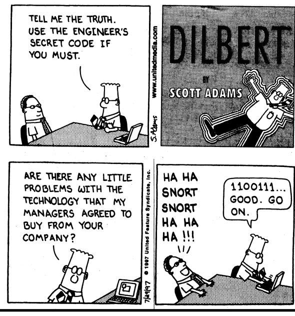
Subtle, but Important, Uses of Binary



Imperative Programs

Taxonomy of Programming Models

- Programming
  - Declarative Programming
    - Functional Programming
    - Logic Programming
  - Imperative Programming
    - Conventional Programming
    - Object-Oriented Programming

Imperative Programming

- View of computation is as sequence of commands or assignments
- (vs. functional: as set of function declarations)
- Most basic operation is the *assignment statement*:

Variable = *Expression*;

x = x+1;

No "referential transparency"

- In a **functional language**
  f(X) * f(X)
  is same value as
  Y = f(X), Y*Y

  } an example of "referential transparency"

- In an **imperative language**, we cannot make this claim; f(X) may have "side-effects" apart from the production of a value. (In some cases, it could even modify X.

Expressive Power

Logic Programming ← most expressive

Functional Programming

Imperative Programming ← least expressive

Expressive Power

- Every imperative program can be expressed as an equivalent functional program
- The general idea:
  - The "**state**" of an imperative program consists of a set of **bindings** of values to variables.
  - A statement, or sequence of statements, in an imperative program can be regarded as a transformation of one state to another.
  - The transformation represented by a statement can be expressed as a function.

Example: Factorial Program

```
int fac(int n)
{
int x, a;
      x = 1; a = 1;
      while( x <= n )
        {
        a = a*x;
        x = x+1;
        }
      return a;
}
```

The *state* is the set of bindings to a, n, and x, which we'll abbreviate (a, n, x), called the *state vector*.

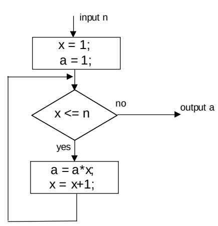
Example:
(a, n, x): (1, 4, 1) -> (1, 4, 2) -> (2, 4, 3) -> (6, 4, 4) -> (24, 4, 5)

24 is returned as fac(4)

Expressing Imperative Programs Functionally (1 of 4)

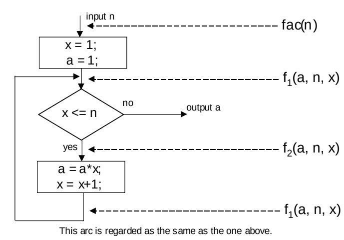
- Think of the program as represented by its flowchart.

```
int fac(int n)
{
int x, a;
x = 1; a = 1;
while( x <= n )
        {
        a = a*x;
        x = x+1;
        }
return a;
}
```



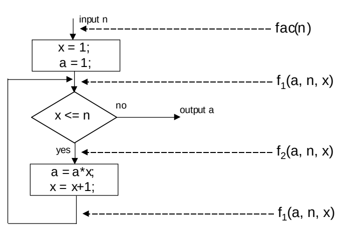
Expressing Imperative Programs Functionally (2 of 4)

- Label each **arc** with the **name** of a **function** having the state vector as an argument, except for the input arc, which gets the input variables as an argument, and the output arc, which need not be labeled.



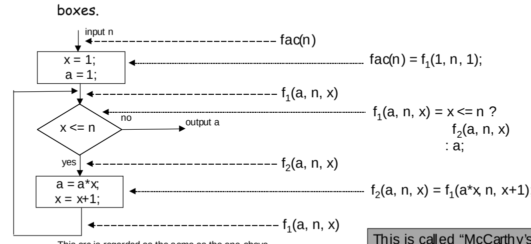
Expressing Imperative Programs Functionally (3 of 4)

- Interpretation of the functions thus introduced:
  - Given the argument values as the state, the function produces the value that the program would eventually produce if it were started in that state at the indicated arc.



Expressing Imperative Programs Functionally (4 of 4)

- Define the functions according to the state transformations in boxes.



This is called "McCarthy's transformation principle in the text.

Simplifying Using Substitution

fac(n) = f_1(1, n, 1);

f_1(a, n, x) = x <= n ?
f_2(a, n, x)
: a;

f_2(a, n, x) = f_1(a*x, n, x+1);

f_1(a, n, x) = x <= n ?
f_1(a*x, n, x+1)
: a;

Try this one

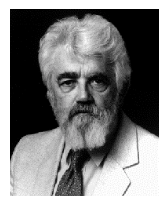
```
int fib(int n)
{
int x, a, b;
x = 1; a = 1; b = 0;
while( x <= n )
        {
        int temp = a+b;
        b = a;
        a = temp;
        x = x+1;
        }
      return a;
}
```

fib(n) =

Recursion -> Iteration?

