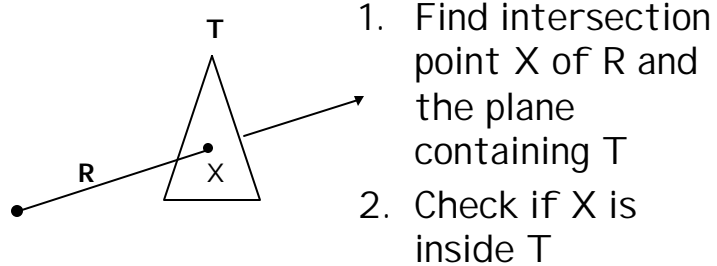
6

## Survival Kit for Project 2

1. Casting Rays
- 2. Computing Intersection Points**
  - **Triangle**
  - Sphere
3. Computing color

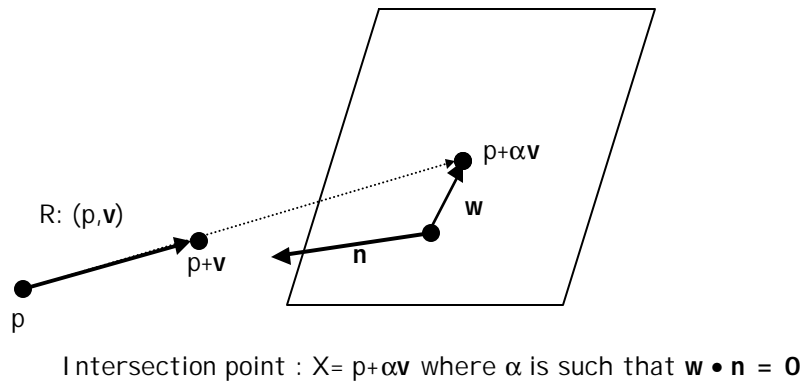
7

## Intersection of R and T



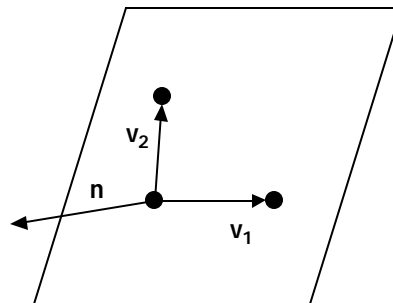
8

## Intersection of a Ray and a Plane



9

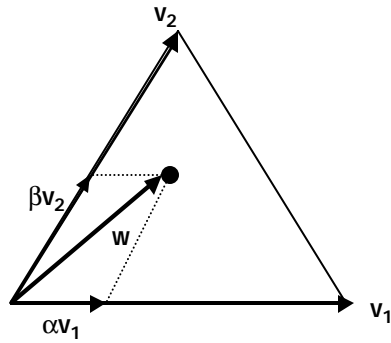
## Normal to a Plane



$$n = v_1 \times v_2$$

10

# Inside Test

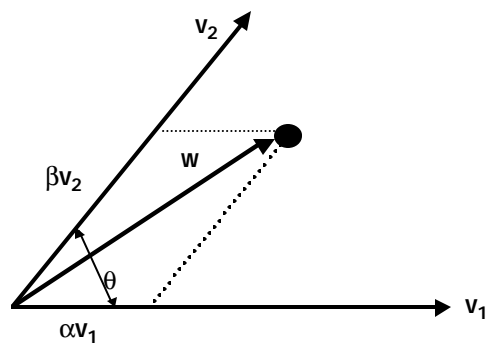


Find  $\alpha$  and  $\beta$  such that  $w = \alpha v_1 + \beta v_2$

Inside iff  $\alpha \geq 0$ ,  $\beta \geq 0$ , and  $\alpha + \beta \leq 1$

11

# Inside Test



To find  $\alpha$  and  $\beta$  solve:

