Computer Graphics: Projection

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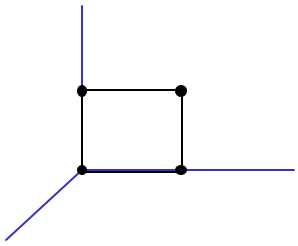
Lecture 7

Graphics Pipeline 1

1. Build Primitives (Model Coordinates)



Polygon in Model Coordinates

Vertices:

v_1= (0,0,0), v_2= (0,1,0), v_3= (1,1,0), v_4= (1,0,0)

Polygon in Homogenous Model Coordinates

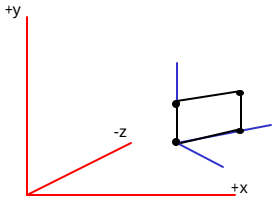
Vertices:

v_1=(0,0,0,1), v_2= (0,1,0,1), v_3= (1,1,0,1), v_4= (1,0,0,1)

Graphics Pipeline 2

1. Build Primitives (Model Coordinates)

2. Assemble Scene (World Coordinates)



Scale by (10,5,5), Rotate -45° about y axis, Translate by (5,2,-10)

Polygon in Homogeneous World Coordinates

For each vertex v_i:
$M_T M_{Rz} M_{Ry}$ v_i = u_i

Graphics Pipeline 3

For the moment we'll ignore clipping

Graphics Pipeline 3

Graphics Pipeline 3

Project into 2d: What does the viewer see?

Depends on:

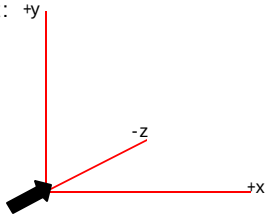
- Type of Projection
- Viewpoint
- Viewing Volume

Projection Types

- Orthographic Projection
  - Distance from viewer does not affect size
  - Used in architectural drawings, etc.
- Perspective Projection
  - Objects that are farther away appear smaller
  - This is how the world appears to us.

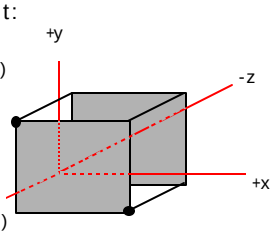
Orthographic Projection

- Viewpoint Default:
  - Position: (0,0,∞)
  - Direction: (0,0,-1)
  - Up (0,1,0)



Orthographic Projection

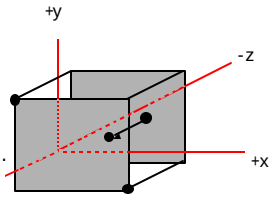
- Viewpoint Default:
  - Position: (0,0,∞)
  - Direction: (0,0,-1)
  - Up (0,1,0)
- Viewing Volume
  - Projection (near) Plane corners: (left,top,near), (right,bottom,far)



Orthographic Projection
(Near Plane at z=0)

Points inside viewing volume project to near plane
(x,y,z) → (x,y,0)

Points outside viewing volume are not seen.



Orthographic Projection Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Projection
(near plane at z=0)

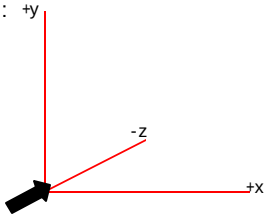
- Orthographic
  (x,y,z,1) → (x,y,0,1)
- Perspective
  (x,y,z,1) → (?,?,0,?)

A second use for that extra dimension

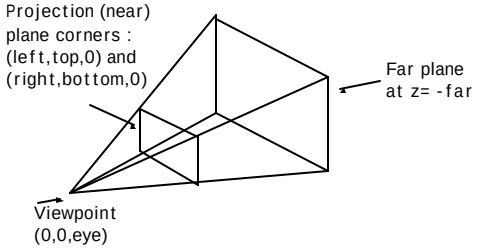
Graphics Pipeline 3

Perspective Projection

- Viewpoint Default:
  - Position: (0,0,eye)
  - Direction: (0,0,-1)
  - Up (0,1,0)
- Viewing Volume



Perspective Projection: frustum

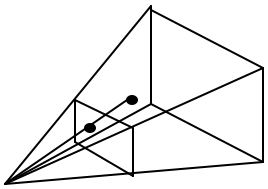


Perspective Projection

- Viewpoint Default:
  - Position: (0,0,eye)
  - Direction: (0,0,-1)
  - Up (0,1,0)
- Viewing Volume
  - Projection (near) Plane corners: (left,top,0), (right,bottom,0)
  - Far plane at z=-far

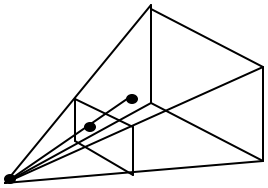
Perspective Projection

$(x,y,z) \rightarrow (?,?,?)$



Perspective Projection

$(x,y,z) \rightarrow (?,?,?)$



This is just the intersection of a line and plane problem

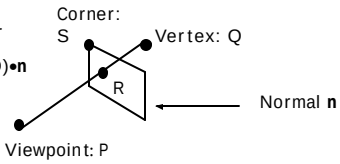
Perspective Projection

Line: $P + \alpha (Q-P)$

Plane: normal **n** and point **S**

R: $P + \alpha (Q-P)$ for

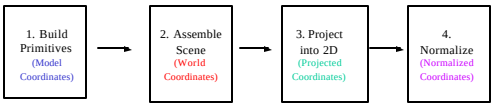
$\alpha = (P-S)\bullet\mathbf{n} \;/\; (P-Q)\bullet\mathbf{n}$



Perspective Projection Matrix

Work in progress

Graphics Pipeline 4



Normalize

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \longrightarrow \begin{bmatrix} x/w \\ y/w \\ z/w \\ 1 \end{bmatrix}$$

Graphics Pipeline 5