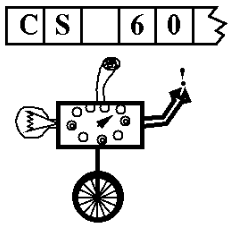
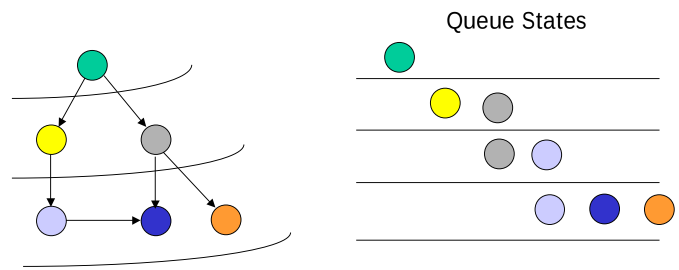


Turing Machines

October 26, 2001

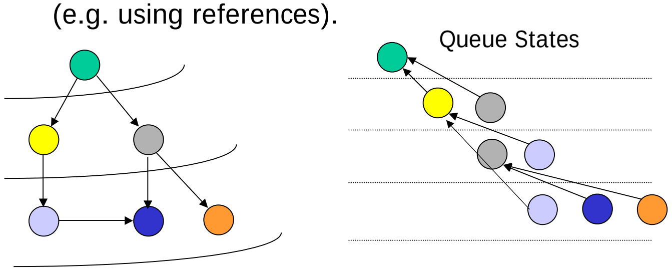
Review: BFS

- Breadth-first search uses a queue



Finding Your Way

- Have states remember their predecessors (e.g. using references).



- This will be handy if a solution is found.

Termination

- To ensure termination, must **remember** states already seen and **refuse to enqueue them**.
  - This probably requires another structure.
- Simple linked list
- Or, better, a hash table (see section 5.5)
  - Much faster
- Note: checking for *game boards* that reoccur, not board/predecessor combinations that reoccur!

Hashing

- Hashing computes a numeric signature for the proposed new item.
- It then uses the signature to access an array (called a hash table) of “equivalence classes” of items.
- Each equivalence class ideally has a relatively *small* number of items in it.
  - The only searching needed is that of searching the small equivalence class, not the whole universe.

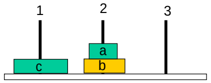
How Java Helps

- Java provides method `hashCode()` of `Object`.
  - This gives a (generally large) number for any object.
  - Warning: by default generally based on *memory address*
- By dividing the hash code by the hash table size, equivalence classes are thereby formed.
  - All objects with the same hash code *modulo* the table size are considered equivalent.
- Java also provides a class `HashSet` that can be used to implement sets.
  - However, for the assignment it may be as simple (or simpler) to make your own hash function and own array of linked lists.

State Representation

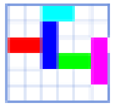
- State representation does not have to be **literal**.
- Any structure that is sufficient to capture all relevant information will do.
- Often there are many choices.
- The choice may impact search efficiency.

Example of State Representation Choices: Towers of Hanoi



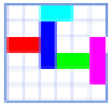
- Mapping from disk to spindle number:
  - ((a 2) (b 2) (c 1))
- List of disks on each spindle, smallest first:
  - ((c) (a b) ( ))
- Solving using rex, for example, the second would probably be more efficient:
  - All manipulation takes place at the tops of the stacks

Example of State Representation Choices: Traffic Jam



- Probably best *not* to replicate entire grid structure as state:
  - Heavyweight states
- Sample solution uses sequence of **offsets** for states:
  - Each offset indicates motion of a car from the original state.

Example of State Representation Choices: Traffic Jam



- Board representation = arrays of offsets:
  - initial state = [0, 0, 0, 0, 0]
    understood: [red, blue, cyan, green, magenta] by position
  - immediate transitions to:
    [0, 0, 1, 0, 0] (move cyan right)
    [0, 0, -1, 0, 0] (move cyan left)
    [0, 1, 0, 0, 0] (move blue down)
    [0, 0, 0, 0, 1] (move magenta down)
    [0, 0, 0, 0, -1] (move magenta up)
- From each array, *and the initial grid*, we can construct the resulting grid.