$$\alpha(v_1 \cdot v_1) + \beta(v_2 \cdot v_1) = w \cdot v_1$$

$$\alpha(v_1 \cdot v_2) + \beta(v_2 \cdot v_2) = w \cdot v_2$$

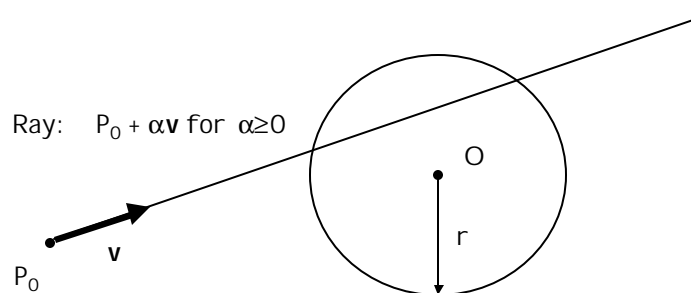
12

## Survival Kit for Project 2

1. Casting Rays
- 2. Computing Intersection Points**
  - Triangle
  - **Sphere**
3. Computing color (no shadows)

13

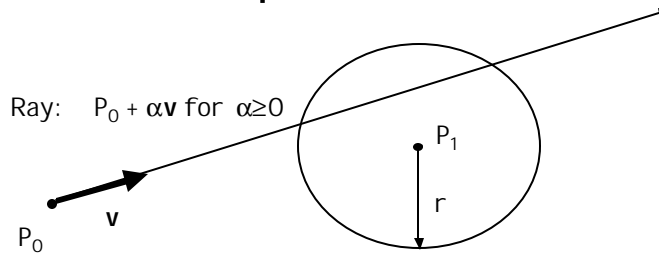
## Intersection of a Ray and a Sphere



$P_0 + \alpha v$  is an intersection point if  $|P_0 + \alpha v - O|^2 = r^2$

14

## Intersection of a Ray and a Sphere



$P_0 + \alpha v$  is an intersection point if  $\|w + \alpha v\|^2 = r^2$ , where  $w = P_0 - P_1$

Solve for  $\alpha$  in  $A\alpha^2 + B\alpha + C = 0$  where

$$A = v \cdot v, B = 2 v \cdot w, C = w \cdot w - r^2$$

15

## Survival Kit for Project 2

1. Casting Rays
2. Computing Intersection Points
  - Triangle
  - Sphere
- 3. Computing color (no shadows)**

16

# Computing Color

(no shadows)

The color of a point on a surface depends on

- Lights in scene
- Material properties of surface
- Geometry of scene

17

# Computing Color

(no shadows)

The contribution of each channel (r,g,b) is the result of five terms

- Ambient
- Diffuse
- Specular
- Emissive
- Transmission

18

## Red Ambient

- The red ambient term is  $r * ar$  where
  - $r$  is the intensity of the red ambient light.  $r$  is specified by the #ambient directive in the ray file.
  - $ar$  is the response of the surface to red ambient light.  $ar$  is specified by the #material directive in the ray file .

19

## Red Diffuse

- The red diffuse term is  $\sum R_{L,D}$  where
  - The summation is taken over all lights L
  - $R_{L,D}$  is the intensity of the red, diffuse reflection of light L at the intersection point

20

## $R_{L,D}$ for directional light L

$$R_{L,D} = dr \cdot r \cdot \max(0, (\mathbf{n} \cdot \mathbf{d})) \text{ where}$$

1.  $dr$  is the response of the surface to red, diffuse light.  $dr$  is specified in the #material directive of the ray file.
2.  $r$  is the red intensity of light L.  $r$  is given in the #light\_dir directive of the ray file.
3.  $\mathbf{n}$  is the unit normal of the surface at the point of intersection.
4.  $\mathbf{d}$  is the vector  $(-1) \cdot \langle dx, dy, dz \rangle / \|\langle dx, dy, dz \rangle\|$ . The direction vector  $\langle dx, dy, dz \rangle$  is specified in the #light\_dir directive of the ray file.