- Is McCarthy's transformation invertible?
  - In some cases, it is possible to go from recursion to iteration, if the program is tail-recursive.
  - In general, it is not possible to transform an arbitrary recursive program to iteration, except in a fairly contrived way:
    - We can always implement recursion using imperative programming and a stack
  - In some sense, this implies that recursive programming is strictly more expressively-powerful than iterative programming.

John McCarthy



A pioneer in artificial intelligence, McCarthy invented LISP, the preeminent AI programming language, and first proposed general-purpose time sharing of computers. Ph.D. Princeton, 1951. Distinctions: NAS, NAE

Link to McCarthy's original paper giving the transformation (IFIP '62).

Funky Faktorial

fac(n) = f_1(1, n, 1);

f_1(a, n, x) = x <= n ?
f_1(a*x, n, x+1)
: a;

Compare to everyone's favorite:

fac(n) = n <= 1 ? 1 : n*fac(n-1);

Tail Recursion (review)

Functions produced by McCarthy's transformation are all "tail-recursive", meaning that the result of the function can be wholly delegated to some other defined function call.

fac(n) = f_1(1, n, 1);

f_1(a, n, x) = x <= n ?
f_1(a*x, n, x+1)
: a;

"tail-recursive"

fac(n) = n <= 1 ? 1 : n*fac(n-1);

non-tail-recursive

Tail Recursion

There is no "messy cleanup" after the inner function is called.

fac(n) = f_1(1, n, 1);

f_1(a, n, x) = x <= n ?
f_1(a*x, n, x+1)
: a;

tail-recursive

Good Housekeeping
Seal of Approval

fac(n) = n <= 1 ? 1 : n*fac(n-1);

non-tail-recursive

Tail Recursion

fac(n) = f_1(1, n, 1);

f_1(a, n, x) = x <= n ?
f_1(a*x, n, x+1)
: a;

tail-recursive

fac(4) => f_1(1, 4, 1) => f_1(1, 4, 2) => f_1(2, 4, 3) => f_1(6, 4, 4) => f_1(24, 4, 5) => 24

fac(n) = n <= 1 ? 1 : n*fac(n-1);

non-tail-recursive

fac(4) => 4*fac(3) => 4*3*fac(2) => 4*3*2*fac(1)=> 4*3*2*1=>4*3*2 => 4*6 => 24

sticky build-up

Tail Recursion

However, tail-recursive functions may be harder to read.

fac(n) = f_1(1, n, 1);

f_1(a, n, x) = x <= n ?
f_1(a*x, n, x+1)
: a;

tail-recursive

fac(n) = n <= 1 ? 1 : n*fac(n-1);

non-tail-recursive

Readers' Digest
Seal of Approval

Which should I use?

- Don't lose sleep over whether to tail-recurse, ***unless***
  - you are processing large data objects and **memory** is a premium, or
  - it is much **costlier** to compute without it, or
  - it's stated on the ***exam*** that you should.
- The compiler must also optimize tail-recursion for this to be effective (currently rex doesn't).
- In development, it might be wise to provide the **clearest** expression of the function first, then later replace it with a tail-recursive version.

Naïve Reverse

The valid rule set:
reverse([ ]) => [ ];

reverse([E | L]) => append(reverse(L), [E]);

is called naïve reverse:

- It's the first reverse coded by the inexperienced.
- It's not tail recursive.
- It's **slow**: takes an extra *factor* of length(L) steps to evaluate.

Accumulators (review)

- Certain arguments of functions, particularly tail-recursive ones, are often designated as "accumulators", e.g.

f_1(a, n, x) = x <= n ?
f_1(a*x, n, x+1)
: a;

accumulator argument

- The idea is that this argument value "accumulates" until the function is ready to return the answer without recursing.

Accumulators for List Processing

- Consider a definition of reverse:

  reverse(L) = reverse(L, [ ]);

  reverse([ ], A) => A;

  reverse([E | L], A) => reverse(L, [E | A]);

- Which argument is an accumulator?
- Is this reverse tail-recursive?

Accumulators for List Processing

reverse(L) = reverse(L, [ ]);

initial accumulation

reverse([ ], A) => A;

final accumulation

reverse([E | L], A) => reverse(L, [E | A]);

intermediate accumulation

Accumulators and Auxiliaries

- Note that when an accumulator is used, it is often in an *auxiliary* function, rather than the main interface function for the user.

- It is bad style to burden the user with the need to know added arguments, such as initial accumulations.