States in Computing Machines

- Broad split:
  - Finite-state systems
    - Finite-state machines (chapter 12)
    - Finite-state systems with a very large number of states (most computers)
  - Infinite-state systems:
    - Turing machines
    - Most programming languages
    - Other structured automata

Turing Machines

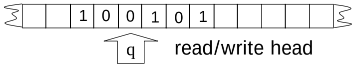
- A very primitive model of computing
  - Conceptually simple to construct
- A very powerful model of computing
  - A Turing machine can compute any function your PC can.

Alan M. Turing

- Celebrated mathematician/computer scientist, 1912-1954
- Code-breaking machine
  (play: "Breaking the Code")
- Reaction-diffusion equations
- Proving programs correct
- Artificial Intelligence
- Theory of computability

Turing Machine Depiction

infinitely-extendable tape (defaults to blank cells)



(finite alphabet)

read/write head

control state (one of a finite set)

Control is determined by a finite set of **rules** (each a 5-tuple):
(current-control-state, current-read-symbol, next-control-state, write-symbol, head-motion)
with the last three components being functions of the first two.

`tm` program on turing

- `/cs/cs60/bin/tm`
- Examples in: `/cs/cs60/tm/*.tm`
- Sample execution:

Turing machine simulator

rule set

```
turing > tm add1.tm
 1 0 1 1 1 1 1 1
 1 1 0 0 0 0 0 0 [_]
end
```

input I typed

output (final tape)

final control state

Rule set add1.tm

(current-control-state, current-read-symbol, write-symbol, head-motion, next-control-state)

start	_	_	left	add1
add1	0	1	right	end
add1	_	1	right	end
add1	1	0	left	add1
end	0	0	right	end
end	1	1	right	end

Assumes that head starts to the right of the non-blank symbols on the tape.

Trace of add1.tm (use -t flag)

```
 1 0 1 1 1 1 1 1 [_]
start
 1 0 1 1 1 1 1 [1]  _
add1
 1 0 1 1 1 1 [1]  0 _
add1
 1 0 1 1 1 [1]  0 0 _
add1
 1 0 1 1 [1]  0 0 0 _
add1
 1 0 1 [1]  0 0 0 0 _
add1
 1 0 [1]  0 0 0 0 0 _
add1
 1 [0]  0 0 0 0 0 0 _
add1
 1 1 [0]  0 0 0 0 0 _
end
 1 1 0 [0]  0 0 0 0 _
end
 1 1 0 0 [0]  0 0 0 _
end
 1 1 0 0 0 [0]  0 0 _
end
 1 1 0 0 0 0 [0]  0 _
end
 1 1 0 0 0 0 0 [0]  _
end
 1 1 0 0 0 0 0 0 [_]
end
```

Other Examples

- See `/cs/cs60/tm/*.tm`
  - E.g., complete binary adder
  - See `add.tm` and `add.doc`
- Can go on to implement more complex operations
  - Multiplication, comparisons
  - Composition of operations
  - Looping, counters...
  - In particular, all of the so-called *partial recursive functions* on natural numbers.

Amazing Fact

- Any specific Turing machine has a finite description (transition table).
  - So, it is possible to put this description on the tape of another Turing machine.
- There exists a Universal Turing Machine
  - Starting with the tape containing a description of any machine T and an input, can simulate the computation of T on that input.
  - i.e., an interpreter for Turing machines.

Church’s Thesis

- a.k.a Turing's thesis, Church-Turing thesis, ...
- Every *intuitively computable function* is computable by some Turing machine.
  - (and hence by a universal turing machine)
- Generally accepted, but cannot be proved.

Implications

- The set of all computable functions can be enumerated (there is a “countable” number of them).
- There are non-computable functions.
- There are problems of interest that are unsolvable.

Non-Proof of Turing’s Thesis

- Proving Turing’s thesis formally would require **formalizing** the notion of computability.
  - This is what Turing set out to do.
  - But that formalization could be argued and to prove or disprove it would require a proof that ***that*** formalization completely captured the notion of computability.
  - We’d then be in a position similar to proving Turing’s thesis.

Disproving Turing’s Thesis

- Conceptually this is possible:
  - Find a function that everyone agrees is computable.
  - Prove that no TM can compute it.
- However, it is highly unlikely.
- *Every* attempt so far to characterize computability is equivalent to the TM.
  - Lambda calculus, partial recursive functions, phrase-structure grammars, register machines, quantum computers, …