21

## $R_{L,D}$ for point light L

$$R_{L,D} = A \cdot dr \cdot r \cdot \max(0, (\mathbf{n} \cdot \mathbf{p})) \text{ where}$$

1.  $dr$ ,  $r$ , and  $\mathbf{n}$  are defined in slide 19
2.  $\mathbf{p}$  is the unit vector **from** the intersection point **to** the light position  $(px, py, pz)$ . Position  $(px, py, pz)$  is given by the #light\_point directive of the ray file.
3.  $A$  is the attenuation term  $1/(ca + la \cdot dist + qa \cdot dist^2)$ .  $ca$ ,  $la$  and  $qa$  are specified by the #light\_point directive of the ray file.  $dist$  is the distance between the intersection point and the light position  $(px, py, pz)$ .

22

## $R_{L,D}$ for spot light L

$$R_{L,D} = A \cdot SP \cdot dr \cdot r \cdot \max(0, (\mathbf{n} \cdot \mathbf{d}))$$

where

1. A is defined in slide 21
2.  $dr$ ,  $r$ ,  $\mathbf{n}$  and  $\mathbf{d}$  are defined in slide 20
3. SP is the "spot light effect." (See Princeton precept notes.)

23

## Red Specular

- The red specular term is  $\sum R_{L,S}$   
where
  - The summation is taken over all lights L
  - $R_{L,D}$  is intensity of the red, specular reflection of Light L at the intersection point

24

## $R_{L,S}$ for directional light L

$$R_{L,S} = sr \cdot r \cdot \max(0, (\mathbf{n} \cdot \mathbf{s}))^N \text{ where}$$

1.  $sr$  is the response of the surface to red, specular light.  $sr$  is specified in the #material directive of the ray file.
2.  $r$  and  $\mathbf{n}$  are defined as in slide 20.
3.  $N$  (not to be confused with  $\mathbf{n}$  - sorry) is given (as "n") in the #material directive of the ray file.
4.  $\mathbf{s}$  is the vector of reflection. See Princeton precept notes.

25

## $R_{L,S}$ for point light L

$$R_{L,S} = A \cdot dr \cdot r \cdot \max(0, (\mathbf{n} \cdot \mathbf{s}))^N \text{ where}$$

1.  $dr$ ,  $r$ , and  $\mathbf{n}$  are defined in slide 20
2.  $A$  is defined in slide 21
3.  $\mathbf{s}$  and  $N$  are defined in slide 24

26

## $R_{L,S}$ for spot light L

$$R_{L,S} = A \cdot SP \cdot dr \cdot r \cdot \max(0, (\mathbf{n} \cdot \mathbf{s}))^N$$

where

1. A is defined in slide 21
2.  $dr$ ,  $r$ ,  $\mathbf{n}$  are defined in slide 20
3. SP is defined in slide 22
4.  $\mathbf{s}$  and  $N$  are defined in slide 23

27

## Red Transmission

- The red light transmitted through the surface is  $R_{\text{neg}} \cdot ktran$  where
  - $ktran$  is the transmission coefficient given in the #material directive
  - $R_{\text{neg}}$  is the light falling on the "backward facing intersection point" of the surface. To compute  $R_{\text{neg}}$  you repeat the lighting calculation but use  $-\mathbf{n}$  as the normal to the surface.

