CS155 – Computer Graphics

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Lecture 6

Class Overview

- Image Processing
- Rendering
- Modeling

Rendering

Model → Image

Scene

- A scene is a collection of geometric primitives located in "world coordinates"
Primitives in 3D
- Point
- Line segment
- Polygon
- Sphere
- Polyhedron
- Curved surface
- Solid object
- Etc.

Geometric Models
- Point P: \((x,y,z)\)
- Line Segment L: \(P_0, P_1\)
- Polygon R: \(P_0, P_1, P_2, \ldots, P_n\)
- Sphere S: \(P, r\)

Line Segment in 3D
Line Segment: \(P_0 + \alpha (P_1-P_0)\) for \(0 \leq \alpha \leq 1\)

Polygon in 3D
Points on and bounded by the line segments formed by \((P_0, P_1, P_2, P_3, P_4)\)

Sphere
All points a distance \(r\) from center \(P\)

Save for later ...
- Polyhedra
- Curved Surfaces
- Solid Objects
- Etc.
Issues

- Scene description
- Viewer description
- Surface appearance

Pinhole Camera Model

How is eye situated?

Eye position

Up

Pointing towards

Eye position (focal point)
Standard Configuration

How is the view plane situated?

How much of the world can be seen?

Example

Issues
  • Scene description
  • Viewer description
  • Surface appearance

Surface appearance
  • Light properties
  • Material properties
  • Geometry
  • Etc.
Issues

- Scene description
  - Specify primitives and their orientation in world coordinates
- Viewer description
  - Specify eye orientation and viewing volume
- Surface appearance
  - Specify light and material properties

Rendering by Ray Casting

For each pixel in the view plane
Cast ray from eye through pixel
Determine first surface in frustum intersected by ray
Compute surface appearance at intersection

Line in 3D

P_0

P_1

Line: P_0 + \alpha (P_1 - P_0) for -\infty < \alpha < \infty

Example Line

(1,3,2)

(2,4,5)

The line consists of the points (1+\alpha, 3+3\alpha, 2+3\alpha) for \alpha in (-\infty,\infty)
Line Segment in 3D

Line Segment: \( P_0 + \alpha (P_1 - P_0) \) for \( 0 \leq \alpha \leq 1 \)

Example Line Segment

The line segment consists of the points \((1+\alpha, 3+3\alpha, 2+3\alpha)\) for \(\alpha\) in \([0,1]\)

Ray in 3D

Ray: \( P_0 + \alpha (P_1 - P_0) \) for \( 0 \leq \alpha \leq \infty \)

Example Ray

The ray consists of the points \((1+\alpha, 3+3\alpha, 2+3\alpha)\) for \(\alpha\) in \([0,\infty)\)

Ray Casting Ray

The ray consists of the points \((e_x, e_y, e_z)\) for \(\alpha\) in \([0,\infty)\)

Standard Configuration

The ray consists of the points \((e_i, e_j, 1+\alpha(k-1))\) for \(\alpha\) in \([0,\infty)\)

\((e_x, e_y, e_z)\)

\((p_x, p_y, p_z)\)
Rendering by Ray Casting

For each pixel in the view plane
Cast ray from eye through pixel
Determine first surface (in frustum) intersected by ray
Compute surface appearance at intersection

Intersection with ray

• Point
• Line segment
• Polygon
• Sphere

Intersection of a line and a point

P lies on L if and only if there exists an α that satisfies the following equations:

\[ x_0 + \alpha(x_1 - x_0) = u \]
\[ y_0 + \alpha(y_1 - y_0) = v \]
\[ z_0 + \alpha(z_1 - z_0) = w \]

AND α is in [0, q]
Intersection of two lines

$L_1: (x_0 + \alpha(x_1-x_0), y_0 + \alpha(y_1-y_0), z_0 + \alpha(z_1-z_0))$
$L_2: (u_0 + \beta(u_1-u_0), v_0 + \beta(v_1-v_0), w_0 + \beta(w_1-w_0))$

for $\alpha, \beta$ in $(-\infty, \infty)$

Intersection of two lines

$L_1$ and $L_2$ intersect if and only if there exists an $\alpha$ and $\beta$ that satisfy the following equations:

$x_0 + \alpha(x_1-x_0) = u_0 + \beta(u_1-u_0)$
$y_0 + \alpha(y_1-y_0) = v_0 + \beta(v_1-v_0)$
$z_0 + \alpha(z_1-z_0) = w_0 + \beta(w_1-w_0)$

Clipping

AND $\alpha$ is in $[1,q]$

Intersection of a line and a triangle

$L: (x_0 + \alpha(x_1-x_0), y_0 + \alpha(y_1-y_0), z_0 + \alpha(z_1-z_0))$

for $\alpha$ in $(-\infty, \infty)$

T: $P_0P_1P_2$

1. Does $L$ intersect the plane containing $T$?
2. If so, does the point of intersection lie in $T$?

Equation of a Plane

This plane is the set of points $(x,y,z)$ where

$ax + by + cz = d$

$d = ax_0 + by_0 + cz_0$, where $(x_0,y_0,z_0)$ is any point on the plane.
Normal to a Plane

\[ \mathbf{n} = \mathbf{w} \times \mathbf{v} \] (i.e. cross product)

Intersection of a line and a plane

L and P intersect if and only if there exists an \( \alpha \) that satisfy the following equations:

\[ A(x_0 + \alpha(x_1 - x_0)) + B(y_0 + \alpha(y_1 - y_0)) + C(z_0 + \alpha(z_1 - z_0)) = D \]

Intersection of a point and a triangle

\[ \mathbf{T} \] is the set of points \( \alpha \mathbf{v} + \beta \mathbf{w} \) where \( \alpha, \beta \) are in \([0,1]\)

Parametric Representation of \( \mathbf{T} \)

The details

- Points: \((x_0, y_0, z_0), (x_1, y_1, z_1), (x_2, y_2, z_2)\)
- Vectors:
  \[ \mathbf{v} = \mathbf{x}_0 \times \mathbf{x}_1 \times \mathbf{x}_2 \] \\
  \[ \mathbf{w} = \mathbf{x}_0 \times \mathbf{x}_2 \times \mathbf{x}_1 \]
- Cross Product: \( \mathbf{n} = \alpha \mathbf{v} \times \mathbf{w} \)
Intersection of a point and a triangle

Point \( P=(x,y,z) \) and triangle \( T: P_0, P_1, P_2 \)
intersect if and only if there exist \( \alpha \) and \( \beta \)
in \([0,1]\) such that
\[
\alpha v_x + \beta w_x = x \\
\alpha v_y + \beta w_y = y \\
\alpha v_z + \beta w_z = z
\]

*Vectors \( v \) and \( w \) are defined as in the last slide.

The details

- Points: \( P_0=(x_0,y_0,z_0), P_1=(x_1,y_1,z_1), P_2=(x_2,y_2,z_2) \)
- Vectors:
  \[
  v = v_x = (x_0-x_1, y_0-y_1, z_0-z_1) \\
  w = w_x = (x_2-x_1, y_2-y_1, z_2-z_1)
  \]

Polygons?

- Oh ... we love those triangles