28

## Recursive Ray Tracing

- The radiosity of a surface point depends on
  - the direct contribution of the light sources
  - light reflected from other objects in the scene
  - light transmitted from other objects in the scene

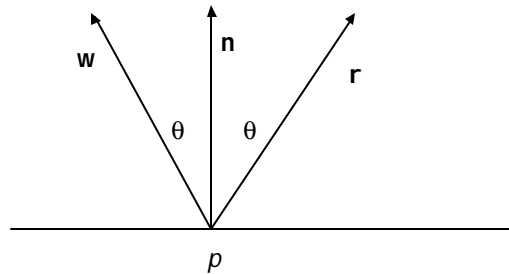
29

## Red Recursive Reflections at point $p$ , $rDepth = m$

- Recursively compute the red light along the ray  $(p, \mathbf{r})$  using  $rDepth = m-1$ 
  - $\mathbf{r}$  is the unit vector in the direction of mirror reflection
- Add the contribution scaled by  $s_r$  to the color at  $p$ .
  - $s_r$  is specified by the `#material` directive

30

## Mirror Reflection



$w$  is the negative of the unit direction vector of the incoming ray

$p$  is the point of intersection with unit normal  $n$

$r$  is the unit vector of mirror reflection:

$$r = 2(n \cdot w)n - w$$

31

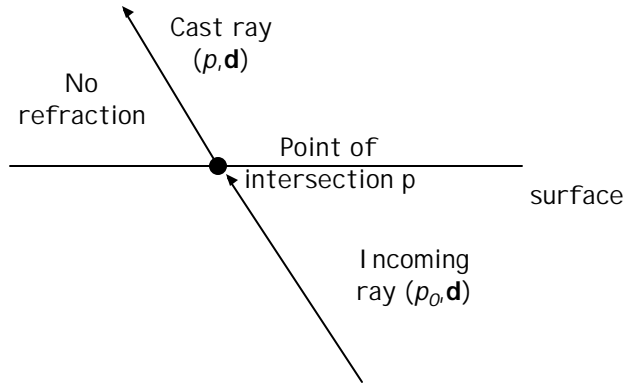
## Red Recursive Transmission at point $p$ , $rDepth = m$ (ignoring Snell's law)

- Recursively compute the red light along the ray  $(p, d)$  using  $rDepth\ m-1$ 
  - $d$  is the unit vector in the direction of the incoming ray
- Add the contribution scaled by  $ktran$  to the color at  $p$ .
  - $ktran$  is specified by the `#material` directive

32

# Transmission

## Ignoring Snell

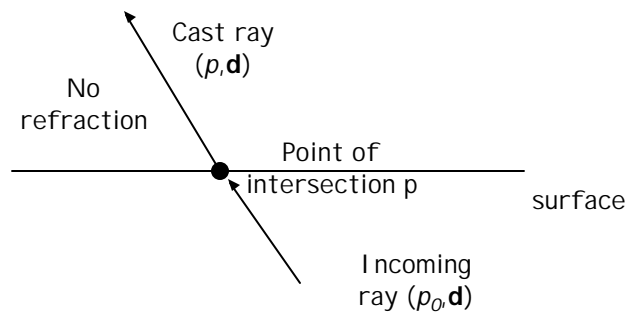


33

# Thin Surface Transmission

## with Snell

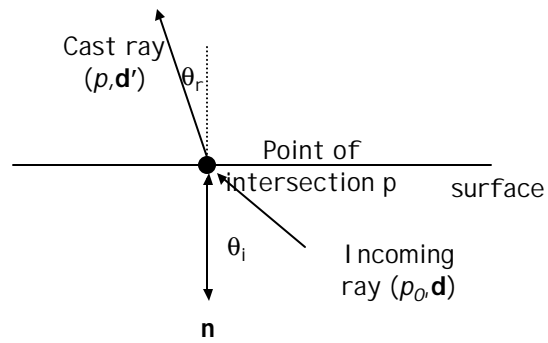
For "thin" (i.e. triangles) objects we can ignore Snell



34

# Thick Surface Transmission

with Snell



$\theta_r$  satisfies:  $\eta_r \sin \theta_r = \eta_i \sin \theta_i$

$$\mathbf{d}' = ((\eta_i/\eta_r) \cos \theta_i - \cos \theta_r)\mathbf{n} - (\eta_i/\eta_r)\mathbf{d}$$

35